THE COGNITIVE - MOTIVATIONAL PROFILES OF STUDENTS DEALING WITH DECIMAL NUMBERS AND FRACTIONS

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This pilot study determined some typical cognitive-motivational profiles of Finnish lower secondary students dealing with fractions and decimal numbers. Forty seven students from grades 7-9 participated in a number concept test, where also the motivational aspects, such as self-efficacy, certainty and tolerance were measured. Four distinctively different profiles were found where the cognitive aspect of task sensitivity and the motivational aspect of tolerance were crucial. The results suggest that if students' cognitive distance to the task demands is too wide, the cognitive conflict is passed unnoticed. In addition moderate sensitivity combined with high estimation of self-efficacy and low tolerance seems to be restrictive to a more radical change and deeper understanding of the concepts.

INTRODUCTION

Conceptual change refers to a situation, where learners' prior knowledge is incompatible with the notion of the new conceptualization and where learners are prone to have systematic errors or misconceptions suggesting that prior knowledge interferes with the acquisition of the new concept. In mathematics, this kind of situation is typical when the students are struggling to learn the concept of rational numbers while their prior thinking of numbers is based on natural numbers (Merenluoto & Lehtinen, 2002; Merenluoto, 2003). In several empirical studies it has been found that humans have an innate cognitive mechanism related to numeral reasoning principles (e.g. Gallistel, & Gelman, 1992; Starkey, 1992) which is based on the discrete nature of objects and strengthened in everyday experiences and linguistic operations (Wittgenstein, 1969) of counting. Later these prior concepts of discreteness are strengthened in order to teach and learn the notion of natural numbers. Because these concepts seem self-evident, self-justifiable or self-explanatory, they easily lead to overconfidence (Fischbein, 1987). As such they might act as an obstacle for conceptual change or lead to mistakes and misunderstandings on more advanced domains of numbers.

The advanced properties of rational numbers as a compact set of numbers are not explicitly taught at the lower levels of mathematics education. These properties are, however, embedded in the representations, rules of operations, and of order for these numbers which are essentially different compared to respective rules of natural numbers. Thus, every extension of the number concept demands new rules to be

Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2004 Vol 3 pp 297–304

learned for operations and the use of a new kind of logic often leading to many different, but systematic problems and misconceptions in mathematics learning (c.f. The multiplier effect, see Verschaffel, De Corte & Van Coillie, 1988).

In several empirical studies it has been found that the misconceptions resulting from problems with prior knowledge of numbers seem to be exceptionally resistant to teaching attempts and that the students a prone to have difficulties with decimal numbers and fractions (e.g. De Corte & Verschaffel 1996; Verschaffel, De Corte & Borghart, 1997). In fact the problem in the school context is that the students are not very well aware of their prior conceptions and are prone to create models, where the prior knowledge is inconsistently combined with the new thinking. In this kind of situation we can speak about a problem of conceptual change. It is possible to claim that in process of a radical change in the thinking of numbers the students are forced to tolerance the ambiguity which comes from newly learned operations and characteristics of numbers while they do not yet fully understand the concepts (c.f. Sorrentino, Bobocel, Gitta & Olson, 1988; Stark, Mandl, Gruber & Renkl, 2002). In fact it is possible that coping with a new complex conceptual system is possible only if the learner has sufficient metacognitive skills to deal with conflicting notions such as when the same number is possible to present in infinite many representations (like fractions) or dealing with the infinity (Merenluoto, 2003).

Motivation seems to be related to the conceptual change in a very complex way. For certain, interest and the feeling of self-efficacy (e. g. Ford, 1992; Schoenfeld, 1987) are the fundamental aspects of high tolerance of ambiguity. However, high self-efficacy and certainty also seems to increase learners' tendency to pass the possible cognitive conflict unnoticed (e.g. Merenluoto & Lehtinen, 2002).

