Eighth International Conference on
Geometry, Integrability and Quantization
June 9–14, 2006, Varna, Bulgaria
Ivaïlo M. Mladenov and Manuel de León, Editors
SOFTEX, Sofia 2007, pp 279–291

CHAPLYGIN GAS AND BRANE

NAOHISA OGAWA

Department of Liberal Arts, Hokkaido Institute of Technology
006-8585 Sapporo, Japan

Abstract. We review the theory of Chaplygin gas. This theory arises from the non-relativistic fluid mechanics with exotic state equation. This condition actually admits that the fluid theory reduces to the simple relativistic geometrical objects. This means that the non-relativistic fluid theory has hidden Poincare symmetry. We will show that the geometrical object is the brane described by the Nambu–Goto action. The application of this Chaplygin gas to the universe model is also briefly reviewed.

1. Introduction

We consider the non-relativistic fluid dynamics without viscosity. In this case the dynamical equation reduces to the usual Euler equation. We further give the exotic state equation to this theory such as

$$P = -2\frac{\lambda}{\rho} \tag{1}$$

with pressure P, density ρ, and some constant λ.

Such a fluid is called a **Chaplygin gas** which turns out to be a special effective theory of the gas fluid. Despites that it seems to be a trivial classical problem, this theory contains marvellous symmetries and beautiful geometrical contents. The symmetry is one-dimensional higher that the Poincare symmetry, and the related geometrical object is the brane with less extrinsic mean curvature. This means that the theory of Chaplygin gas is equivalent to the relativistic brane theory [10–13].

In this paper, we briefly sketch the relation to brane picture, then we find many types of solutions, and we show the symmetries of this theory. According to the relation to relativistic brane, we find the equivalence to the Born–Infeld theory. From this picture, we find different types of solutions. In the final section, we discuss the application of exotic state equation to the universe model, called Chaplygin cosmology. The quantization of this theory is not yet done, but it is a very attractive open problem.

2. Fluid Mechanics

Let us start with the non-relativistic Euler equation in $D+1$-dimensional space time

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{\nabla P}{\rho}\cdot \tag{2}$$

We assume the irrotational condition,

$$\boldsymbol{v} = \nabla\theta. \tag{3}$$

Making use of the state equation (1) the equation (2) can be obtained from the action

$$S = \int \mathrm{d}^{D+1}x \left[\theta\dot{\rho} - \left(\frac{\rho}{2}(\nabla\theta)^2 + \frac{\lambda}{\rho}\right)\right]. \tag{4}$$

After eliminating the variable ρ by using the equation of motion, we can simplify the action to the form

$$S = -2\sqrt{\lambda}\int \mathrm{d}^{D+1}x\sqrt{\dot{\theta} + \frac{(\nabla\theta)^2}{2}}\cdot \tag{5}$$

The Euler–Lagrange equation for this action is

$$\frac{\partial}{\partial t}\frac{1}{\sqrt{\dot{\theta} + (\nabla\theta)^2/2}} + \nabla\cdot\left(\frac{\nabla\theta}{\sqrt{\dot{\theta} + (\nabla\theta)^2/2}}\right) = 0. \tag{6}$$

3. Brane Solution

Let us consider the solution for θ. First we consider the static solution. Its equation is

$$\mathrm{Div}(\boldsymbol{n}) = 0 \tag{7}$$

where

$$\boldsymbol{n} \equiv \frac{\nabla\theta}{|\nabla\theta|}\cdot \tag{8}$$

The unit vector $\boldsymbol{n}$ above is normal to the hypersurface

$$\theta(x^1, x^2, \ldots, x^D) = 0. \tag{9}$$

Let us now consider the meaning of the equation (7). For that purpose we introduce the hypersurface Ω defined by equation (9), and set the curved coordinate $q^1, \ldots, q^{D-1}$ on this hypersurface as shown in Fig. 1. Then we can define the

