RF01: AFFECT IN MATHEMATICS EDUCATION - EXPLORING THEORETICAL FRAMEWORKS[1]

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This article brings into a dialogue some of the theoretical frameworks used to study affect in mathematics education. We shall present affect as a representational system, affect as one regulator of the dynamic self, affect in a socio-constructivist framework, and affect as embodied. We also evaluate these frameworks from different perspectives: mathematical thinking, students with special needs, and methodology.

INTRODUCTION

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Affect has been a topic of interest in mathematics education research for different reasons (McLeod, 1992). One branch of study has focused on the role of emotions in mathematical thinking generally, and in problem solving in particular. Another branches have focused on the role of affect in learning, and on the role of affect in the social context of the classroom. Affective variables can be seen as indicative of learning outcomes or as predictive of future success. Partly because of this diversity in the research areas, but also partly because of the different epistemological perspectives of researchers, there is considerable diversity in the theoretical frameworks used in the conceptualisation of affect in mathematics education.

McLeod (1992) identified three concepts used in the research on affect in mathematics education: beliefs, attitudes and emotions. He made distinctions among these and described emotions as the most intense and least stable, beliefs as the most stable and least intense, and attitudes as somewhere in between on both dimensions. Beliefs were seen as the most 'cognitive', and emotions as the least so. Later DeBellis and Goldin (1997) added a fourth element, values. Most research on affect in mathematics education has used one or more of these four concepts. However, the theoretical foundation beneath these concepts is not quite clear.

Attitude has perhaps the longest history in mathematics education. Yet several authors (e.g. Di Martino & Zan, 2001; Hannula, 2002a) quite recently point out that attitude is an ambiguous construct, that it is often used without proper definition, and that it needs to be developed theoretically. Regarding beliefs, Furinghetti and Pehkonen (2002) asked a virtual panel to evaluate the definitions given for this concept in the literature. Their main finding was that no definition could be accepted by all experts in the panel. Hence, there is not one concept 'belief' used in the field,

[1] This article has been produced in collaboration and hence the authorship is divided. The first author and the main contributor(s) of each section is mentioned at the beginning of the respective section.

but many closely related ones; some of them are discussed in the recent book edited by Leder, Pehkonen, and Törner (2002).

Emotion is probably the most fundamental concept when we wish to discuss affect. Researchers who have studied the psychology of emotions have used different approaches, and there is no final agreement upon what emotions are. However, there is large agreement on certain aspects. First, emotions are seen in connection with personal goals. Emotions are also seen to involve physiological reactions, as distinct from non-emotional cognition. Third, emotions are also seen to be functional, i.e. they have an important role in human coping and adaptation. (E.g. Goldin, 2000; Lazarus, 1991; Mandler, 1989; Power & Dalgleish, 1997)

However, there is no agreement on how many basic emotions there are, or what they would be - or even if there are any basic emotions. It is well known that emotions are not only consequences of cognitive processing; they also affect cognition in several ways: emotions bias attention and memory and activate action tendencies (e.g. Power & Dalgleish, 1997). However, there is no detailed understanding of this interaction.

Value is the concept that has probably been least used of the four, and thus the relevant research is in its formative stages. However, values education is a dominant theme in various educational systems' goals around the world, and it is important to explore what a research focus on values in mathematics education can offer to our concerns about affect (Bishop, 2001). Of particular interest at the present time is the relationship between beliefs and values, with the focus of the first being on principles and propositions, and with the second being on choices, priorities and actions.

These four concepts do not cover the whole field of affect. Terms such as motivation, feeling, mood, conception, interest, anxiety, and view have also been used in this field. Motivation is an important concept, but surprisingly little research has been done explicitly on motivation in mathematics - at least within PME (Hannula, 2002b). What is its relation to the above-mentioned four concepts?

