# THE DISTRIBUTIONS OF VAN HIELE LEVELS OF GEOMETRIC THINKING AMONG 1ST THROUGH 6TH GRADERS

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## INTRODUCTION

Geometry is one of the most important topics in mathematics (Ministry of Education of Taiwan (MET), 1993, 2000, 2003; National Council of Teachers of Mathematics (NCTM), 1989, 1991, 1995, 2000). Geometry curriculum is developed and designed according to the van Hiele model of geometric thought in Taiwan (MET, 1993, 2000, 2003).

In 1957, the van Hiele model was developed by two Dutch mathematics educators, P. M. van Hiele, and his wife (van Hiele, 1957). Several studies have been conducted to discover the implications of the theory for current K-12 geometry curricula, and to validate aspects of the van Hiele model (Burger & Shaughnessy, 1986a; Eberle, 1989; Fuys, Geddes, & Tischler, 1988; Gutierrez, Jaime, & Fortuny, 1991; Ma & Wu, 2000; Mayberry, 1983; Molina, 1990; Senk, 1983; Pegg, 1985; Pegg & Davey, 1989, 1991; Usiskin, 1982; Wu, 1994, 1995, 2003; Wu & Ma, 2005a, 2005b). Besides the researches of Wu & Ma (2005a, 2005b), most of researchers focus on the geometry curricula of secondary school. To discover the implications of the van Hiele theory for elementary school students, however, it is also very important. The focus of this study is at the elementary level. This research report is one of the six sessions from the project "A Study of perceptual apprehensive, operative apprehensive, sequential apprehensive and discursive apprehensive for elementary school students (POSD)", funded by National Science Council of Taiwan (NSCTW, Grant No. NSC92-2521-S-142-004).

The main objectives of this study were to determine the distributions of van Hiele levels of geometric thinking among 1st through 6th graders.

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## THEORICAL FRAMEWORK

There are five levels of the van Hiele's geometric thought: "visual", "descriptive", "theoretical", "formal logic", and "the nature of logical laws" (van Hiele, 1986, p. 53). These five levels have two been labelled in two different ways: Level 1 through Level 5 or below level 1 through Level 4. Researchers have not yet come to a conclusion of which one to use. In this study, these five levels were called Level 1 through Level 5, and the focus of this study was on Level 1, visual.

At level 1, students learned the geometry through visualization. According to van Hiele (1986), "Figures are judged by their appearance. A child recognizes a rectangle by its form and a rectangle seems different to him than a square (p. 245)." At this first level students identify and operate on shapes (e.g., squares, triangles, etc.) and other geometric parts (e.g., lines, angles, grids, etc.) based on the appearance.

> [At the second level,] figures are bearers of their properties. That a figure is a rectangle means that it has four right angles, diagonals are equal, and opposite sides are equal. Figures are recognized by their properties. If one tells us that the figure drawn on a blackboard has four right angles, it is a rectangle even if the figure is drawn badly. But at this level properties are not yet ordered, so that a square is not necessarily identified as being a rectangle. (van Hiele, 1984b, p. 245)
>
> [At the third level,] Properties are ordered. They are deduced one from another: one property precedes or follows another property. At this level the intrinsic meaning of deduction is not understood by the students. The square is recognized as being a rectangle because at this level definitions of figure come into play. (van Hiele, 1984b, p. 245)

Mayberry (1983) designed an oral instrument to investigate pre-service teachers' levels of reasoning. She found (a) evidence supporting the hierarchy of the van Hiele levels, (b) students at different levels for different concepts, and (c) cases of students did not reach level 1 (visual) reasoning skills. The focuses of this study were the elementary school students and first three van Hiele levels, to explore what kind of the results comparing with Mayberry's research.

## METHODS AND PROCEDURES

### Participants

The participants were 5,581 elementary school students who were randomly selected from 25 elementary schools in 23 counties/cities in Taiwan. There were 2,717 girls and 2,864 boys. The numbers of participants, from 1$^{st}$ to 6$^{th}$ grades, were 910, 912, 917, 909, 920, 1,013 students, respectively.