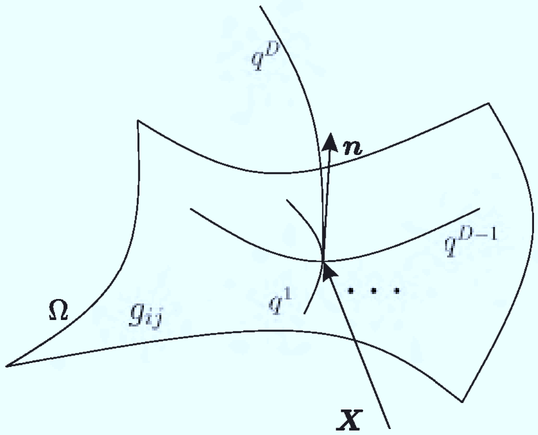

Figure 1. Surface Ω with coordinates and metric

metric tensor on Ω as

$$g_{ij} = \frac{\partial \boldsymbol{X}}{\partial q^i} \cdot \frac{\partial \boldsymbol{X}}{\partial q^j} \tag{10}$$

where Latin indices (i, j) run from 1 to $D-1$, and $\boldsymbol{X} = \boldsymbol{X}(q^1, \ldots, q^{D-1})$ specifies a point on Ω. We further introduce another coordinate q^D in normal direction by using the relation

$$\boldsymbol{n} = \frac{\partial \boldsymbol{x}}{\partial q^D}$$

where $\boldsymbol{x}$ specifies the point of D-dimensional Euclidean space. Making use of this coordinate, the hypersurface Ω is defined by $q^D = 0$. The set of coordinates $\{q^1, \ldots, q^D\}$ covers all D-dimensional Euclidean space $\boldsymbol{x} = \boldsymbol{x}(q^1, \ldots, q^D)$.

The line element in this D-dimensional space is given by

$$\mathrm{d}S^2 = \tilde{g}_{\mu\nu} \mathrm{d}q^\mu \mathrm{d}q^\nu \tag{11}$$

where

$$\tilde{g}_{\mu\nu} \equiv \frac{\partial \boldsymbol{x}}{\partial q^\mu} \cdot \frac{\partial \boldsymbol{x}}{\partial q^\nu} = \begin{pmatrix} \tilde{g}_{ij} & 0 \\ 0 & 1 \end{pmatrix}. \tag{12}$$

The Greek indices $\mu, \nu, \ldots$ run from 1 to D. And we have the relation

$$g_{ij} = \tilde{g}_{ij}(q^D = 0).$$

The extrinsic curvature of the hypersurface Ω is defined by

$$\kappa_{ij} = \frac{\partial \boldsymbol{X}}{\partial q^i} \cdot \frac{\partial \boldsymbol{n}}{\partial q^j}. \tag{13}$$

The extrinsic mean curvature is given by

$$\begin{aligned}\kappa &= g^{ij}\frac{\partial \boldsymbol{X}}{\partial q^i}\cdot\frac{\partial \boldsymbol{n}}{\partial q^j} = \tilde{g}^{\mu\nu}\frac{\partial \boldsymbol{x}}{\partial q^\mu}\cdot\frac{\partial \boldsymbol{n}}{\partial q^\nu}\bigg|_\Omega - \tilde{g}^{DD}\frac{\partial \boldsymbol{x}}{\partial q^D}\cdot\frac{\partial \boldsymbol{n}}{\partial q^D}\bigg|_\Omega \\ &= \delta^{AB}\frac{\partial \boldsymbol{x}}{\partial x^A}\cdot\frac{\partial \boldsymbol{n}}{\partial x^B}\bigg|_\Omega + (\text{last term}) = \mathrm{Div}(\boldsymbol{n})\end{aligned} \tag{14}$$

where the last term vanishes due to the fact that $\boldsymbol{n} = \partial\boldsymbol{x}/\partial q^D$.
Therefore the meaning of the equation (7), $\kappa = 0$, is that Ω is a minimal hypersurface just like the soap film which boundary is a closed line [19].

