PROPERTIES OF BIHARMONIC SUBMANIFOLDS IN SPHERES

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Abstract. In the present paper we survey the most recent classification results for proper biharmonic submanifolds in unit Euclidean spheres. We also obtain some new results concerning geometric properties of proper biharmonic constant mean curvature submanifolds in spheres.

1. Introduction

Biharmonic maps \( \phi : (M, g) \to (N, h) \) between Riemannian manifolds are critical points of the bienergy functional

\[
E_2(\phi) = \frac{1}{2} \int_M \| \tau(\phi) \|^2 v_g
\]

where \( \tau(\phi) = \text{tr} \nabla d\phi \) is the tension field of \( \phi \) that vanishes for harmonic maps (see [17]). The Euler-Lagrange equation corresponding to \( E_2 \) is given by the vanishing of the bitension field

\[
\tau_2(\phi) = -J^\phi(\tau(\phi)) = -\Delta \tau(\phi) - \text{tr} R^N(d\phi, \tau(\phi))d\phi
\]

where \( J^\phi \) is formally the Jacobi operator of \( \phi \) (see [24]). The operator \( J^\phi \) is linear, thus any harmonic map is biharmonic. We call proper biharmonic the non-harmonic biharmonic maps. Geometric and analytic properties of proper biharmonic maps were studied, for example, in [2, 25, 27].