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FINITE GROUP ACTIONS IN SEIBERG–WITTEN THEORY

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Abstract. Let X be a closed and oriented Riemannian four-manifold with $b_2^+(X) > 1$. We discuss the Seiberg–Witten invariants of X and finite group actions on spin^c structures of X. We introduce and comment some of our results on the subject.

1. Introduction

In the past twenty years, the symbiosis between mathematics and theoretical physics has always been a source of unexpected and profound results.

Even if we do not make attempt to relate it chronologically, the story begun with the Donaldson's gauge theory aiming a nonabelian generalization of the classical electromagnetic theory.

As results of it the nonsmoothability of certain topological four-manifolds, exotic smooth structures on $\mathbb{R}^4$, and nondecomposability of some four-manifolds have been established.

The computation of Donaldson invariants however is highly nontrivial.

In 1994, the monopole theory in four-manifolds gave a rise to the Seiberg–Witten invariant which is much simpler than the Donaldson theory, also had almost the same effects on the Donaldson theory, and was used for a proof of the Thom conjecture.

At almost the same time the Gromov–Witten invariant of symplectic manifolds was introduced. Using it we may compute the number of algebraic curves, representing a two-dimensional homology class in a symplectic manifold.

In 1995 Taubes [26] proved that for symplectic four-manifolds the Seiberg–Witten invariant and the Gromov–Witten invariant are the same.

In 1982 Freedman [15] classified the simply connected closed topological four-manifolds.

In 1983 Donaldson [12] proved the nonsmoothability of certain topological four-manifolds.

In 1994, Witten [29] had introduced the Seiberg–Witten theory which simplifies the Donaldson's gauge theory.

In 1994 Kontsevich and Manin [20] introduced the Gromov–Witten invariant and applied it to enumerative problems of algebraic geometry.

In this article we want to introduce the Seiberg–Witten invariant and its fundamental consequences. We would like to survey finite group actions in the Seiberg–Witten theory, and some of our results which were already published (see the references at the end of the paper).

2. Review of Seiberg–Witten Invariant

Let X be a closed, oriented four-manifold with $b_2^+(X) > 1$. Let $L \longrightarrow X$ be a complex line bundle with $c_1(L) = c_1(X) \mod 2$.

Let $W^{\pm}$ be the twisted spinor bundles on X associated with the line bundle L. Let $\sigma : W^+ \otimes T^*X \longrightarrow W^-$ be the Clifford multiplication. There is a correspondence $\tau : W^+ \times W^+ \longrightarrow \mathrm{End}(W^+)_\circ$ given by $\tau(\phi, \phi) = (\phi \otimes \overline{\phi}^t)_\circ$ which is a traceless endomorphism of W^+.

The Levi-Civita connection on X combined with a connection A on L induces a Dirac operator

$$D_A : \Gamma(W^+) \longrightarrow \Gamma(W^-)$$

along with the Seiberg–Witten (SW) equations

$$D_A\phi = 0, \qquad F_A^+ = -\tau(\phi, \phi) \tag{1}$$

whose solutions correspond to the absolute minima of some functional.

The gauge group $C^\infty(X, \mathrm{U}(1))$ of L acts on the space of solutions (A, ϕ) of the SW-equations and quotient $\mathfrak{M}(L)$ of the space of solutions of the equations modulo the gauge group is called the moduli space associated to the spin^c structure L on X.

If we perturb the SW-equations or find a generic metric on X, the moduli space $\mathfrak{M}(L)$ is a compact orientable d-manifold where $d = \frac{1}{4}[c_1(L)^2 - (2\chi + 3\sigma)]$.

If $x_\circ$ is some fixed base point in X, the evaluation at $x_\circ$ gives a representation $\rho : C^\infty(X, \mathrm{U}(1)) \longrightarrow \mathrm{U}(1)$, which induces a $\mathrm{U}(1)$-bundle $E \longrightarrow \mathfrak{M}(L)$.

If the dimension of $\mathfrak{M}(L)$ is even, i.e., $d = 2s$, then the Seiberg–Witten invariant of L is given by

$$\mathrm{SW}(L) = \langle c_1(E)^s, \mathfrak{M}(L)\rangle.$$

The fundamental properties of the SW-invariants are as follows.