4. Time Dependent Solution

Now we look for the time dependent solution. Since our theory is just the non relativistic fluid mechanics, it has the Galilean invariance. The Galilean boost is defined by

$$\boldsymbol{x}' = \boldsymbol{x} - \boldsymbol{V}_0 t. \tag{15}$$

The corresponding Lie transformation for the velocity potential is given by the following relation [14–16]

$$\theta(\boldsymbol{x}, t) \to \theta'(\boldsymbol{x}, t) = \theta(\boldsymbol{x} + t\boldsymbol{V}_0, t) - \boldsymbol{x}\cdot\boldsymbol{V}_0 - \frac{1}{2}\boldsymbol{V}_0^2 t. \tag{16}$$

We can easily find its validity by taking the space derivative, i.e., it reduces to the relation $\boldsymbol{v}' = \boldsymbol{v} - \boldsymbol{V}_0$.
By using this boost, we can construct time dependent solution from time independent solution $\theta_s(\boldsymbol{x})$

$$\theta(\boldsymbol{x}, t) = \theta_s(\boldsymbol{x} + t\boldsymbol{V}_0) - \boldsymbol{x}\cdot\boldsymbol{V}_0 - \frac{1}{2}\boldsymbol{V}_0^2 t. \tag{17}$$

Note that we have hidden symmetry as follows. The time independent solution θ_s satisfies the equation

$$\nabla\cdot\left(\frac{\nabla\theta_s}{|\nabla\theta_s|}\right) = 0. \tag{18}$$

The above equation is invariant under the transformation with any function $F(x)$ satisfying $\mathrm{d}F/\mathrm{d}x \neq 0$

$$\theta_s \to \theta_s' = F(\theta_s). \tag{19}$$

Therefore, more general solution for the Chaplygin gas is given as

$$\theta(\boldsymbol{x}, t) = F(\theta_s(\boldsymbol{x} + t\boldsymbol{V}_0)) - \boldsymbol{x}\cdot\boldsymbol{V}_0 - \frac{1}{2}\boldsymbol{V}_0^2 t. \tag{20}$$

We have obtained $(D, 1) = (\text{space}, \text{time})$ dimensional solution for Chaplygin gas from $D-1$ dimensional minimal hypersurface. Two examples of two-dimensional

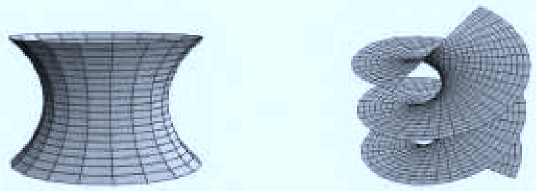

Figure 2. Catenoid and helicoid

minimal hypersurfaces are shown in Fig. 2 [9, 19]. The corresponding solution generated by the catenoid is

$$\theta_s(x, y, z) = z - A \operatorname{arccosh}\left(\frac{\sqrt{x^2 + y^2}}{A}\right) \tag{21}$$

and that one associated with the helicoid is given by

$$\theta_s(x, y, z) = z - \arctan\left(\frac{y}{x}\right)\cdot \tag{22}$$

5. Another Solution

We look for the solution in the form

$$\theta = t - \sqrt{2} f(x^1, x^2, \ldots, x^D). \tag{23}$$

By putting this expression into (6), we obtain

$$\frac{\partial}{\partial x^A}\left(\frac{\partial_A f}{\sqrt{1 + (\nabla f)^2}}\right) = 0 \tag{24}$$

where $A = 1, 2, \ldots, D$. The meaning of this equation is as follows. Let us consider the D-dimensional hypersurface in $(D+1)$-dimensional Euclidean space in the form

$$z = f(x^1, x^2, \ldots, x^D). \tag{25}$$

This is shown in Fig. 3. The infinitesimal area of this hypersurface $\mathrm{d}S$ is related to the projected area $\mathrm{d}x^1\mathrm{d}x^2\cdots\mathrm{d}x^D$ on the flat D-dimensional hypersurface

$$\cos\phi\,\mathrm{d}S = \mathrm{d}x^1\mathrm{d}x^2\cdots\mathrm{d}x^D \tag{26}$$

where ϕ is the angle between $\boldsymbol{n}$ and the positive z direction. Therefore $\cos\phi = n_z$ and we obtain the hypersurface area

$$S = \int \frac{\mathrm{d}x^1\mathrm{d}x^2\cdots\mathrm{d}x^D}{n_z}\cdot \tag{27}$$