The research on conceptual change (e.g. Carey, 1985; Chi, Slotta & DeLeeuw, 1994; Duit, 1999; Hatano & Inagaki, 1998; Karmiloff-Smith, 1995; Vosniadou, 1994; 1999) has this far mainly dealt with cognitive factors, but especially during the last few years several researchers have agreed that these processes can not be explained in mere cognitive terms, but also motivational aspects (e.g. Pintrich, Marx, & Boyle, 1993; Linnenbrink & Pintrich, 2002), should be considered.

The aim of this pilot study is participate to this discussion and to analyse the relations of cognitive and motivational factors in students dealing with decimal numbers and fractions.

METHOD

The participants in the pilot study were students on grades 7-9 at Finnish comprehensive school, grade 7 (n = 15), grade 8 (n = 17) and grade 9 (n = 15), the percentage of girls was 40 %, 18 % and 67 % respectively. All the students had the same teacher.

In the beginning of *the procedure* the students were asked to fill in a questionnaire on own estimation of how much they had understood of the mathematics taught at school, their self-efficacy and tolerance with difficult problems in mathematics. The teacher was also asked to estimate the same variables for each of the students. *Phase 2.* The students were given a two-paged rational number concept test with 26 tasks testing sorting of numbers, identification of different representations of numbers (see Table 1), the density of numbers on the number line, and basic calculations. They were also asked to estimate their certainty on the answers with a 5-point-likert-scale, from 1 (a wild guess) to 5 (as sure as I know that 1 + 1 = 2) and to pick the most difficult and easy problem on each page. All the variables were calculated as percentages of maximum.

Students' *achievement level* in mathematics was estimated by the teacher on the scale from 5 to 10. Besides analyzing the answers to the tasks qualitatively the *test scores* (representing also the students' cognitive sensitivity to the tasks) were also scored from 0 to 1, where zero was given, if the answer was incorrect. The reliability of respective *certainty scores* was: alpha .799.

Tolerance of ambiguity was measured using two components: 1) *Estimated tolerance* was measured with the teacher's and students' answers to statement with a 5-point-likert-scale "If the task feels difficult I/the student do/does not do it", alpha .619; 2) *Test tolerance* was measured as the number of tasks done (score 1 per each task) and as the quality and thoroughness of explanations in the tasks (score 0-2 per each task).

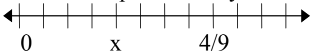
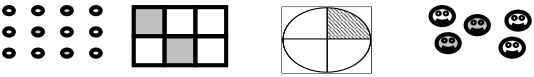
Students' *self efficacy in mathematics* was measured with a 5-point-likert-scale with statements such as "I am good in solving problems", "I'm doing well in mathematics at school", "I like difficult problems, then I can struggle to solve them", five items alpha .832. *Experience of understanding* the students were given a rectangle (1 cm x 10 cm) and asked to color as large part as they estimated to have well understood about the mathematics they had been faced this far at school. The colored portion was measured as percentages.

RESULTS

The results refer to major problems with rational numbers (Table 1). According to the results the students had a high tendency to a mistaken transfer from natural numbers to the domain of rational numbers, such as giving an answer of "one" when asked, how many decimal numbers there are between numbers 0.50 and 0.52. The mean score for all the students was 52 per cent (SD 21) and the mean of certainty estimations was 64 per cent (SD 15). Between the grades there were no statistical differences in test scores or certainty scores. Instead, the statistical differences were related to the achievement levels of the students (high, average, low), the task scores, $F(2, 44) = 18.5$, $p = .000$; $\eta = .452$, and certainty scores, $F(2, 44) = 6.66$, $p = .003$; $\eta =$

.232. The difference was due to the difference between the high and low achieving students (Scheffe, $p < .001$).

