ABSTRACTION, SCAFFOLDING AND EMERGENT GOALS
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This paper reports on the relation between the construction of mathematical knowledge and scaffolded discourse. We work within an operational model of ‘abstraction in context’ which views abstraction as a vertical reorganisation of previously constructed knowledge into new knowledge. We extend this model by considering human mediation, the functions of scaffolding interventions and emergent goals. We exemplify our arguments by considering verbal data from two students engaged in tasks concerned with the graphs of the absolute value of linear functions and discuss interrelations between human mediation, scaffolding interventions and emergent goals.

INTRODUCTION

The term ‘abstraction’ has been largely influenced by empiricist accounts which treat abstraction as decontextualised higher-order knowledge involving generalisations achieved through the recognition of commonalties isolated in a large number of particular instances (see Ohlsson and Lehtinen (1997) for a critique). Many, of late, have found this view wanting and proposed alternative accounts which call attention to the importance of social and contextual factors (Noss & Hoyles, 1996; van Oers, 2001; Hershkowitz, Schwarz & Dreyfus, 2001). We focus on Hershkowitz et al.’s (ibid.) account (referred to as HSD hereafter) as it offers an operational model for an empirical investigation of the abstraction process.

HSD view abstraction as a vertical reorganisation of previously constructed mathematical knowledge structures into the new ones. Such reorganisation occurs in an activity through three epistemic actions: recognising, building-with and constructing. They argue that abstractions develop through three stages: (i) the need for a new structure, (ii) the construction of new structures by means of three epistemic actions and (iii) the consolidation of the newly constructed structures. HSD provide empirical evidence regarding the stages (i) and (ii) but merely assumes the importance of stage (iii). HSD call for further investigation into the validity of their model. To this aim we designed a study to subject HSD’s account of abstraction to empirical scrutiny and our findings extended this model in several dimensions.

Monaghan & Ozmantar (in press) make a small but important refinement to the HSD model of abstraction by viewing an abstraction as a consolidated construction that can be used to create new constructions. In this paper we focus on the construction stage and consider human mediation, scaffolding interventions and emergent goals. We briefly describe the study, present student verbal data with a commentary and discuss interrelationships.
THE STUDY

The main study, of which this paper reports one aspect, set out to investigate the validity of HSD’s model with a particular focus on scaffolding and social interaction. Our focus on scaffolding stemmed from a realisation that interventions, from a knowledgeable agent (e.g. interviewer) providing students with purposeful help and regulate them towards the achievement of mathematical abstraction, are important (and often uncommented on) aspects of many studies including HSD’s (e.g. van Oers, 2001). The metaphor of scaffolding, coined by Wood et al. (1976), refers to sensitive and supportive interventions given to learners to achieve a particular level of competence not readily available to the learners’ unassisted efforts. Such interventions require a tutor’s actions to be ‘contingent’ (Wood, 1991) in supporting learners through cycles of monitoring and analysing their performance in relation to task’s demands and then assisting them depending on their progress (Scott, 1998).

In the main study, we employed 20 Turkish (aged 17-18) students who worked on tasks concerned with the absolute value of linear functions. Students were selected from 134 on the basis of a diagnostic test. This test was designed to identify students who had the necessary knowledge to tackle the tasks but were not acquainted with the content. Of the students, 14 worked in pairs and 6 worked individually. Four pairs and three individuals were scaffolded in their work and the rest were not. All students worked on four tasks on four successive days without time limitation. Tasks 1, 2 and 4 were designed to allow students to construct a method(s) to sketch the graphs of, respectively, \( f(x), f(|x|) \) and \( f(|x|) \), given the linear graph of \( f(x) \). Task 3 was designed to give students the opportunity to consolidate their constructions in task 1 and 2.