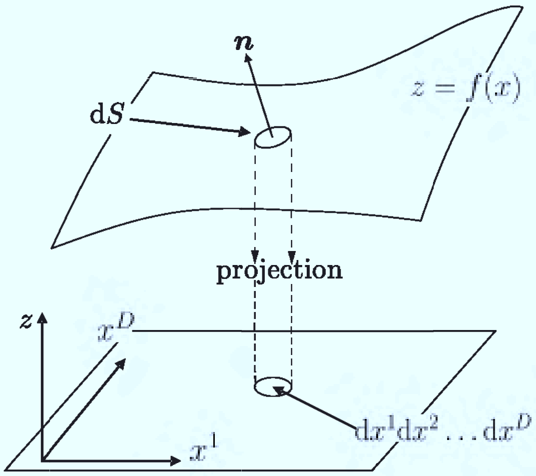

Figure 3. D-dimensional hypersurface

The normal vector $\boldsymbol{n}$ can be calculated as

$$\boldsymbol{n} = \frac{\nabla_{D+1}(z-f)}{|\nabla_{D+1}(z-f)|} = \frac{\boldsymbol{k} - \nabla f}{\sqrt{1+(\nabla f)^2}} \tag{28}$$

where ∇_{D+1} is $(D+1)$-dimensional differential operator and $\boldsymbol{k}$ is the unit vector in z direction. Then we have

$$S = \int \sqrt{1+(\nabla f)^2}\, \mathrm{d}x^1 \mathrm{d}x^2 \cdots \mathrm{d}x^D. \tag{29}$$

The Euler–Lagrange equation for this action (surface area) gives the equation (24). This time we have obtained $(D, 1)$-dimensional solution for Chaplygin gas from D-dimensional minimal hypersurface [19].

6. Symmetries of Chaplygin Gas

The theory has two kinds of hidden symmetries. One is Dilatation (time rescaling) with the scalar parameter λ

$$\theta'(t, \boldsymbol{x}) = \mathrm{e}^{\lambda}\theta(\mathrm{e}^{\lambda}t, \boldsymbol{x}) \tag{30}$$

and the other one is the highly non-linear transformation called field dependent transformation [1, 2] generated by the vector parameter $\boldsymbol{\omega}$

$$\begin{aligned} T(t, \boldsymbol{x}) &= t + \boldsymbol{\omega}\cdot\boldsymbol{x} + \frac{1}{2}\boldsymbol{\omega}^2\theta(T, \boldsymbol{R}) \\ \boldsymbol{R}(t, \boldsymbol{x}) &= \boldsymbol{x} + \boldsymbol{\omega}\theta(T, \boldsymbol{R}) \\ \theta'(t, \boldsymbol{x}) &= \theta(T, \boldsymbol{R}). \end{aligned} \tag{31}$$

This transformation mixes the field and space time parameters.

Actually the theory includes the following symmetries [1,2,10–13]:

- Time and Space translations: $D+1$ generators
- Space rotations: $D(D-1)/2$ generators
- Galilean boost: D generators
- Field translation: one generator
- Time rescaling (dilatation): one generator
- Field dependent transformation: D generators.

This means that the total number of generators is $(D+2)(D+3)/2$ which is the same as that of $(D+2)$-dimensional Poincare generators, and they really induce the Poincare algebra.