Theorem 1. *Let X be a closed smooth oriented four-manifold with $b_2^+(X) > 1$.*

1) *There is only a finite number of spin*^{c} *structures* L *for which* $\mathrm{SW}(L) \neq 0$.
2) *If* $X = X_1 \sharp X_2$ *with* $b_2^+(X_i) > 0$, $i = 1, 2$, *then* $\mathrm{SW}(L) = 0$ *for all spin*^{c} *structures* L *on* X.
3) *For a spin*^{c} *structures* L *on* X, *the* $\mathrm{SW}(L)$ *is independent of the metrics on* X, *and depends only on the class* $c_1(L)$.
4) *If* f *is a diffeomorphism of* X, *then* $\mathrm{SW}(L) = \pm\mathrm{SW}(f^*L)$.
5) *If* X *admits a metric of positive scalar curvature, then* $\mathrm{SW}(L) = 0$ *for all spin*^{c} *structures* L *on* X.
6) *If* X *is a closed symplectic four-manifold with canonical complex line bundle* K_X, *then* $\mathrm{SW}(K_X) = \pm 1$.
7) *A complex curve in a Kähler surface has minimum genus in its homology class.*

3. Finite Group Actions on spin^c Structures

Let X be a closed, oriented, Riemannian four-manifold. Let P be the principal bundle of oriented orthonormal frames associated to the tangent bundle TX of X. Let $L \to X$ be a complex line bundle on X satisfying $c_1(L) \equiv w_2(TX) \mod 2$. There is a one-to-one correspondence between the set of spin^c structure on X and the set of elements of $H^1(X, \mathbb{Z}_2) \otimes 2H^2(X, \mathbb{Z})$.

Let P_L be the principal $\mathrm{U}(1)$ bundle associated to the bundle L. Let G be a finite group. Let G act on X by orientation preserving isometries. The induced action of G on the frame bundle P commutes with the right action of SO(4) on P. Choose an action of G over the principal $\mathrm{U}(1)$ bundle $P_L \to X$ which is compatible with the action of G on X, and commutes with the canonical right action of $\mathrm{U}(1)$ on P_L. If the induced action of G on the product $P \times P_L$ lifts to an action of G on the associated principal spin^c bundle $\tilde{P}$ which commutes with the right action of Spin^c(4) on $\tilde{P}$, then the action of G on $\tilde{P}$ is called a spin^c action on the spin^c structure $\tilde{P}$. Thus the spin^c action of G on $\tilde{P} \to X$ induces a diffeomorphism on X, P, P_L and $\tilde{P}$ for each element in G and induces bundle automorphisms on P, P_L and $\tilde{P}$ which cover the action of G on X.

Proposition 1. *If the action of a finite group* G *on a spin*^{c} *structure* $\tilde{P}$ *associated to a line bundle* L *over* X *is a spin*^{c} *action, then for each element* $h \in G$, h *acts on* P, P_L *and* $\tilde{P}$ *as a bundle automorphism which cover the action of* h *on* X.

Let X be a closed, oriented, Riemannian four-manifold. Let $\tilde{P}$ be a principal spin^c bundle associated to the line bundle $L \to X$. Let the action of a finite group G on $\tilde{P} \to X$ be a spin^c action. For each element $h \in G$ there are liftings $h : P \to P$ and $\tilde{h} : \tilde{P} \to \tilde{P}$. We define an action of G on the twisted $\frac{1}{2}$-spinor bundle $W^+ = \tilde{P} \times \mathbb{C}^2$ by $h(\tilde{p}, a) = (\tilde{h}\tilde{p}, a)$ for each $h \in G$ and $(\tilde{p}, a) \in W^+$.

Since the action of G on $\tilde{P}$ commutes with the right action of Spin^c(4) on $\tilde{P}$, this action is well defined.

On the $W^+ = \tilde{P} \times \mathbb{C}^2$, the action of Spin^c(4) on $\mathbb{C}^2$ is given by $(q_1, q_2, \mathrm{e}^{\mathrm{i}\theta})a = q_1 a \mathrm{e}^{\mathrm{i}\theta}$ for each element

$$[q_1, q_2, \mathrm{e}^{\mathrm{i}\theta}] \in \mathrm{Spin}^c(4) = (\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_2, \qquad a \in \mathbb{C}^2.$$

If $\tilde{h}\tilde{p} = \tilde{p}\alpha$ for some $\alpha = [q_1, q_2, \mathrm{e}^{\mathrm{i}\theta}] \in \mathrm{Spin}^c(4)$, then

$$h(\tilde{p}, a) = (\tilde{h}\tilde{p}, a) = (\tilde{p}\alpha, a) = (\tilde{p}, \alpha a) = (\tilde{p}, q_1 a \mathrm{e}^{\mathrm{i}\theta}).$$