One important problem in the recent research on affect is the understanding of the interaction between affect and cognition. This problem is addressed in several ways, since here is great variation in the theoretical frameworks that mathematics education researchers have used. Goldin (2000) interprets affect as a representational system - parallel to cognitive systems - that encodes important information regarding problem solving. Some other approaches emphasize the social dimension. Socio-constructivists see affect primarily grounded in and defined by the social context (Op ‘t Eynde, De Corte, Verschaffel, 2001). Discursive practice theory emphasizes positions that are made available by the practices at play in the social context, and that enable and constrain the emotions that can be experienced and expressed (Evans, 2000). In the Vygotskian framework emotions become one dimension of the Zone of Proximal Development (Nelmes, 2003). Quite a different approach is to look at the recent findings of neuroscience and see how that informs our view of affect in mathematical thinking (Schlöglmann, 2002). Embodied view (Drodge & Reid, 2000), self-regulation (Malmivuori, 2001), and psychoanalytic theory (Evans, 2000) provide yet other theoretical frameworks for conceptualising affect.

All these different approaches have their value, and there is space for a multitude of approaches. One approach may be more suitable for certain research questions, while other questions require different theoretical tools. However, there is also a need to increase coherence in this field. Undeniably, there is the need for discussion. We invited Gerald Goldin, Peter Op 't Eynde, Marja-Liisa Malmivuori and David Reid & Laurinda Brown to each present a description of one theoretical framework that they have used to conceptualise affect. We have also invited three persons to evaluate the usefulness of these frameworks from different perspectives: Shlomo Vinner from the perspective of mathematical thinking, Melissa Rodd from the perspective of students with special needs, and Jeff Evans from the methodological perspective.

CHARACTERISTICS OF AFFECT AS A SYSTEM OF REPRESENTATION

Gerald A. Goldin [1st theoretical framework]

The prevailing view of mathematics as a purely intellectual endeavour, where emotion has no place, is perhaps just one reason for the relatively little attention devoted to research on affect in mathematics education. The methodological difficulty of designing and carrying out reliable empirical studies in this domain also poses an obstacle. We do not now have a precise, shared language for describing the affective domain, within a theoretical framework that permits its systematic study. Let us consider some ingredients of a possible theoretical framework for discussing mathematical affect, based partially on joint work with Valerie DeBellis (DeBellis, 1996; DeBellis & Goldin, 1997, 1999; Goldin, 2000, 2002, and references therein).

Affect as a system of representation and communication

The idea that affect has a basic *representational* function seems to be a less-than-usual perspective in psychology. More often emotions are described merely as accompanying cognition, or occurring in parallel with cognitive activity. Usually they are regarded as *consequences of* cognition, and often as having immediate *consequences for* cognition, either facilitating or impeding cognitive activity. Going beyond these evident features, we propose to regard affect much more fundamentally as one of several internal, mutually-interacting *systems of representation* within the individual human being. That is, the affective system functions *symbolically* so as to encode essential information. Loosely speaking, our emotional feelings and the complex structures involving them have *meanings,* even when we may not be consciously aware of those meanings, or able to articulate them.

Among the kinds of information commonly encoded affectively are: (1) information descriptive of the external physical and social environment in relation to the individual [as when *fear* may signify immediate danger or threat, and *relief* signify that a transition from danger to safety has occurred]; (2) information regarding the individual's own cognitive/affective configurations [as when *surprise* may signify the unexpectedness of an event as it occurs, or *frustration* signify lack of perceived progress in achieving a goal]; (3) information about other peoples' cognitive and affective configurations [as when *attraction* to another may encode the other person's favourable interest in one's own personality]; and (4) information about social and cultural expectations in relation to the individual [as when *pride* may signify fulfilment of societal role expectations, or *shame* signify failure to fulfil them]. Each description of such an encoding suggests a sense in which affect is *situated* [see the discussion by Op 't Eynde below].