7. Relativistic Brane

The fact that the theory has the Poincare symmetry, means that it can be interpreted as a relativistic object. To see this point clearly, we start from the relativistic action of D-brane in $(D+1,1)$ space time. The action is the Nambu–Goto action given by

$$S = -\int \mathrm{d}^{D+1}q \sqrt{(-1)^D \det\left[\eta_{\mu\nu}\frac{\partial X^\mu}{\partial q^a}\frac{\partial X^\nu}{\partial q^b}\right]} \tag{32}$$

where X is the target space variable with its indices $(\mu,\nu,\dots)=0,1,\dots,D+1$, and metric: $\eta=\mathrm{diag}(1,-1,-1,\dots,-1)$. The local coordinates on the brane are given by q with indices $(a,b,\dots)=0,1,\dots,D$ as shown in Fig. 4.

To fix the gauge degrees of freedom, and to check the non-appearance of ghost field, we go to the path integral formulation. Since our theory include a Hamiltonian, momentum constraints and the gauge constraint χ, we have

$$Z = \int DX \int DP \prod_a \delta(T_a) \prod_b \delta(\chi^b) |\det\{T_a,\chi^b\}| \exp\left(\mathrm{i}\int P_\mu \dot{X}^\mu \mathrm{d}^{D+1}q\right). \tag{33}$$

Here P is the momentum conjugate to X. The Hamiltonian and momentum constraints are

$$T_0 = P_\mu P^\mu - (-1)^D G G^{00}, \qquad T_i = P_\mu \frac{\partial X^\mu}{\partial q^i}. \tag{34}$$

The metric field is given by

$$G_{ab} = \eta_{\mu\nu}\frac{\partial X^\mu}{\partial q^a}\frac{\partial X^\nu}{\partial q^b}, \qquad G = \det(G_{ab}). \tag{35}$$

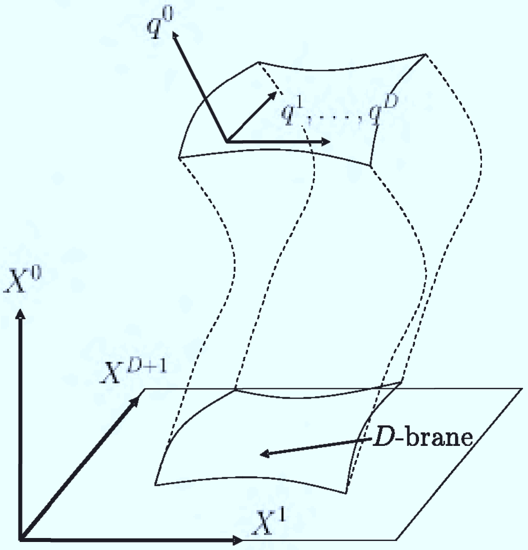

Figure 4. Relativistic D-brane

Now we fix the gauge in two different ways. First we use the light cone gauge defined by

$$\chi^0 \equiv q^0 - \frac{1}{\sqrt{2}}(X^0 + X^{D+1}), \quad \chi^k \equiv q^k - X^k, \quad k = 1, \ldots, D. \tag{36}$$

By inserting above equations into (33), we obtain the new action

$$S = \int \mathrm{d}^{D+1}q \left[\theta\dot{\rho} - \left(\frac{\rho}{2}(\nabla\theta)^2 + \frac{1}{2\rho}\right)\right] \tag{37}$$

where the relations $\theta \equiv \frac{1}{\sqrt{2}}(X^0 - X^{D+1})$ and $\rho \equiv \frac{1}{\sqrt{2}}(P^0 - P^{D+1})$ have been used.

By using the equation for ρ, we obtain the action

$$S_{\text{chap}} = -\sqrt{2}\int \mathrm{d}^{D+1}q\sqrt{\dot{\theta} + \frac{(\nabla\theta)^2}{2}}. \tag{38}$$

This is just the theory of Chaplygin gas as we have already seen.