### Instrument

The instrument used in this study, Wu's Geometry Test (WGT), was specifically designed for this project due to there were no suitable Chinese instruments available. This instrument was designed base on van Hiele level descriptors and sample responses identified by Fuys, Geddes, and Tischler (1988). There were 25 multiple-

choice questions of the first van Hiele level; 20 in the second and 25 in the third. The test is focus on three basic geometric figures: triangle, quadrilateral and circle.

The scoring criteria were based on the van Hiele Geometry Test (VHG), developed by Usiskin (1982), in the project "van Hiele Levels and Achievement in Secondary School Geometry" (CDASSG Project). In the VHG test, each level has five questions. If the student answers three, four, or five the first level questions correctly, he/she has reached the first level. If the students (a) answered three questions or more correctly from the second level; (b) met the criteria of the first level; and (c) did not correctly answer three or more questions, from levels 3, 4, and 5, they were classified as in second level. Therefore, using the same criteria set by Usiskin (1982), the passing rate of this study was set at 60%. If the scores of the students did not follow the criteria, the cases were labelled "jump phenomenon" by the authors.

## Validity and Reliability of the Instrument

The attempt to validate the instrument (WGT) involved the critiques of a validating team. The members of this team included elementary school teachers, graduate students majored in mathematics education, and professors from Mathematics Education Departments at several pre-service teacher preparation institutes. The team members were given this instrument, and provide feedback regarding whether each test item was suitable. They also gave suggestions about how to make this test better.

In order to measure the reliability of the WGT, 289 elementary school students ( from grades 1-6) were selected to take the WGT. These students were not participants in this study. The alpha reliability coefficient of the first van Hiele level of WGT was .67 ($p < .001$), 0.88 ( $p < .01$ ) level 2, and 0.94 ( $p < .01$ ) level 3, using SPSS® for Windows® Version 10.0.

## Procedure

The one-time WGT was given to 1st to 6th graders during April 2004. The class teachers of the participants administered the test in one mathematics class. The answers were graded by the project directors.

The distribution of the questions is in Table 1.

| | Number of the questions |
|---|---|
| van Hiele level 1 | Q 1 to Q25 |
| van Hiele level 2 | Q26 to Q45 |
| van Hiele level 3 | Q46 to Q70 |

Table 1: The distribution of questions in level 1 to 3

## RESULTS

### Overall performance on basic figures

Based on the questions of triangle, 43.0% of the elementary school students were at van Hiele level 1, 28.0% at level 2, and 5.2% at Level 3. The students who were at level 1 of questions of quadrilateral were 25.9%, 28.0% at level 2, and 5.5% at level 3. For the questions of circle, 35.7% of the elementary school students were at van Hiele level 1, 45.5% at level 2, and 7.7% at level 3 (See Table 2).

The percentage of students did NOT meet the criteria of level 1 (below level 1), for the triangle were 20.8%, 30.3% for quadrilateral, and 7.7% for circle. It seemed that the circle concept is the easiest one for students, followed by triangle concept, and quadrilateral concept. It was worth to mention that the percentage of students appeared "jump phenomenon", for the triangle were 2.9%, 10.3% for quadrilateral, and 3.3% for circle.

| | Triangle | | Quadrilateral | | Circle | |
|---|---|---|---|---|---|---|
| | Frequency | Percent | Frequency | Percent | Frequency | Percent |
| Below level 1 | 1163 | 20.8 | 1691 | 30.3 | 431 | 7.7 |
| Level 1 | 2402 | 43.0 | 1447 | 25.9 | 1995 | 35.7 |
| Level 2 | 1561 | 28.0 | 1563 | 28.0 | 2538 | 45.5 |
| Level 3 | 293 | 5.2 | 305 | 5.5 | 432 | 7.7 |
| Jump | 162 | 2.9 | 575 | 10.3 | 185 | 3.3 |
| Total | 5581 | 100.0 | 5581 | 100.0 | 5581 | 100.0 |