Of course, to say that information is *represented* does not necessary imply it is *true.* This pertains whether the representational system is ordinary spoken language, visual imagery, or affect. The feeling of fear may occur when danger is only imagined, not actual. One may be unaware that one's emotional feelings are in this sense non-veridical; or one may know it perfectly well, as when feelings occur while reading an engaging novel. Furthermore, in real-life examples, the information encoded by emotional feelings typically cuts across more than one of the above categories, and is highly nuanced. For example, the feeling of *pride* may signify not only the actual fulfilment of sociocultural expectations, but also the proud feelings attributed by the person to others (e.g., the individual's parents or teachers), as well as the high value placed by the individual on the opinions of particular others. The meanings of affect often have to do with *complex, self-referential* information, such as "what I think someone else thinks of me."

In doing mathematics, the affective system likewise encodes information relevant to mathematical problems, and especially relevant to *the person in relation to the mathematical activity.* The feeling of *bewilderment* in approaching a problem in mathematics may simultaneously suggest that certain standard problem interpretations or problem-solving strategies do not work, because the problem is nonroutine [information about the mathematical structure of the problem], and the person's lack of specific *knowledge* or even more specifically, inability to formulate a subgoal [information about the state of the problem solver]. The feeling of *anxiety* may represent certain beliefs the person holds about his or her inability to do mathematics, or about possibly negative opinions others may form if he or she is unsuccessful. Affective states may evoke heuristic strategies; thus frustration [encoding the absence of apparent progress in solving a mathematical problem after repeated efforts] may evoke a major change in strategy [for example, a decision to try a special case, or to solve a simpler, related problem].

Cognitive representational systems function partly but very importantly by evoking affect and the information it encodes. This applies specifically to the internal verbal/ syntactic systems, imagistic systems, formal notational systems, and strategic/ heuristic systems of representation discussed elsewhere in formulating a model for mathematical problem-solving competence. In short, affective representation is not auxiliary to cognition; it is centrally intertwined with it. Affective configurations routinely signify, evoke, enhance or subdue, and otherwise interact with cognitive configurations, in ways that are highly context-dependent and person-specific.

In addition to its internal, representational function, affect also provides a *language for communication* among human beings. Here we highlight those “messages” conveyed through steady or intermittent eye contact and pupil dilation (or the absence thereof), facial expressions, gestures, posture and “body language”, interjections, intonation, song, way of breathing, laughter, tears, blushing, and so forth. Much of the communication that thus takes place happens tacitly among individuals; we find it difficult or impossible to say what it is specifically in someone else’s expression, half-open lips, intermittent-to-steady gaze, or raised eyebrow, that gives the impression they are curious, or amused, or very serious. Very likely, in everyday life, our judgments are often wildly inaccurate. Yet the system works extraordinarily well, underpinning and motivating virtually all human activity. Each individuals’ affect normally interacts with and evokes affect in others, so that information is exchanged and people in pairs or groups can share affect and function effectively together.

While there is considerable evidence of “emotion” in the world of other mammals, affective communication in the complexity that we experience it seems to be (like natural language) an essentially human phenomenon. It seems plausible that our system of affect, and the communication it makes possible, evolved as human beings evolved. Perhaps it enabled humans to function effectively at the tribal level, as well as in family groups, and facilitated the evolution of children who *learn* for many years before becoming biological adults. If this perspective is correct, it should not surprise us if the development of powerful affective structures turns out to be the key to effective mathematics learning and teaching.

Affective pathways: Local and global affect

Local affect refers to the changing states of emotional feeling experienced by individuals as they engage in mathematical (or other forms of) activity. Recurrent sequences of such states, one leading to the next depending on the context, may be termed *affective pathways.*

For instance, in a highly idealized example, a student approaching a problem in mathematics may initially experience *curiosity,* followed by a sense of *puzzlement* or *bewilderment* if the problem is unfamiliar or difficult. Repeated unsuccessful attempts may evoke *frustration.* Perhaps after one or several changes of strategy, the student experiences some *encouragement* as progress seems to occur, *elation* at a new insight or breakthrough, followed by *satisfaction* with having solved a difficult problem or understood a new mathematical concept. Alternatively, the student’s frustration may lead to *anxiety, anger, fear,* and/or *despair,* evoking avoidance strategies and defence mechanisms—a very different pathway.