On the other hand, we can use another gauge fixing condition called a Cartesian gauge

$$\chi^0 \equiv q^0 - X^0, \quad \chi^k \equiv q^k - X^k, \quad k = 1, \ldots, D. \tag{39}$$

Then we have the action

$$S = \int \mathrm{d}^{D+1}q \left[\theta\dot{\rho} - \sqrt{1+\rho^2}\sqrt{1+(\nabla\theta)^2}\right] \tag{40}$$

where this time $\theta \equiv X^{D+1}$, $\rho \equiv P^{D+1}$ were used. By using the equation for ρ we obtain also the action

$$S_{\mathrm{BI}} = -\int \mathrm{d}^{D+1}q \sqrt{1-\eta^{\mu\nu}\frac{\partial\theta}{\partial q^\mu}\frac{\partial\theta}{\partial q^\nu}}. \tag{41}$$

This action is called **Born–Infeld action** [1,2,5,6,9–16,19].

We have two theories due to the different choices of the gauge. These two theories have the same origin and there should be some relation between their solutions. This point is clarified by Jackiw and Polychronakos [14–16]. Let θ_{Chap} and θ_{BI} be the respevtive solution in each of these theories. Then these solutions are related to each other by the equations

$$T(t,x) + \theta_{\mathrm{Chap}}(T,x) = \sqrt{2}t \tag{42}$$

$$\theta_{\mathrm{BI}}(t,x) = \sqrt{2}T(t,x) - t. \tag{43}$$

First we define the field T by the first equation, then the second equation follows. This relation is easily understood from the difference in the two choices for the gauge, i.e., (36), (39) and the definitions of the fields $\theta_{\mathrm{Chap}} \equiv \frac{1}{\sqrt{2}}(X^0 - X^{D+1})$ and $\theta_{\mathrm{BI}} \equiv X^{D+1}$. Since there exists an explicit relation between the solutions of the two theories we can find the solution from the Born–Infeld theory. Let us compare the equation (41) and the action of $(D+1)$-dimensional minimal hypersurface (similar to (29))

$$S_{\mathrm{ms}} = -\int \mathrm{d}^{D+1}q \sqrt{1+(\nabla f)^2}. \tag{44}$$

By noting $\eta = \mathrm{diag}(1,-1,-1,\ldots,-1)$, we find the relation between the two theories

$$\theta_{\mathrm{BI}}(q^0,\boldsymbol{q}) = f(\mathrm{i}q^0,\boldsymbol{q}). \tag{45}$$

This means that $(D,1)$ Chaplygin or Born–Infeld solution is produced from $(D+1)$-dimensional minimal hypersurface [19]. Before finishing this section we give some conclusions concerning all previous discussions, namely

- Chaplygin gas is deeply related to the minimal hypersurfaces
- $D-1$, D, $D+1$ minimal hypersurfaces gives $(D,1)$ solutions of Chaplygin gas
- The origin of Chaplygin gas and Born–Infeld theories is the relativistic membrane theory.

8. Application to the Universe Model

It was believed for a long time that the expansion of universe has been decelerating. This is because Einstein equation requires it should decelerate unless unknown energy with negative pressure does not exist. In 1998, however, super nova Ia data showed that the universe expansion is really accelerating [7,20,21]. To explain this fact, some new universe models have been discussed. One of them is the Chaplygin cosmology [3, 4, 8, 17, 18]. To see this deceleration property of universe, we first consider the homogeneous and isotropic universe which requires the metric of the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2 \mathrm{d}\sigma^2. \tag{46}$$

Here σ is the spatial parameter given by

$$\mathrm{d}\sigma^2 = \frac{\mathrm{d}r^2}{1 - Kr^2} + r^2 \mathrm{d}\Omega^2 \tag{47}$$

where $\mathrm{d}\Omega^2$ is the metric on the two-dimensional sphere. The value of K coincides with the scalar curvature of our three-dimensional space. More exactly, $K > 0$ means a closed universe S_3, $K < 0$ means an open universe, i.e., a hyperbolic space, and $K = 0$ means flat universe. By putting above relation into Einstein equation without cosmological term, we have the **Friedmann equation**

