

# ENHANCING MATHEMATICAL LITERACY WITH THE USE OF METACOGNITIVE GUIDANCE IN FORUM DISCUSSION

*Bracha Kramarski*

*Nava Mizrachi*

School of Education  
Bar-Ilan University  
Ramat-Gan 52900  
Israel

*The purpose of the present study was to investigate the effects of forum discussion embedded within metacognitive guidance on mathematical literacy. In particular the study compares two learning environments: (a) Forum discussion with metacognitive guidance (FORUM+META); and (b) Forum discussion without metacognitive guidance (FORUM). Participants were 43 seventh-grade students (boys and girls) who practiced online problem solving in two classes. It was found that students who were exposed to FORUM+META discussion outperformed students that were not exposed to metacognitive guidance (FORUM discussion) on mathematical literacy. The effects were observed on various aspects of solving real-life tasks: (a) Understanding the task; (b) Using mathematical strategies; (c) Processing information; and (d) Using mathematical reasoning.*

“Mathematical literacy is defined in PISA as the capacity to identify, understand and engage in mathematics as well as to make well-founded judgments about the role that mathematics plays in an individual’s current and future life as a constructive, concerned and reflective citizen” (OECD/PISA, 2003, p. 20). The definition revolves around the wider uses of mathematics in people’s lives and is not limited to mechanical operations. “Mathematical literacy” is used here to indicate the ability to put mathematical knowledge and skills to use rather than just mastering them within a school curriculum. To “engage in” mathematics covers not just simple calculation (such as deciding how much change to give someone in a shop) but also wider uses, including taking a point of view and appreciating things expressed numerically (such as having an opinion about a government’s spending plan). Mathematical literacy is assessed by giving students tasks based on situations which represent the kind of problems encountered in real-life.

From a motivational perspective, such tasks are challenging tasks that are relevant to the students’ world and daily life (OECD/PISA, 2003). Although real-life tasks are important, little is known at present on how to enhance students’ ability to solve such

tasks. The question: What characteristics should a learning environment have to facilitate the construction of students' mathematical literacy, merits further research.

E-learning is currently one of the popular environments of training and learning. In particular, participating in forum discussion has been touted as offering immense potential for improving the effectiveness of learning. Forum discussion allows asynchronous exchanges. It also permits one-to-one as well as one-to-many interactions. The learners are motivated, they are able to learn independently, and they can transfer and apply the knowledge to real-life situations (e.g., Deaudelin & Richer, 1999). However, this technology, as has been the case with prior technologies, raises the question about the pedagogical way of how to use that technology.

A review of pedagogical approaches shows that the majority put emphasize on the development of Self-Regulated approaches to learning (e.g., Butler & Winne, 1995; Mevarech & Kramarski, 1997; Kramarski & Mevarech, 2003; OECD/PISA, 2003). PISA describes Self Regulated Learning (SRL) as a style of activities for problem solving that includes: Evaluating goals, thinking of strategies and choosing the most appropriate strategy for solving the problem. The method of Mevarech & Kramarski, (1997), called **IMPROVE** emphasizes the importance of providing each student with the opportunity to construct mathematical meaning by involving him or her in self questioning that focus on: (a) *comprehending* the problem (e.g., "What is the problem all about?"); (b) constructing *connections* between previous and new knowledge (e.g., "What are the similarities/differences between the problem at hand the problems you have solved in the past? and why?"); (c) use of *strategies* appropriate for solving the problem (e.g., "What are the strategies/tactics/principles appropriate for solving the problem and why?"; and (d) *reflecting* on the processes and the solution (e.g., "What did I do wrong here?"; "Does the solution make sense?").

Generally speaking, researchers reported positive effects of metacognitive guidance on students' mathematical reasoning in cooperative learning (e.g., Schoenfeld, 1992; Mevarech & Kramarski, 1997; Kramarski & Mevarech, 2003) and in different computerized environments (e.g., Kramarski & Ritkof, 2002). Most of that studies examined the effects of metacognitive guidance on solving mathematical problems based on school curriculum and less on tasks based on situations which represent the kinds of problem encountered in real-life. Given these studies, the purpose of the present study was to investigate the effects of forum discussion embedded within metacognitive guidance on mathematical literacy. In particular the study compares two learning environments: (a) Forum discussion with metacognitive guidance (FORUM+META); and (b) Forum discussion without metacognitive guidance (FORUM). The effects will be observed on various aspects of solving a real-life task.

## Method

Participants were 43 *seventh-grade* students (boys and girls, mean age 13.4) who practiced problem solving in two classes. One class (n=20) was exposed to forum discussion embedded with metacognitive guidance (FORUM+ META); and the other class (n=23) were exposed to forum discussion without metacognitive guidance (FORUM).

### Treatments

Students in the two conditions practiced for six weeks (90 min. a week) of problem solving with the solution of three real-life tasks. The study was implemented in pairs. Students were encouraged to think about the task, explain it to each other, and approach it from different perspectives.

Since each treatment was composed of two components: The use of forum discussion with or without metacognitive guidance. We first describe each component separately, and then how the components were combined.