We present the verbal data of two students (H&S) working with a tutor/interviewer (the first author) who aimed to scaffold H&S’s work through a range of interventions, from asking for explanations to giving feedback, explanations and directions, if needed. H&S worked on task 4 which involved five open questions. Question 1 (Q1) asked to sketch the graph of \( f(x) = |(|x|-4)| \) and report on the patterns. Students were then asked, Q2, to compare the graphs of \( f(x) = x-4 \) and \( f(x) = |(|x|-4)| \). Q3 presented graph of \( f(x) = x+3 \) and asked to use it in sketching the graph of \( |f(|x|)| \). In Q4 four linear functions without equations were presented and students were asked to sketch the graphs of \( |f(|x|)| \) for each of these. Q5 asked students to explain how to obtain the graph of \( |f(|x|)| \) from the graph of an arbitrary linear function \( f(x) \).

VERBAL DATA

For the first two questions H&S substituted values for \( x \) to accurately draw the graphs of \( f(x) \) and \( |f(|x|)| \). They recognised symmetries in the W-shaped graph of \( |f(|x|)| \) for Q1 and commented on similarities and differences between the graphs of \( f(x) \) and \( |f(|x|)| \). They moved on to question 3 and again substituted values for \( x \) to accurately draw the graphs of \( f(x) \) and \( |f(|x|)| \), which was V-shaped. H&S then compared the two absolute valued graphs obtained in Q1 and Q3 in relation to the original linear graphs of \( f(x) \) by focusing on specific line segments, rays and symmetries:
133H: Look I think the first part \([f(x)\) at \(x>0\)] always remains the same… oh does it?
134S: Yes
135H: But in the first question there is \(\ldots\) a line segment
136S: This graph is also symmetric in the \(y\)-axis. But I don’t know how it helps us!
137H: We know that the part of \(f(x)\) over the \(x\)-axis remains the same, right?
138S: Yes \(\ldots\) and also \(\ldots\) they are taken symmetrically in the \(x\)-axis.
139H: But wait! \(\ldots\) it [graph of \(|f(x)|\) for question 3] doesn’t obey this rule…
140S: Yeah I know, there was a line segment in the first graph
141H: I don’t think we can ever understand how to use \(f(x)\) to draw the graph of \(|f(x)|\).
142S: The first graph was something like W-shaped… but this graph is V-shaped.
143H: They are totally different! How can we speak in a general way? Even this question made things worse rather than helping us.
144S: We’d better stick to substituting… we can answer the next question by substituting.

Until this point the interviewer intentionally limited his assistance in order to observe how far H&S could progress on their own. The interviewer (having monitored that H&S had tried, and had given upon, to develop a method to sketch the graphs of \(|f(x)|\) and believing that they were losing confidence in their abilities) intervened and suggested that they return to the first question. He brought their earlier constructions of \(|f(x)|\) and \(f(|x|)\) to their attention and suggested that they keep these in mind.

165I: if you pay a closer attention to the equation… I mean look at the expression itself, \(|f(|x|)|\), it is a combination of these two \([|f(x)|\) and \(f(|x|)|\). Do you see that?
166H: Yes, that’s right \(\ldots\)
167S: Yeah, this \(|f(|x|)|\) is a combination of \(f(|x|)\) and \(|f(x)|\) \(\ldots\)
168I: Ok, let’s think about it and consider what you know. How can we use our knowledge to obtain this graph [of \(|f(x)||]?\)
169S: Look it makes sense now \(\ldots\)
170H: Yeah, I think it makes sense! If \(|f(|x|)|\) is a combination of \(f(|x|)\) and \(|f(x)|\), can we think about it like a computation with parentheses?
171I: Computation with parentheses?
172H: I mean for example when we are doing computations with some parentheses like… let’s say for example, \((7-(4+2))\), then we follow a certain order…
173S: Right, I understood what you mean… we need to first deal with the parenthesis inside of the expression, is that what you mean?
174H: Yeah, I think it is somehow similar, I can sense it but I am unable to clarify…
175S: I know what you mean but how could we determine the parenthesis in here?
176I: You both made an excellent point. OK, let’s think about it together! In the expression of \(|f(|x|)|\), can we think about the absolute value sign at the outside of the whole expression as larger parenthesis, which includes another one just inside.
Following 165I intervention, H proposed an analogy with arithmetic in relation to the expression of \(|f(|x|)|\). But H&S were unclear as to how to “determine the parenthesis” (175S), for which the interviewer (176I) gave an explanation to which H reacted:

177H: Aha, I got it… I know what we will do.
178I: Could you please tell us?
179H: We can consider \(f(|x|)\) as if it was the smaller parenthesis!
180I: Smaller parenthesis?
181H: I mean it should be the first thing that we need to deal with
182S: Yeah, I agree… I think we should begin with the graph of \(f(|x|)\) and first draw it
183H: But what next?
184S: Then we can use the absolute value at the outside… in the similar way of doing computations.
185H: But we will be drawing graphs! Can we really do this?
186S: I am not too sure if we can… but it sounds plausible…
187I: What you are doing here is not computation (…) but you are making an analogy (…) and I see no problem with that… let’s draw the graph by considering what we’ve just talked about and then decide if it will work or not, huh?

In the above excerpts, H&S planned how to use the structures of \(|f(x)|\) and \(f(|x|)\) in sketching the graph of \(|f(|x|)|\). The interviewer encouraged (187I) H&S to use these ideas in sketching the target graph, which they later successfully did, in two steps, through the successive application of their earlier constructions of \(f(|x|)\) and \(|f(x)|\) to the given graph of \(f(x)\). By doing so, H&S were enriched with a new method to view the graphs of \(|f(|x|)|\), which we call the ‘two-step method’ that H explained as follows:

244H: when drawing \(f(|x|)\), part of \(f(x)\) at the positive \(x\) remains unchanged… then this part is taken symmetry in the \(y\)-axis and err and also part of \(f(x)\) at the negative \(x\) is cancelled. After that, we apply absolute value to this graph, and for this… negative values of \(y\) are taken symmetry in the \(x\)-axis and thus we obtain the graph of \(|f(|x|)|\).

DISCUSSION

It is clear from the excerpts that H&S constructed a new method unavailable to them before and that the interviewer assisted H&S in their construction. Closer inspection of student-interviewer interaction suggested that we focus further attention on three particular issues: human mediation, functions of interviewer interventions and emergent goals. We discuss these issues below under discrete headings but point out that they are interrelated. These considerations, we believe, extend the analytic power of the HSD model of abstraction with particular regard to the construction stage.

Human mediation

Vygotsky (1981) proposed that higher mental processes and human actions in general are mediated by technical and psychological tools and by other humans: “it is through the mediation of others…that the child undertakes activities. Absolutely everything in the behaviour of the child is merged and rooted in social relations” (cited in Ivic,
We take it as given that the interviewer’s interventions mediated H&S’s construction of the ‘two-step method’; he acted as a knowledge artefact which the students made essential use of to produce their construction. H’s act of recognition, for example, “$|f(|x|)|$ is a combination of $f(|x|)$ and $f(|x|)$” (170H) was interviewer-mediated: it followed the interviewer’s prompt (165I) after which she exclaimed “it makes sense!” (170H). Here H’s utterance is not a simple repetition of the interviewer’s utterance of 165 as she used this in connecting the expression of $|f(|x|)|$ with computational precedence (building-with) and even gave an example (172H). Thus, in H’s utterance, not only is the act of recognising but also the resulting building-with is mediated by the interviewer’s intervention in 165 and 168.