By the definition of the spin^c action, the group G acts on X and L as orientation-preserving isometries. The induced actions of G on p and P_L commute with right actions of $\mathrm{SO}(4)$ and $\mathrm{U}(1)$ on P and P_L. Thus the action of G on $\tilde{P}$ commutes with the lift of the connections to $\tilde{P}$. If ∇ is the connection of W^+ associated to the Levi-Civita connection on P and a Riemannian connection A on P_L, then we have the formulas

$$h(\nabla_v s) = \nabla_{hv} hs, \qquad h(Ds) = h\left(\sum_i \mathrm{e}_i \cdot \nabla_{\mathrm{e}_i} s\right) = \sum_i h\mathrm{e}_i \cdot \nabla_{h\mathrm{e}_i} hs.$$

So $hD = Dh$, where $h \in G$, $v \in TX$, $s \in \Gamma(W^+)$ and $D : \Gamma(W^+) \to \Gamma(W^-)$ is the Dirac operator associated to the connection ∇. Thus the Dirac operator D is a G-equivariant elliptic operator. The G-index of D is a virtual representation

$$L(G, X) = \ker D - \operatorname{coker} D \in R(G).$$

For each element $h \in G$, the Lefschetz number is defined by

$$L(h, X) = \operatorname{trace}(h|_{\ker D}) - \operatorname{trace}(h|_{\operatorname{coker} D}) \in \mathbb{C}.$$

Theorem 2 (Atiyah–Singer). *Let X^h be the set of the fixed points of h in X, and let $i : X^h \to X$ be the inclusion and N^h the normal bundle of X^h in X. Then the Lefschetz number $L(h, X)$ is*

$$L(h, X) = (-1)^k \frac{\mathrm{ch}_h(i^*(W^+ - W^-))\mathrm{td}(TX^h \otimes \mathbb{C})}{e(TX^h)\mathrm{ch}_h(\Lambda_{-1}N^h \otimes \mathbb{C})}[X^h]$$

where $k = \frac{1}{2}\dim X^h$.

4. Finite Group Actions and Seiberg–Witten Equations

Let X be a closed, oriented, Riemannian four-manifold. Let a finite group G acts on X as orientation preserving isometries. Let $\pi : L \to X$ be a complex line bundle satisfying $c_1(L) \equiv w_2(TX) \mod 2$. Let the group G acts on L such that the projection π is a G-map. Choose a metric on L on which G acts by isometries. Let $\mathcal{A}(L)$ be the set of all Riemannian connections on L. The space $\mathcal{A}(L)$ is an affine space modelled on $\Omega^1(\mathrm{i}\mathbb{R})$. The set of all bundle automorphisms of L forms a group $\mathcal{G}(L)$ which is identified with the set of smooth maps from X into S^1. The group $\mathcal{G}(L)$ acts on $\mathcal{A}(L) \times \Gamma(W^+)$ by $g(A,\phi) = (A - g^{-1}\mathrm{d}g, g^{\frac{1}{2}}\phi)$ for $g \in \mathcal{G}(L)$, $(A,\phi) \in \mathcal{A}(L) \times \Gamma(W^+)$. The $\left(\pm\frac{1}{2}\right)$-spinor bundles $S^\pm$ of X and the square root $L^{\frac{1}{2}}$ of the bundle L may not exist globally but exist locally. Since $c_1(L) \equiv w_2(TX) \mod 2$, the twisted $\left(\pm\frac{1}{2}\right)$-spinor bundles $W^\pm = S^\pm \otimes L^{\frac{1}{2}}$ do exist globally.

For each $h \in G$, $g \in \mathcal{G}(L)$, $\nabla \in \mathcal{A}(L)$, $v_i \in \Gamma(TX)$, $\sigma \in \Gamma(L)$, $\varphi \in \Omega^k$ and $\phi \in \Gamma(W^+)$, we define the actions of G on these as follows:

1) $h(\nabla)_v\sigma = h(\nabla_{h^{-1*}v}(h^{-1}\sigma))$, $h(\sigma) = h\sigma h^{-1}$
2) $h(\varphi)_{v_1,\dots,v_k} = h(\varphi_{h^{-1*}v_1,\dots,h^{-1*}v_k})$
3) $h(g) = h \circ g \circ h^{-1}$
4) $h(\phi) = \phi h^{-\frac{1}{2}}$, where $h^{\frac{1}{2}} : X \to \mathrm{U}(2)$ is given by the lift $h^{\frac{1}{2}}(x) = \tilde{h}(x)$ of $h(x) : P_{h^{-1}(x)} \to P_x$.