As they recur, such affective pathways lead to the construction of *global affect* within the individual—long-term affective structures that in the first case might facilitate future enthusiasm, engagement, expectations of success, and a positive mathematical self-concept, but in the second case might lead to future distaste, avoidance, expectations of failure, and a negative self-concept.

Subdomains of affective representation: A tetrahedral model

McLeod and his collaborators proposed three components of the affective domain, which DeBellis and I proposed extending to four, creating a sort of tetrahedral model. In order of increasing stability in the individual over time, and degree of cognitive involvement, they are: (1) *emotions* or emotional feelings; the rapidly-changing states of feeling experienced during mathematical (or other) activity; emotions may range from the mild to the very intense, and are seen as local and contextually- embedded; (2) *attitudes,* orientations or predispositions toward having certain sets of feelings in particular contexts (e.g., mathematical contexts); attitudes are seen as moderately stable, involving a balance of interacting affect and cognition; (3) *beliefs,* discussed a bit further below, which involve the attribution of some sort of truth to systems of propositions or other cognitive configurations; beliefs are often highly stable, highly cognitive, and highly structured, but affect is nevertheless interwoven with them; and (4) *values,* including ethics and morals, the deep personal "truths" held by individuals that help to motivate priorities; values are stable, usually highly affective as well as cognitive, and may also be highly structured.

Each vertex of the tetrahedron (emotions, attitudes, beliefs, and values) may be understood as interacting dynamically with the others in an individual. For example, emotions influence attitudes, beliefs, and values; one mechanism for this influence is the construction of global structures as a result of the recurrence of certain affective pathways. In addition, each vertex interacts interestingly with the corresponding component in the affective domain of other individuals.

Affective competencies and affective structures

In the study of mathematical cognition we discuss *competencies*—the capabilities of an individual to perform particular tasks, take particular cognitive steps, or process information in particular ways in particular representations. Related cognitive competencies form complex, cognitive structures. Analogously, *affective competencies* refer to the capabilities of an individual to make effective use of affect during mathematical activity—for example, to act on curiosity, or to take frustration as a signal to alter strategy. Likewise we see the need to characterize and discuss the most important *affective structures* in relation to mathematics—for example mathematical intimacy [structures of emotional feelings, attitudes, beliefs, and values associated with intimate and vulnerable engagement in mathematical activity]; mathematical integrity [affective structures associated with the commitment to truth and understanding in mathematical activity], and mathematical self-identity [affective structures associated with the sense of self, "who I am" in relation to mathematics; see the related comments by Malmivuori below]. It is well-known that many students and adults have global affective structures that impede mathematical learning—in common parlance, "math anxiety"—but we do not have a straightforward way to change this. Thus it is important to study *mechanisms of change in global affect,* in analogy perhaps with how acts of forgiveness or self-forgiveness can permanently transform structures of anger, resentment, or guilt.

Meta-affect

An idea that has assumed a central role in our thinking is *meta-affect,* referring to affect about affect, affect about and within cognition that may again be about affect, the monitoring of affect both through cognition and affect. Our hypothesis is that meta-affect is *the most important aspect of affect.* It is what enables people, in the right circumstances, to experience *fear* as *pleasurable* (e.g., in experiencing a terrifying ride on a roller coaster). Towers of meta-affect occur often, and when they do they are usually very powerful—thus one may feel *guilt* about one's *anger* about the *pain* of perceived rejection by a parent whom one *loves.* At the core, perhaps, may be the love; but the negative meta-affect transforms it into something painful, and the anger and guilt contribute to an enduring if dysfunctional structure.