### Forum discussion

The students that were exposed to forum discussion practiced problem solving of real-life tasks once a week in the computer lab (90 min).

The teacher did not interfere in the discussion, she encouraged the students to participate in the discussion, to send assignments each other, to reflect on the solutions and to submit questions regarding the solution process. The students were also encouraged to ask their friends for help when they encountered difficulties in understanding and correcting the solution, if needed. In addition, the students were asked to send the final solution to the teacher, in the forum or as an attachment file using word or excel.

### Metacognitive guidance

The metacognitive guidance was based on the IMPROVE technique suggested by Mevarech & Kramarski (1997). The method utilized a series of four self-addressed metacognitive questions.

The *comprehension questions* were designed to prompt students to reflect on the problem/task before solving it. In addressing a comprehension question, students had to read the problem/task aloud, describe the task in their own words, and try to understand what the task/concepts mean. The *comprehension questions* included questions such as: “What is the problem/task all about?”; “What is the question?”; “What is the meaning of the mathematical concepts?”

The *connection questions* were designed to prompt students to focus on similarities and differences between the problem/task they work on and the problem/task or set of problems/tasks that they had already solved. For example: “How is this problem/task different from/similar to what you have already solved? Explain why”.

The *strategic questions* were designed to prompt students to consider which strategies are appropriate for solving the given problem/task and for what reasons. In addressing the strategic questions, students had to describe the *what* (e.g., "What strategy/tactic/principle can be used in order to solve the problem/task?") the *why* (e.g., "Why is this strategy/tactic/principle most appropriate for solving the problem/task?") and *how* (e.g., "How can I organize the information to solve the problem/task; and "How can the suggested plan be carried out?").

The *reflection questions* were designed to prompt students to *reflect* on their understanding and feelings during the solution process (e.g., "What am I doing?"; "Does it make sense?"; "What difficulties/feelings I face in solving the task?"; "How can I verify the solution?"; "Can I use another approach for solving the task?").

FORUM+META discussion: Under this condition, students studied according to the IMPROVE method and FORUM discussion described above. They were encouraged to use the metacognitive questions during their discussion, in their written explanations when they solved the mathematical tasks and in their reflection on their friends' solution.

FORUM discussion: Under this condition, students studied the same as under the FORUM+META condition but they were not exposed to the metacognitive guidance.

### **Measures**

Two measures were used in the present study to assess students' mathematical problem solving: (a) a pre-test that focused on students' mathematical knowledge prior to the beginning of the study: (b) a post-test that assessed students' mathematical ability to solve real-life task.

Pre-test of Mathematics Prior Knowledge: To control for possible differences prior to the beginning of the study, a 38-item pretest was administered to all students at the beginning of the school year. The test covered arithmetic knowledge taught prior to the beginning of the study in the following content: Whole numbers, fractions, decimals and percents. The test was based on multiple-choice items regarding basic factual knowledge and open-ended computation problems. In addition students were asked to explain their reasoning.

Scoring: For each item, students received a score of either 1 (correct answer) or 0 (incorrect answer), and a total score ranging from 0 to 38. The scores were translated to percents. The Kuder Richardson reliability coefficient was  $\alpha = .87$ .

Mathematics Post-test: The Pizza Task- Your classmates organize a party. The school will provide the soft drinks, and you are asked to order the pizza. The class budget is NIS 85.00. Of course, you want to order as many pizzas as you can. Here are proposals of three local pizza restaurants and their prices. Compare the prices and

suggest the cheapest offer to the class treasurer. Write a report to the class treasurer in which you justify your suggestion. The task is provided on Appendix A.

The Pizza Task has all the characteristics of “Mathematical literacy” that is needed to be engaged in solving such task. The situation is most familiar to junior high school students, the mathematical data is rich, and there is no ready-made algorithm for solving it. Quite often children at this age go to restaurants that offer pizzas having different prices, sizes, and supplements; and quite often they have to decide which kind of pizza is more consuming. The Pizza Task requires the use of a variety of sources of information (e.g., prices, size, number of supplements) and it has many different correct solutions. The solvers have to make computations, use different representations, and apply knowledge regarding geometry, fractions and ratio.

Scoring: Students’ responses were scored on criteria based on the model of Cai, Lane & Jakabcsin (1996) for analyzing open-ended tasks: (a) Understanding the problem; (b) Processing information; (c) Using mathematical strategies; and; (d) Mathematical reasoning. Each criterion was scored between 0 (no response or incorrect response) to 4 (full correct response).

A full correct answer regards to referring to all the relevant data in each of the three proposals, making the calculations correctly, organizing of the information in a table, diagram, or an algebraic expression, making a correct suggestion based on the given information, and justifying the suggestion by explaining one’s mathematical reasoning. In addition, the type of arguments that were provided in mathematical reasoning were analyzed on five criteria:

(1) Providing the final result; (2) Providing the computation process; (3) Using daily mathematical terms; (4) Using formal mathematical expressions and; (5) Using logic mathematical argument. Inter-judge reliability of categories was .89.