Proposition 2. *The spaces $\mathcal{A}(L)$, $\mathcal{G}(L)$, Ω^k, $\Gamma(L)$ and $\Gamma(W^+)$ are closed under the action of G.*

For each connection $A \in \mathcal{A}(L)$ on L we have a Dirac operator

$$D_A : \Gamma(W^+) \to \Gamma(W^-)$$

whose symbol is given by the Clifford multiplication. The Clifford multiplication produces an isomorphism

$$\varrho : \Lambda^+ \otimes \mathbb{C} \to \mathrm{End}(W^+)_\circ$$

between the complexified self-dual two-forms and the traceless endomorphisms of W^+. There is a pairing $\tau : W^+ \times \hat{W}^+ \to \mathrm{End}(W^+)_\circ$ defined by $\tau(\phi_1,\phi_2) = \phi_1 \circ \phi_2^t - \frac{1}{2}\operatorname{tr}(\phi_1 \circ \phi_2^t)\,\mathrm{Id}$. The Seiberg–Witten equations are defined by

$$D_A\phi = 0, \qquad \varrho(F_A^+) = \tau(\phi,\bar{\phi})$$

where $(A,\phi) \in \mathcal{A}(L) \times \Gamma(W^+)$ are invariant under the action of G.

Proposition 3. *If the action of G on the spin^c structure W^+ is a spin^c action, then G acts on the solutions of the Seiberg–Witten equations.*

Let $\mathcal{G}(L)^G = \{g \in \mathcal{G}(L)\,;\, h \circ g \circ h^{-1} = g, h \in G\}$ be the G-invariant subgroup of $\mathcal{G}(L)$. Since G acts on L as orientation preserving isometries and the structure group of L is the abelian group $\mathrm{U}(1)$, $h \circ g \circ h^{-1} = g$ on the fixed point set X^G, for each element $h \in G$ and $g \in \mathcal{G}(L)$. Let $\mathcal{SW}(L)$ be the set of solutions of the Seiberg–Witten equations.

If $b_2^+ > 0$, then the space of solutions to the Seiberg–Witten equations does not contain (A, ϕ) which $\phi \equiv 0$ for a generic metric of perturbation of the second equation by an imaginary-valued G-invariant self-dual two-form on X.

5. Some Results of Finite Group Action on Seiberg–Witten Invariants

Let X be a closed symplectic four-manifold. The tangent bundle TX of X admits an almost complex structure which is an endomorphism $J : TX \to TX$ with $J^2 = -I$. The almost complex structure J defines a splitting

$$T^*X \otimes \mathbb{C} = T^{1.0} \otimes T^{0.1}$$

where J acts on $T^{1.0}$ and $T^{0.1}$ as multiplication by $-\mathrm{i}$ and i, respectively. The **canonical bundle** K_X on X associated to the almost complex structure J is defined by $K_X = \Lambda^2 T^{1.0}$.

A symplectic structure ω on X is defined as a closed two-form with $\omega \wedge \omega \neq 0$ everywhere. An almost complex structure J on X is said to be compatible with the symplectic structure ω if $\omega(Jv_1, Jv_2) = \omega(v_1, v_2)$ and $\omega(v, Jv) > 0$ for a non-zero tangent vector v.

The space of compatible almost complex structure of a given symplectic structure on X is non-empty and constructible. If an almost complex structure J is compatible with ω. Then for any $v, w \in TX$, $g(v, w) = \omega(v, Jw)$ defines a Riemannian metric on X. For such a metric on X, the symplectic structure ω is self-dual and $\omega \wedge \omega$ gives the orientation on X. On the other hand, any metric on X for which ω is self-dual can define an almost complex structure J which is compatible with the symplectic structure ω.

Let (X, ω) be a closed, symplectic, four-manifold. A **diffeomorphism** σ on X is **symplectic**, **anti-symplectic** if σ satisfies $\sigma^*\omega = \omega$ or $\sigma^*\omega = -\omega$, respectively.

An **involution** σ on X is **symplectic**, **anti-symplectic** if and only if it satisfies $\sigma_* J = J\sigma_*$ or $\sigma_* J = -J\sigma_*$, respectively, for some compatible almost complex structure J on X with the symplectic structure ω. If (X, ω) is a Kähler surface with Kähler form ω, then an involution σ on X is symplectic, anti-symplectic if and only if it is holomorphic, anti-holomorphic, respectively.

Now we assume that X is a closed, smooth and oriented four-manifold with a finite fundamental group. Let G be a finite group.