Table 1
Examples of the tasks used in the test with the frequency and percentage of correct answers

	Number of correct answers
1. How many decimal numbers there are between 0.50 and 0.52 on the number line?	3 (6 %)
2. Which decimal number is the next after 0.60? 3. Mother bought half a kilo of grapes costing 2.10 euros per kg. How much is her change from 10 euros?	29 (62 %)
4. Sort following fractions from the smallest to the largest: $\frac{1}{2}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{2}{100}$ $\frac{6}{4}$	23 (49 %)
5. Sam had run one eighth of his route and there was 3.5 km left. How long is the route?	17 (36 %)
6. Which number is represented by ‘x’? 	13 (28 %)
7. 1/2 x 1/8 =	16 (34 %)
8. 4 : 1/3 =	12 (26 %)
9. The identification task:  3/12 1/4 0.333… 1/3 2/8 2/5 0.25 2/6 Draw a line to connect the picture to the number presenting the coloured portion of it. (It is possible that there are several numbers for the same picture.)	The mean of correct connections: 6.7 (56 %)

Typical to all the students was that they had significantly more difficulties in dealing with fractions than in dealing with decimal numbers, paired samples difference in test scores, $t = 9.25$; $p =.000$, and in certainty scores, $t = 11.90$; $p =.000$.

The only gender related difference was in students' self-efficacy estimations, $F(1, 46) = 5.38$, $p = .025$; $\eta = .107$, and in the estimations of understanding of school mathematics, $F(1, 46) = 13.84$; $p = .001$; $\eta = .239$, the estimations of the boys were higher than the same for the girls.

Table 2 shows the high correlations between the cognitive and motivational variables measured in the test. Achievement level in mathematics had the highest correlations to test score, self-efficacy, understanding of mathematics, and estimated tolerance, but lower correlations to the measured test tolerance and certainty scores. In addition, the correlation of the achievement level in mathematics to the test tolerance was higher for the girls (.599) than for the boys (.320). The students had a clear tendency to over estimate their certainty and tolerance with difficult tasks.

Table 2
Correlations between the cognitive and motivational factors measured in the test

	1.	2.	3.	4.	5.	6.
1. Achievement level in mathematics estimated by the teacher						
2. Estimated understanding of school mathematics	.669**					
3. Estimated self-efficacy	.719**	.778**				
4. Estimated test tolerance	.711**	.684**	.624**			
5. Test tolerance	.395*	.279	.338*	.308*		
6. Test score	.710**	.498**	.474**	.528**	.430**	
7. Certainty score	.480**	.543**	.423**	.410**	.288	.541**

** p< .01; *p< .05

To analyze the typical cognitive - motivational profiles for the students a cluster analysis was used on the variables in Table 2. Four distinctive different profiles were found.

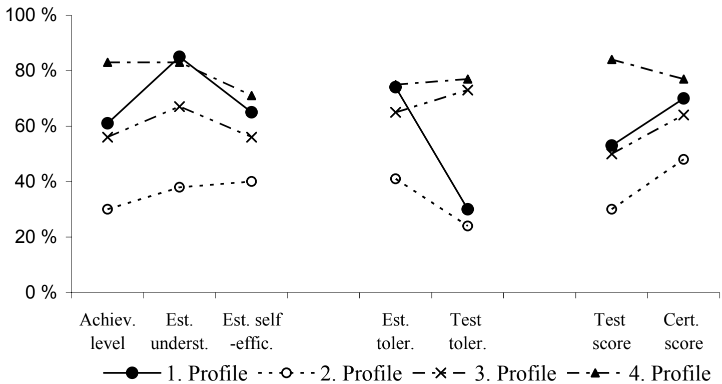


Figure 2. Typical profiles of the students in the cognitive-motivational variables measured in the test

The *first profile* was the most typical for the boys in the study (sixteen students, fourteen boys and two girls). Typical to this profile was significantly high estimations of their own understanding, tolerance with difficult tasks, and certainty compared to their average achievement level, low test tolerance, and moderate test scores. Their clear over estimation of certainty (see Figure 2) suggests some kind of illusion of understanding. A typical student on this profile was Carl (name changed). He was an average student in mathematics and liked mathematics. He also explained that he is good in mathematics, but did not want difficult problems. But then again he found mathematics discouraging. Thus he gave conflicting answers. His test score was 38 per cent of the maximum, but his certainty estimations optimistically 84 per cent. He gave correct answers in sorting the decimal numbers, in the word problems (Table 1) and in the calculations with decimal numbers but had problems in all the tasks where fractions were used.