Table 2: The overall distributions of levels 1 to 3

### The distributions of van Hiele levels of Triangle Concepts

| | | | Triangle | | | | Total |
|---|---|---|---|---|---|---|---|
| | | | Below level 1 | Level 1 | Level 2 | Level 3 | |
| Grade | 1 | Count | 472 | 438 | 0 | 0 | 910 |
| | | % within Grade | 51.9% | 48.1% | .0% | .0% | 100.0% |
| | 2 | Count | 341 | 571 | 0 | 0 | 912 |
| | | % within Grade | 37.4% | 62.6% | .0% | .0% | 100.0% |
| | 3 | Count | 151 | 494 | 239 | 0 | 884 |
| | | % within Grade | 17.1% | 55.9% | 27.0% | .0% | 100.0% |
| | 4 | Count | 78 | 448 | 359 | 0 | 885 |
| | | % within Grade | 8.8% | 50.6% | 40.6% | .0% | 100.0% |
| | 5 | Count | 68 | 262 | 449 | 96 | 875 |
| | | % within Grade | 7.8% | 29.9% | 51.3% | 11.0% | 100.0% |
| | 6 | Count | 53 | 189 | 514 | 197 | 953 |
| | | % within Grade | 5.6% | 19.8% | 53.9% | 20.7% | 100.0% |
| Total | | Count | 1163 | 2402 | 1561 | 293 | 5419 |
| | | % within Grade | 21.5% | 44.3% | 28.8% | 5.4% | 100.0% |

Table 3: The percentage analyzed by grades and levels based on triangle

The percentage of students appeared "jump phenomenon", for the triangle were 2.9% (See Table2). Thus, there were 5,419 (97.1%) students who could be assigned to levels 1 to 3. The distributions of van Hiele level of triangle concepts from grades 1 to 6 of each figure were shown as Table 3.

Based on the questions of triangle, 48.1% of the grades 1 were at van Hiele level 1, 62.6% grade 2. The grades 3 were at level 1 of the triangle concept were 55.9%, 27.0% at level 2. The grades 4 were at level 1 of the triangle concept were 50.6%, 40.6% at level 2. The grades 5 were at level 1 of the triangle concept were 29.9%, 51.3% at level 2, and 11.0% at level 3. The grades 6 who were assigned at level 1 of the triangle concept were 19.8%, 53.9% at level 2, and 20.7% at level 3 (See Table 3).

**The distributions of van Hiele levels of Quadrilateral Concepts**

The percentage of students appeared "jump phenomenon", for the quadrilateral were 10.3% (See Table2). Thus, there were 5,006 (89.7%) students who could be assigned to levels 1 to 3. The distributions of van Hiele level of quadrilateral concepts from grades 1 to 6 of each figure were shown as Table 4.

| | | | Quadrilateral | | | | Total |
|---|---|---|---|---|---|---|---|
| | | | Below level 1 | Level 1 | Level 2 | Level 3 | |
| Grade | 1 | Count | 627 | 283 | 0 | 0 | 910 |
| | | % within Grade | 68.9% | 31.1% | .0% | .0% | 100.0% |
| | 2 | Count | 488 | 424 | 0 | 0 | 912 |
| | | % within Grade | 53.5% | 46.5% | .0% | .0% | 100.0% |
| | 3 | Count | 288 | 330 | 225 | 0 | 843 |
| | | % within Grade | 34.2% | 39.1% | 26.7% | .0% | 100.0% |
| | 4 | Count | 127 | 263 | 443 | 0 | 833 |
| | | % within Grade | 15.2% | 31.6% | 53.2% | .0% | 100.0% |
| | 5 | Count | 86 | 84 | 441 | 113 | 724 |
| | | % within Grade | 11.9% | 11.6% | 60.9% | 15.6% | 100.0% |
| | 6 | Count | 75 | 63 | 454 | 192 | 784 |
| | | % within Grade | 9.6% | 8.0% | 57.9% | 24.5% | 100.0% |
| Total | | Count | 1691 | 1447 | 1563 | 305 | 5006 |
| | | % within Grade | 33.8% | 28.9% | 31.2% | 6.1% | 100.0% |

Table 4: The percentage analyzed by grades and levels based on quadrilateral

Based on the questions of quadrilateral, 31.1% of the grades 1 were at van Hiele level 1, 46.5% grade 2. The grades 3 were at level 1 of the quadrilateral concept were 39.1%, 26.7% at level 2. The grades 4 were at level 1 of the quadrilateral concept were 31.6%, 53.2% at level 2. The grades 5 were at level 1 of the quadrilateral concept were 5.8%, 28.2% at level 2, and 37.0% at level 3. The grades 6 who were assigned at level 1 of the quadrilateral concept were 8.0%, 57.9% at level 2, and 24.5% at level 3 (See Table 4).