But what effect did this mediation have on H&S’s developing construction? One could argue that the interventions ‘facilitated’ H&S’s mathematical actions. However, our analysis suggests that these interventions brought about crucial transformations in H&S’s ways of seeing, talking and acting which went far beyond ‘mere’ facilitation. When H&S failed to develop a ‘better’ method than substitution (143H&144S), the interviewer intervened and brought the structures of $|f(|x|)|$ and $f(|x|)$ to the focus of their attention. Following the interviewer’s suggestion of considering $|f(|x|)|$ as a combination of $f(|x|)$ and $f(|x|)$ (165I), a transformation is apparent in the students’ seeing (seeing “precedence of operations” in the expression $|f(|x|)|$; see 170H-175S), talking (talking about the graphs of $f(|x|)$ and $f(|x|)$ in $|f(|x|)|$; see 177H-186S) and acting (merging the graphs of $f(|x|)$ and $f(|x|)$ into a single graph; see 244H). The importance of these transformations resulting from the interviewer’s mediation can be better appreciated when we compare H&S’s earlier considerations of the graphs until 144 where they merely focused on the ostensible features of the graphs such as “line segments”, “parts” and “symmetries” (133H-144S) which did not lead H&S to construct a new method and in fact they eventually declared their intention to give up developing a method other than substitution (143S-144H).

But what functions did the interventions serve in leading to these transformations? We attend to this question in next section.

**Functions of the interviewer interventions**

We focus on three functions of interventions that appear important in explaining these transformations: reducing uncertainty, direction of attention and regulation.

In the protocol excerpts, reducing the students’ uncertainty appears to be a crucial function of the interventions. During the construction process uncertainty seems to be inevitable as construction requires not only that students recognise and use available knowledge structures but also that they reorganise them, put them together and forge new connections amongst them. Furthermore, all of these actions need to be carried out in an ‘unfamiliar situation’ which increases learner uncertainty (see Wood, 1991). Indeed construction is the process through which students become familiar with the new structure, which presupposes students’ unfamiliarity with the to-be-constructed structure before construction. Students have no clear picture of the construction to be
formed (for otherwise it would already be constructed) so they confront uncertainty, albeit at varying degrees, when striving to construct something unfamiliar to them.

We can observe the influence of the interviewer interventions in the reduction of the students’ uncertainty during their progression towards the target construction. H&S’s uncertainty about the aptness of their proposals and explanations appeared during this task. For instance, following their suggested analogy to computational precedence (170H-174H), they were uncertain as to how to “determine the parenthesis” in the expression of $|f(|x|)|$ (175S). They also expressed their uncertainty as to the aptness of approaching the graphs of $|f(|x|)|$ through the successive application of $f(|x|)$ and $|f(x)|$ (185H&186S). The interviewer played a crucial role in handling H&S’s uncertainty when he intervened, for example in 187I, to give positive feedback (“I see no problem”), specified a target (“let’s draw the graph … and then decide”) and helped H&S to continue their work, which led them to construct the two-step method.

The second function of the interventions was directing the students’ attentions and efforts. The management of attention in collaborative learning environments is critically important during new learning (Barron, 2003; van Oers, 2001). Mason & Spence (1999) attribute a pivotal role to shifts in one’s attention in doing and learning mathematics and they argue that:

… coming to know is essentially a matter of shifts in the structure of attention, in what is attended to, in what is stressed and what consequently ignored with what connections … Knowing is not a simple matter of accumulation … [but] rather a state of awareness, of preparedness to see in the moment (p.151)

However, if students are not aware of the importance and necessity of the knowledge artefacts at their disposal, they are unlikely to make use of them as they (or their attention) are ‘blocked’. This was the case at times for H&S, e.g. when they initially focused on specific “line segments”, “parts” and “symmetries” (see 133H-140S), they failed to recognise the connection between their knowledge of $f(x)$ and $f(|x|)$ and a construction of $|f(|x|)|$. It is with this ‘connection’ that the interviewer’s interventions to direct H&S’s attention are particularly important. In H&S’s work, the interviewer first brought $|f(x)|$ and $f(|x|)$ to their attention and helped them recall what they knew about these functions. Later he drew H&S’s attention to the expression of $|f(|x|)|$ and suggested viewing this as a combination of $f(|x|)$ and $|f(x)|$ (165I). Only after ensuring that $f(x)$ and $f(|x|)$ were the focus of the students’ attention (166H&167S) did he invite them to work out an idea as to how to use $|f(x)|$ and $f(|x|)$ to obtain $|f(|x|)|$ (168I).