The students on the *second profile* (seven boys and four girls) were low achievers in mathematics. In sorting the decimal numbers, many of them used a rule: "the number is small if it has many decimals" and they typically sorted the fractions by the nominators or denominators. In the identification task (Table 1) they found only the four obvious connections with high certainty and chose this task to the easiest task on the page. These results refer to a low cognitive sensitivity to the demands of the tasks thus suggesting a wide cognitive distance to the concepts of rational numbers.

The *third profile*, where eleven students were classified on (seven boys and four girls) was especially characterized with high tolerance where there were no statistical difference between the estimated tolerance and test tolerance as it was the case in the profiles one and two. But like in previous profiles, they over estimated their certainty in the tasks. These were high and average achievers in mathematics. A typical student on this profile was Eric (name changed) who liked mathematics and difficult problems. He had some intuition of several possible answers to the question of the next number after the number 0.60, giving the answers 0.61 and 0.60001. He found eight of the 12 correct connections in the identification task, but failed in all other tasks where fractions were used.

To the *fourth (high) profile* were classified eight students (two boys and six girls). These were high achieving students in mathematics, with the highest test scores and sensitive certainty scores (no statistical difference to the scores). This was the only profile where the teachers estimation of students' self-efficacy in mathematics was significantly higher than their own, t(repeated measures) = -3.88; p = .005. Only one student in this profile had problems in sorting the fractions. Seventh grader Ann (name changed) was one of the three students in the whole group, who's answer to the questions pertaining to the density of numbers on the number line (like tasks 2-3, Table 1) suggested deeper understanding of the rational numbers. She was a high achiever in mathematics who estimated that she had understood everything taught at school and liked mathematics. She answered that she is good in mathematics and that mathematics is easy and useful, but needs work. She gave correct answers to the majority of the tasks. Her answers to the most difficult tasks referred to a quite high sensitivity to fundamental feature of density of rational numbers, when she explained that between number 0.50 and 0.52 there exist two or more numbers "because they can be tenths, hundreds, etc." And when asked about the next number after .60, she explained: “may be my answer is wrong, but in the decimal numbers there can be very many numbers after the comma. Thus, I can not answer to that question”. This was the best answer to this question in the whole group.

DISCUSSION

The results of the test refer to major problems with decimal numbers and especially with handling the fractions. The answers of even the best students suggested mostly operational level of understanding indicating to an enrichment kind of learning (c.f. Vosniadou, 1999). Only in the answers of a few students there were some indications of deeper level of thinking suggesting a preliminary state of conceptual change. The identification task and the tasks of sorting the numbers were the best indicators of the quality of conceptual understanding in the test (see also Sowder, 1992).

The high correlations between the cognitive and motivational factors refer to the importance of considering the motivational aspects in the research on conceptual change. But they also refer to the complex interaction of these variables in learning where more research is needed. The students' achievement level in mathematics had a significant relation to the high sensitivity to the cognitive demands and high tolerance of ambiguity that seems to be optional for the conceptual change. However, similar to our earlier empirical results from the upper secondary level (Merenluoto & Lehtinen, 2002) also in this data from lower secondary levels of mathematical education, the moderate operational understanding of the concepts has a tendency to prevent the students' from noticing the cognitive conflict. The results also confirm that in the attempts to teach for conceptual change it is crucial to consider the cognitive distance between students’ prior knowledge and the new phenomenon to be learned.

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