ANOTHER PERSPECTIVE ON THE RELATION BETWEEN
CLASSICAL AND QUANTUM INTEGRABILITY

JEAN-CLAUDE ZAMBRINI† and NENAD MANOJLOVIĆ‡

† GFM, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal
‡ Área Departamental de Matemática, F.C.T., Campus de Gambelas
Universidade do Algarve, 8000-117 Faro, Portugal

Abstract. We describe a framework suggesting how to “deform” in
Planck’s constant \( \hbar \) the classical concept of integrability. The key point
is to use well defined counterparts of Feynman’s measure on the space
of paths of the mechanical system. Then we introduce the associated
deformation of quantum conservation laws. The method is tested on el-
ementary systems and provide, indeed, more information than expected.

1. Motivation

This is a brief report on some qualitative aspects of the relations between
the classical notions of integrability and their quantum counterparts. We wish
to advocate an approach to this question which is certainly not mainstream
but seems to us conceptually natural, as well as to provide a few arguments
showing why we believe that it is indeed promising. The full realization of
this research program may, however, take years so we will somehow abuse
the hospitality of Professor Ivailo Mladenov, hoping to be able to prove in a
future Varna Conference on Geometry, Integrability and Quantization, that the
program sketched in the 2001 edition was, indeed, sound!

In mathematical physics, the motivation for studying together the two
above-mentioned notions of integrability can be traced back to the need to
define the quantization of classical systems whose behavior is conflicting as
much as possible with the one of integrable systems, the ergodic ones. This
is one way to approach “quantum chaos”, not optimal however since it is not
really an intrinsic quantum mechanical definition [1, 2]. But, at least, this per-