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{c^2 K}{a^2} \tag{48}$$

and the energy conservation equation

$$\frac{\partial(\rho c^2 a^3)}{\partial t} = -P \frac{\partial a^3}{\partial t}. \tag{49}$$

Using these two equations, we obtain the deceleration parameter of the universe

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}\left(1 + \frac{3P}{\rho}\right) \frac{8\pi G\rho}{3c^2 \left(\frac{\dot{a}}{a}\right)^2}. \tag{50}$$

Then we see that

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} > 0 \tag{51}$$

holds under the condition

$$P > -\frac{1}{3}\rho. \tag{52}$$

To obtain the accelerating universe, we need the exotic matter satisfying

$$P < -\frac{1}{3}\rho. \tag{53}$$

Such a "matter" is called a **dark energy**.

What happens to our universe if we suppose that our matter field satisfies the state equation of Chaplygin gas? This problem was discussed by several authors [3,4,8, 17,18], under the name Chaplygin cosmology. In this review we briefly sketch the discussion by Kamenshchik *et al* [17].

Taken together the Chaplygin state equation and the energy conservation equation (49) give us

$$\rho = \sqrt{A + \frac{B}{a^6}} \tag{54}$$

where B is the integration constant.

From this equation for early universe, i.e., when $a^6 \ll B/A$ one gets

$$\rho = \frac{\sqrt{B}}{a^3}. \tag{55}$$

This relation with energy conservation law (49) requires

$$P = 0. \tag{56}$$

That is, the universe is dust dominated and decelerating, i.e., $q > 0$. For example in the case of the Einstein–de Sitter model $K = 0$, $a(t) = a_0(t/t_0)^{2/3}$. The decelerating property of universe in its beginning is important for the universe has time of life, i.e., if it is accelerating exponentially, we do not have to consider its birth.

On the other hand, for the large value of the cosmological radius, i.e., if $a^6 \gg B/A$

$$\rho = \sqrt{A}. \tag{57}$$

When combined with energy conservation law (49) this relation requires (in $c = 1$ unit)

$$P = -\sqrt{A}. \tag{58}$$

The situation is the same as in the case of the empty universe with a cosmological constant

$$\Lambda = \sqrt{A}. \tag{59}$$

Then by using the Friedmann equation (48) with the condition

$$a^2 \gg \frac{3c^4|K|}{8\pi G\Lambda}$$

we obtain

$$a(t) \sim a_0 \exp\left[\sqrt{\frac{8\pi G\Lambda}{3c^2}}t\right]. \tag{60}$$

So we have inflation for the large universe. In this sense, Chaplygin gas interpolates two phases of the universe. First it expands in power law (deceleration) and later expands exponentially (inflation).

Let us compare the Chaplygin cosmology with the observational data. The present observational fact can also be described by the dust matter with cosmological constant. In such a case

$$\begin{aligned} P &= P_\Lambda + P_M = -\Lambda \\ \rho &= \rho_\Lambda + \rho_M \ = \Lambda + \rho_M . \end{aligned} \tag{61}$$

Then the observational data at present time can be represented as

$$\left[\frac{\Lambda}{\rho_M}\right]_{\text{present}} \sim \frac{7}{3}. \tag{62}$$

If we believe in the Chaplygin cosmology $P = -A/\rho$, we obtain

$$A = \Lambda(\Lambda + \rho_M) \sim 1.43\Lambda^2 \tag{63}$$

and then we have also

$$\sqrt{A} \sim 1.2\Lambda.$$

Since $\sqrt{A}$ is the cosmological constant for large scale universe, the above result shows the increasing cosmological constant [17]. Such a "dynamical cosmological constant" or dark energy is usually called Quintessence [7]. Therefore, the Chaplygin cosmology can be one of picks of the Quintessence.

9. Summary

In this article, we have considered various aspects of the Chaplygin gas. To summarise the previous discussions, we will list them again:

- Chaplygin gas has a geometrical meaning of a minimal hypersurface
- Hidden Poincare symmetry is included
- Theory of the relativistic brane is equivalent to the Chaplygin gas and Born–Infeld theory
- Chaplygin's state equation works as a model for accelerating universe.