Theorem 3 ([6]). *If G acts smoothly and freely on the four-manifold X which has a non-vanishing SW-invariant, then the quotient X/G cannot be decomposed as a smooth connected sum $X_1 \sharp X_2$ with $b_2^+(X_i) > 0$, $i = 1, 2$.*

Theorem 4 ([6]). *Let X be a closed and symplectic four-manifold with a finite fundamental group, $c_1(X)^2 > 0$ and $b_2^+(X) > 3$.*

If $\sigma : X \longrightarrow X$ is a free anti-symplectic involution on X, then the SW-invariants vanish on the quotient X/σ. In particular, the X/σ does not have any symplectic structure.

Theorem 5 ([7]). *Let X be a manifold with a nontrivial SW-invariant, $b_2^+(X) > 1$, and let Y be a manifold with negative definite intersection form. If $n_1, \ldots, n_k$ are even integers such that $4b_1(Y) = 2n_1 + \cdots + 2n_k + n_1^2 + \cdots + n_k^2$ and $\pi_1(Y)$ has a nontrivial finite quotient, then the connected sum $X \sharp Y$ has a nontrivial SW-invariant but does not admit any symplectic structure.*

Theorem 6 ([10]). *Let (X, w) be a closed, symplectic four-manifold and let σ be an anti-symplectic involution on (X, w) with fixed loci $X^\sigma = \amalg \Sigma$ as a disjoint union of Lagrangian surfaces. If one of the components of X^σ is a surface of genus $g > 1$ and $b_2^+(X/\sigma) > 1$, then the quotient X/σ has vanishing SW-invariants.*

Example. Let $X = (\Sigma_g \times \Sigma_g, w \oplus w)$, and an anti-symplectic involution

$$f : \Sigma_g \longrightarrow \Sigma_g \quad \text{with} \quad f^* w = -w.$$

Let $\sigma_f : \Sigma_g \times \Sigma_g \longrightarrow \Sigma_g \times \Sigma_g$ be given by

$$\sigma_f(x, y) = (f^{-1}(y), f(x)).$$

Then σ_f is an anti-symplectic involution with fixed points $(\Sigma_g \times \Sigma_g)^{\sigma_f} \simeq \Sigma_g$. By the Hirzebruch signature theorem we have

$$b_2^+(X/\sigma_f) = \frac{1}{2}(b_2^+(X) - 1) = g^2 > 1, \quad \text{if} \quad g > 1.$$

In [6] Cho shows that if the cyclic group $\mathbb{Z}_2$ acts smoothly and freely on a closed, oriented, smooth four-manifold X with a finite fundamental group and with a non-vanishing Seiberg–Witten invariant, then the quotient $X/\mathbb{Z}_2$ cannot be decomposed as a smooth connected sum $X_1 \sharp X_2$ with $b_2^+(X_i) > 0$, $i = 1, 2$. In [28] Wang shows that if X is a Kähler surface with $b_2^+(X) > 3$ and $K_X^2 > 0$ and if $\sigma : X \to X$ is an anti-holomorphic involution (i.e., $\sigma_* \circ J = -J \circ \sigma_*$) without fixed point set then the quotient $X/\sigma \equiv X'$ has vanishing Seiberg–Witten invariant.

Theorem 7 ([9]). *Let X be a Kähler surface with $b_2^+(X) > 3$ and $H_2(X, \mathbb{Z})$ have no two-torsion. Suppose that $\sigma : X \longrightarrow X$ is an anti-holomorphic involution with fixed point set Σ which is a Lagrangian surface with genus greater than 0 and $[\Sigma] \in 2H_2(X, \mathbb{Z})$. If $K_X^2 > 0$ or $K_X^2 = 0$ and $g(\Sigma) > 1$, then the quotient X/σ has a vanishing Seiberg–Witten invariant.*

Let X be a closed smooth four-manifold with $\mathbb{Z}_p$ action, where p is a prime. Suppose $H_1(X, \mathbb{R}) = 0$ and $b_2^+(X) > 1$.

Theorem 8 ([13]). *Suppose $\mathbb{Z}_p$ acts trivially on $H^2(X^+, \mathbb{R})$ of self-dual harmonic two-forms and for any $\mathbb{Z}_p$-equivariant spin^c-structure L on X, the index of the equivariant Dirac operator*

$$D_A : \Gamma(W^+) \longrightarrow \Gamma(W^-)$$

has the form

$$\mathrm{ind}_{\mathbb{Z}_p}(D_A) = \sum_{j=0}^{p-1} k_j t^j \in R(\mathbb{Z}_p) = \mathbb{Z}[t]/(t^p = 1).$$

Then the Seiberg–Witten invariant satisfies

$$\mathrm{SW}(L) = 0 \pmod{p} \quad \textit{if} \quad k_j \leq \frac{1}{2}(b_2^+(X) - 1), \quad j = 0, 1, \ldots, p-1.$$

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