## **Results**

Table 1 presents the mean scores, adjusted mean scores, and standard deviations on problem solving of the Pizza real-life task by time and treatment. Table 2 presents the frequencies and  $\chi^2$  test for providing arguments on the solution of the Pizza task by type of argumentation and treatment.

A one way ANCOVA and MANCOVA were carried out on the total score and on the various measures of the real-life task with the pretest scores used as a covariant.

Table 1: Mean Scores and Standard Deviations on the Pretest and on the Pizza Task, by Treatment

	FORUM+ META N=20	FORUM N=23	F
<b>Pretest<sup>a</sup>: Prior knowledge</b>			
M	70.38	65.55	F(1,42)=0.5
SD	23.04	20.60	
<b>Posttest<sup>b</sup>: Pizza Task (total)</b>			
M	3.52	1.87	F(1,41)= 31.19**
Adjusted M	3.44	1.86	
SD	0.30	1.04	
<b>Understanding the Task<sup>b</sup></b>			
M	3.90	2.36	F(1, 41)=21.9**
Adjusted M	4.00	2.30	
SD	0.31	0.84	
<b>Using Mathematical Strategies<sup>b</sup></b>			
M	3.60	2.39	F(1, 41)=29.8**
Adjusted M	3.67	2.33	
SD	0.50	0.50	
<b>Processing Information<sup>b</sup></b>			
M	3.40	2.32	F(1, 41)=22.7*
Adjusted M	3.50	2.30	
SD	0.75	0.58	
<b>Mathematical Reasoning<sup>b</sup></b>			
M	3.32	1.70	F(1, 41)= 27.2**
Adjusted M	3.42	1.59	
SD	0.65	0.70	

Note. <sup>a</sup> Range: 0-100, <sup>b</sup> Range: 0-4, \*p< .001; \*\* p< .0001

Results (Table 1) indicated that prior to the beginning of the study no significant differences were found between the two treatments on their prior knowledge. But on the post-test it was found that students who were exposed to the FORUM+META discussion significantly outperformed their counterparts (FORUM discussion) in solving the real-life task on all the four criteria.

Further results (Table 2) indicated that students that participated in FORUM+META discussion used significantly more Logic-Mathematical arguments and formal mathematical expressions than students from the FORUM discussion (75%, 90%; 34.8%, 30.4%, respectively). Whereas, most of the FORUM discussion students based their reasoning on repeating their final result without explaining “why” they got it (65.2%; 30%, respectively for the FORUM vs. FORUM+META).

Table 2: Frequencies and  $\chi^2$  test for providing arguments on the solution of the Pizza task by type of argumentation and treatment

ARGUMENTATION	FORUM+	FORUM	$\chi^2$
	META		
	N=20	N=23	
Providing the final result	6 (30 %)	15 (65.2 %)	5.31*
Providing the computation process	14 (70%)	7 (30.4%)	6.32*
Using daily mathematical terms	14 (70%)	14 (60.9%)	0.39
Using formal mathematical expressions	18 (90%)	7 (30.4%)	15.60***
Using logic – mathematical argument	15 (75%)	8 (34.8%)	6.96**

\*  $p < .05$ ; \*\* $p < .01$ ; \*\*\*  $p < .001$

### Conclusions and Discussion

The findings showed that the metacognitive students were better able to solve a real-life task and communicate their reasoning. This is probably due to the fact that the metacognitive guidance embedded in FORUM discussion trained students to think about which strategies are appropriate for solving the task and why. By doing so, students suggested different kinds of representations, compared the strategies, and analyzed each strategy.

Our findings indicate that using FORUM discussion is not sufficient for enhancing mathematical literacy. There is a need to structure mathematical discussion, and that features of discussion such as giving reasons must be practiced and reinforced. This conclusion is in line with other studies that showed that asking students to answer *why* questions during the solution processes helped them to elaborate and retain information (Schoenfeld, 1992). These findings support other conclusions on the importance of integrating pedagogical uses with advanced technology, in particular metacognitive guidance (e.g., Deaudelin & Richer, 1999; Kramarski & Ritkof, 2002).

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#### APPENDIX A: The Pizza Task

TYPE OF PIZZA	PRICE PER PIZZA	DIAMETER	PRICE FOR SUPPLEMENTS
<b>PIZZA BOOM</b>			
PERSONAL PIZZA	3.50 N.I.S	15	4.00 N.I.S
SMALL	6.50 N.I.S	23	7.75 N.I.S
MEDIUM	9.50 N.I.S	30	11.00 N.I.S
LARGE	12.50 N.I.S	38	14.45 N.I.S
EXTRA LARGE	15.50 N.I.S	45	17.75 N.I.S
<b>SUPER PIZZA</b>			
SMALL	8.65 N.I.S	30	9.95 N.I.S
MEDIUM	9.65 N.I.S	35	10.95 N.I.S
LARGE	11.65 N.I.S	40	12.95 N.I.S
<b>SPECIAL PIZZA</b>			
SMALL	6.95 N.I.S	25	1 N.I.S
LARGE	9.95 N.I.S	35	1.25 N.I.S