References

[1] Bazeia D. and Jackiw R., *Nonlinear Realization of a Dynamical Poincare Symmetry by a Field-Dependent Diffeomorphism*, Ann. Phys. **270** (1998) 246–259.

[2] Bazeia D., *Galileo Invariant System and the Motion of Relativistic d-Branes*, Phys. Rev. D **59** (1999) 085007.

[3] Biesiada M., Goldlowski W. and Szydlowski M., *Generalized Chaplygin Gas Models tested with SNIa*, Astrophysics Journal **622** (2005) 28–38.

[4] Bilic N., Gary B. and Raoul D., *Unification of Dark Matter and Dark Energy: The Inhomogeneous Chaplygin Gas*, Phys. Lett. B **535** (2002) 17–21.

[5] Bordemann M. and Hoppe J., *The Dynamics of Relativistic Membranes I: Reduction to 2-dimensional Fluid Dynamics*, Phys. Lett. B **317** (1993) 315–320.

[6] Bordemann M. and Hoppe J., *The Dynamics of Relativistic Membranes II: Nonlinear Waves and Covariantly Reduced Membrane Equations*, Phys. Lett. B **325** (1994) 359–365.

[7] Caldwell R., Dave R. and Steinhardt P., *Cosmological Imprint of an Energy Component with General Equation of State*, Phys. Rev. Lett. **80** (1998) 1582–1585.

[8] Fabris J., Goncalves S. and de Souza P., *Density Perturbations in an Universe Dominated by the Chaplygin Gas*, Gen. Rel. Grav. **34** (2002) 53–63.

[9] Gibbons G., *Born–Infeld Particles and Dirichlet p-Branes*, Nucl. Phys. B **514** (1998) 603–639.

[10] Hoppe J., *Quantum Theory of a Relativistic Surface*, In: Constraints Theory and Relativistic Dynamics, Workshop in Firenze, World Scientific, 1986, pp 267–276.

[11] Hoppe J., *Quantum Theory of a Massless Relativistic Surface and a Two Dimensional Bound State Problem*, Elem. Part. Res. J. (Kyoto) **80** (1989) 145–202.

[12] Hoppe J., *Some Classical Solutions of Relativistic Membrane Equations in 4 Space-Time Dimensions*, Phys. Lett. B **329** (1994) 10–14.

[13] Hoppe J., *Conservation Laws and Formation of Singularities in Relativistic Theories of Extended Objects*, hep-th/9503069.

[14] Jackiw R. and Polychronakos A., *Dynamical Poincare Symmetry Realized by Field-Dependent Diffeomorphism*, Contribution to Faddeev Festschrift, Steklov Mathematical Institute Proceedings, hep-th/9809123, 21 pp.

[15] Jackiw R. and Polychronakos A., *Fluid Dynamical Profiles and Constants of Motion from d-Branes*, Commun. Math. Phys. **207** (1999) 107–129.

[16] Jackiw R. and Polychronakos A., *Super Symmetric Fluid Mechanics*, Phys. Rev. D **62** (2000) 085019.

[17] Kamenshchik A., Moschella U. and Pasquier V., *An Alternative to Quintessence*, Phys. Lett. B **511** (2001) 265–268.

[18] Nesseris S. and Perivolaropoulos L., *A Comparison of Cosmological Models Using Recent Supernova Data*, Phys. Rev. D **70** (2004) 043531.

[19] Ogawa N., *A Note on Classical Solution of Chaplygin-Gas as D-Brane*, Phys. Rev. D **62** (2000) 085023.

[20] Perlmutter S., Aldering G., Goldhaber G. et al., *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, The Astronomical Journal **517** (1999) 565–586.

[21] Riess A., Filippenko A., Challis P. *et al*, *Observational Evidence from Supernovae for an Accelerating Universe and Cosmological Constant*, The Astronomical Journal **116** (1998) 1009–1038.