SEIBERG-WITTEN EQUATIONS ON $\mathbb{R}^6$

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Abstract. It is known that Seiberg-Witten equations are defined on smooth four dimensional manifolds. In the present work we write down a six dimensional analogue of these equations on $\mathbb{R}^6$. To express the first equation, the Dirac equation, we use a unitary representation of complex Clifford algebra $\mathbb{C}l_{2n}$. For the second equation, a kind of self-duality concept of a two-form is needed, we make use of the decomposition $\Lambda^2(\mathbb{R}^6) = \Lambda^2_1(\mathbb{R}^6) \oplus \Lambda^2_6(\mathbb{R}^6) \oplus \Lambda^2_8(\mathbb{R}^6)$. We consider the eight-dimensional part $\Lambda^2_8(\mathbb{R}^6)$ as the space of self-dual two-forms.

1. Introduction

The Seiberg-Witten equations defined on four-dimensional manifolds yield some invariants for the underlying manifold. There are some generalizations of these equation to higher dimensionnal manifolds. In [2, 7] some eight-dimensional analogies were given and a seven-dimensional analog was presented in [5]. In this work we write down similar equations to Seiberg-Witten equations on $\mathbb{R}^6$.

2. spin$^c$–structure and Dirac Operator on $\mathbb{R}^{2n}$

Definition 1. A spin$^c$-structure on the Euclidian space $\mathbb{R}^{2n}$ is a pair $(S, \Gamma)$ where $S$ is a $2^n$–dimensional complex Hermitian vector space and $\Gamma : \mathbb{R}^{2n} \to \text{End}(S)$ is a linear map which satisfies

$$\Gamma(v)^* + \Gamma(v) = 0, \quad \Gamma(v)^*\Gamma(v) = |v|^21$$

for every $v \in \mathbb{R}^{2n}$.

The $2^n$-dimensional complex vector space $S$ is called spinor space over $\mathbb{R}^{2n}$.

From the universal property of the complex Clifford algebra $\mathbb{C}l_{2n}$ the map $\Gamma$ can be extended to an algebra isomorphism $\Gamma : \mathbb{C}l_{2n} \to \text{End}(S)$ which satisfies $\Gamma(\tilde{x}) = \tilde{x}$ for every $\tilde{x} \in \mathbb{C}l_{2n}$.