The interventions also had a regulative function which often took the form of setting goals through, mainly, direct requests, inviting H&S to focus on certain aspects of the task e.g. “let’s draw the graph” (187I) and “how can we use our knowledge to obtain this graph” (168I). The goals were important in focusing the student’s attention and in reducing their uncertainty; it was, to a large extent, through their efforts to satisfy these goals that H&S moved closer to the target construction. These goals were not predetermined but ‘emerged’ in the course of interaction. We are convinced of the importance of such emergent goals in scaffolded discourse. These goals are
dialectically shaped by the interviewer’s understanding of the students’ development at certain stages in the activity and the students’ understanding of the interventions in the context of a particular task. We attend to this issue next.

**Emergent goals in scaffolded discourse during the construction**

HSD’s model of abstraction is an activity theoretic model. Leont’ev’s (1981) exposition of activity theory argues that the main goal of an activity is realised by an aggregate of actions subordinated to partial goals which can be distinguished from, yet are constitutive to, the main goal. Saxe (1991), which is activity theoretic in all but name, considers practice-linked emergent goals, little and often unconscious goals which come into being and fade away. Saxe’s goals are not static constructions but rather are “emergent phenomena shifting and taking new forms as individuals use their knowledge and skills alone and in interaction with others to organise their immediate contexts” (ibid., p.17). Our use of the term ‘emergent goals’ has similarities to Leont’ev’s partial goals and Saxe’s emergent goals: they are emergent goals for the interviewer but are partial goals for the students.

Ozmantar (2004) argues that emergent goals in scaffolded discourse are contingent upon dialectically interrelated parameters: the task, the interviewer’s interventions, the students’ interpretations and prior emergent goals. Viewing emergent goals in relation to the construction of knowledge in scaffolded discourse is a complex matter. The complexity stems, to a considerable extent, from the differences in the participants’ (i.e. interviewer and students) understandings: the interviewer has a clear vision of the target construction and the possible ways to achieve this but the students do not. This affects the way in which the participants interpret the task and the main goal of the activity. For example, the main ‘goal of the task’ was for the students to construct a method to sketch the graph of \(|f(|x|)|\), given the graph of \(f(x)\). This was the goal of the interviewer but it was not necessarily seen and interpreted in the same way by the students. When H&S encountered difficulties in developing a method at the end of Q3, they decided to “stick to substitution” which they could use to sketch the graphs in Q4 (144S). This suggests that H&S’s goal was to answer the questions and complete the task, not to develop a general method. The emergent goals we speak of in this scaffolded task arose from the motives of the interviewer, to coordinate the students’ partial goals with his interpretation of the main goal of the task.

Emergent goals in scaffolded discourse belong to the agent in focus; the emergent goals of the interviewer generate emergent goals for the student(s). Consider some of the interviewer’s emergent goals: to draw H&S’s attention to \(|f(|x|)|\) as a combination of \(|f(|x|)|\) and \(|f(|x|)|\) (165l&168I) and to understand H’s analogy (171I). Corresponding student emergent goals are: to make sense of \(|f(|x|)|\) as a combination of \(|f(|x|)|\) and \(|f(|x|)|\) (166H, 167S &170H) and to explain the analogy of computational precedence.

Important questions arise: How are the differences in the structure of emergent goals reconciled in the discourse? To what extent should the emergent goals of the different parties be compatible for the interventions to be fruitful? Are the differences obstacles or essential dynamics of the discourse? We do not have immediate answers
to these questions. It is clear, however, that H&S constructed new knowledge (the main goal of the activity) through the fulfilment of a series of emergent goals which are distinguishable from, yet subordinated to, the main goal itself. It is also clear from the protocol excerpts that H&S achieved the construction of the two-step method through their efforts to realise these emergent goals.

References


