FLUX CONJECTURE ON SYMPLECTIC SUBMANIFOLDS

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Abstract. Let $(M, \omega)$ be a closed symplectic $2n$-dimensional manifold. According to the well-known result by Donaldson [5] there exist $2m$-dimensional symplectic submanifolds $(V^{2m}, \omega)$ of $(M, \omega)$, $1 \leq m \leq n - 1$, with $(m - 1)$-equivalent inclusions. In this paper, we have found a relation between the flux group and the kernel of the Lefschetz map. We have also some properties of the flux groups for all symplectic $2m$-submanifolds $(V^{2m}, \omega)$ where $2 \leq m \leq n - 1$.

1. Introduction

Let $(M, \omega)$ be a compact symplectic manifold and $\text{Symp}_0(M)$ denote the identity component of the symplectomorphism group $\text{Symp}(M)$ of $(M, \omega)$. Recall that the flux homomorphism

$$F_\omega : \pi_1(\text{Symp}_0(M)) \to H^1(M, \mathbb{R})$$

can be defined as follows. For an element $\phi \in \pi_1(\text{Symp}_0(M))$ and any homology class $\alpha \in H_1(M, \mathbb{R})$ set

$$(F_\omega(\phi), \alpha) = (\omega, \phi_t\alpha)$$

where $\phi_t\alpha$ denotes the trace of a loop $\alpha$ under the isotopy $\{\phi_t\}$ representing $\phi$ and $(\cdot, \cdot)$ is the natural pairing. It is well known that $\phi$ is represented by a Hamiltonian loop if and only if $F_\omega(\phi) = 0$. Define the flux group $\Gamma_M$ of $M$ by the image of the flux homomorphism, i.e.,

$$\Gamma_M = \text{im}\{F_\omega : \pi_1(\text{Symp}_0(M)) \to H^1(M, \mathbb{R})\} \subset H^1(M, \mathbb{R}).$$

The importance of this notion is due to the fact that the Hamiltonian diffeomorphism group $\text{Ham}(M)$ is closed in $\text{Symp}_0(M)$ if and only if $\Gamma_M$ is a discrete subgroup of $H^1(M, \mathbb{R})$. The statement that $\Gamma_M$ is discrete is known as the flux conjecture. Then we obtain a relation between the flux group $\Gamma_M$ of $M$ and