**The distributions of van Hiele levels of Circle Concepts**

The percentage of students appeared "jump phenomenon", for the circle were 3.3% (See Table2). Thus, there were 5,396 (96.7%) students who were at levels 1 through 3. The distributions of van Hiele level of circle concepts from grades 1 to 6 of each figure were shown as Table 5.

| | | | Circle | | | | Total |
|---|---|---|---|---|---|---|---|
| | | | Below level 1 | Level 1 | Level 2 | Level 3 | |
| Grade | 1 | Count | 220 | 690 | 0 | 0 | 910 |
| | | % within Grade | 24.2% | 75.8% | .0% | .0% | 100.0% |
| | 2 | Count | 97 | 815 | 0 | 0 | 912 |
| | | % within Grade | 10.6% | 89.4% | .0% | .0% | 100.0% |
| | 3 | Count | 58 | 285 | 546 | 1 | 890 |
| | | % within Grade | 6.5% | 32.0% | 61.3% | .1% | 100.0% |
| | 4 | Count | 20 | 132 | 717 | 2 | 871 |
| | | % within Grade | 2.3% | 15.2% | 82.3% | .2% | 100.0% |
| | 5 | Count | 22 | 35 | 644 | 159 | 860 |
| | | % within Grade | 2.6% | 4.1% | 74.9% | 18.5% | 100.0% |
| | 6 | Count | 14 | 38 | 631 | 270 | 953 |
| | | % within Grade | 1.5% | 4.0% | 66.2% | 28.3% | 100.0% |
| Total | | Count | 431 | 1995 | 2538 | 432 | 5396 |
| | | % within Grade | 8.0% | 37.0% | 47.0% | 8.0% | 100.0% |

Table 5: The percentage analyzed by grades and levels based on cycle

Based on the questions of circle, 75.8% of the grades 1 were at van Hiele level 1, 89.4% grade 2. The grades 3 were at level 1 of the circle concept were 32.0%, 61.3% at level 2, and 0.1% at level 3. The grades 4 were at level 1 of the circle concept were 15.2%, 82.3% at level 2, and 0.2% at level 3. The grades 5 were at level 1 of the circle concept were 4.1%, 74.9% at level 2, and 18.5% at level 3. The grades 6 who were assigned at level 1 of the circle concept were 4.0%, 66.2% at level 2, and 28.3% at level 3 (See Table 5).

## CONCLUSION:

More than half of (up to 51.9%) graders 1 did NOT met the criteria of the first level (below level 1) based on the triangle, 68.9% on quadrilateral, and 24.2% on circle. It seems that the circular concept is the easiest for students; on the other hand, the concept of quadrilateral is the most difficult to students. This result consisted with the research of Wu & Ma (2005a).

The results of this study found that the higher grades get the higher van Hiele levels. Based on the questions of triangle and quadrilateral, no students of graders 1 to 4 were at level 3 and no grades 1 to 2 were at level 2. Based on these three basic figures (triangle, quadrilateral, circle), most of grades 1 to 2 were at level 1, and grades 3 to 6 were at level 2. Only grade 5 and 6 could meet the level 3.

Comparing with Mayberry's (1983) research, this study found, for elementary school students, (a) evidence supporting the hierarchy of the van Hiele levels, (b) students at different levels for different concepts of basic figures, and (c) cases of students did not reach level 1 of basic figures.

The results of this study identified the easiest and the most difficult concepts of basic figures for students, it is important to investigate the reason(s) behind this result. The authors of this study are interested to investigate why elementary students have difficulties in quadrilateral. One reason might be that quadrilaterals, except squares

and rectangle, are rarely shown in the textbook of grade 1, and in their daily lives. Researchers might consider this as their research interests.

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