Consideration of meta-affect suggests that the most important affective goals in mathematics are *not* to eliminate frustration or to make all mathematical activity easy and fun. Rather they are to develop meta-affect where the feelings *about* emotions associated with impasse or difficulty are productive! Beliefs and values also play a role here, as they influence the ecological function of the emotion in the individual's personality. For example, the feeling of frustration with a mathematical problem *could* and *should* indicate that problem is nonroutine and interesting. The feeling should carry with it anticipation of possible elation at understanding something new, so that the *frustration itself* is experienced as interesting, curious, and anticipatory of joy in success. Related "cognitive" beliefs and values in relation to mathematics—belief that success is in fact likely, the value placed on achieving a challenging goal—can contribute to the construction of powerful meta-affect.

Belief systems, meta-affect, and sociocultural contexts

Finally, let us comment briefly on beliefs, systems of belief, and meta-affect. We have noted that beliefs establish meta-affective contexts for the experience of emotion. Reciprocally, affect stabilizes beliefs. The beliefs to which people hold fast may or may not be true, but they are *comfortable.* To say this is not to assert that they are necessarily pleasant; a belief may be somewhat painful [e.g., the belief by a child that she is "no good in math"], but it may be helping to shore up defences against greater hurt [e.g., being "no good in math" the child cannot be *expected* to perform well, and so will not disappoint her teachers or her parents].

Systems of beliefs allow for redundancy and mutual support, further stabilizing them. "Math is for boys," "You have to be really smart to do math," or "You have to be sort of a nerd to like math," may fit together well with "I'm no good in math." *Socially or culturally shared* beliefs and affective structures contribute substantially to the way in which meta-affect and belief systems sustain each other. In general, the strongest affect and most stable belief systems are those such as nationalist fervour, or religious reverence, that are shared and culturally embedded.

All of this suggests we give considerable explicit attention to the affective dimension in understanding the persistence of belief systems that are counterproductive to powerful mathematical learning and teaching.

A DYNAMIC VIEWPOINT: AFFECT IN THE FUNCTIONING OF SELF-SYSTEM PROCESSES

Marja-Liisa Malmivuori [2nd theoretical framework]

Newly rediscovered theoretical constructs, such as metacognition, consciousness and self-regulation, afford opportunities to consider cognition as more closely linked to affect and behaviour in learning and education. The role of personal constructive and self-regulatory aspects of affective responses in social, contextual and situational environments is emphasized in the suggested dynamic viewpoint. More generally, the view connects these aspects closely to the functioning, qualities and development of students' self-systems and self-system processes in respect to learning mathematics. The qualities and functioning of significant self-system processes ultimately determine the power and role of affect in students´ personal learning or performance processes in mathematical situations. The perspective applies recent cognitive, socio-cognitive, constructivist, as well as phenomenological views of learning and links affect strongly, naturally and in a dynamical way to cognition. Moreover, the chosen conceptualisations and developed learning model try to overcome the restrictions often caused by the use of traditional and static affective concepts.

Affect in personal learning processes

This view considers affective factors and emotional experiences as essential features of *personal learning processes* and functioning. In addition to affective experiences, we use also such terms as affective arousals, states and responses, each of which relates both to biophysical, mental and expressive human aspects or processes. Students as historical and social individuals or *selves* constitute, evaluate, develop, and regulate themselves and their own affective experiences and learning processes in relation to mathematics. These are essential aspects of personal functioning and development. With respect to powerful affective arousals and experiences especially important are students' self-perceptions in social contexts and situations. The related highly influential affective responses can be called '*self-affects*'. They are connected with students' experiences of self-esteem, self-worth, and/or personal control with respect to mathematics, which can be described as the aspect of 'how one feels about one's worth' (cf., Harter, 1985). The significant relationship between the self and affect is acknowledged in the classical psychological theories. Within education research domain it appears in the close measured relationship between students' self-concept, self-esteem, self-confidence or self-efficacy and their highly intense responses, such as anxiety, and further the qualities of their motivational or learning outcomes (e.g., Covington & Roberts, 1994; Schunk, 1989).

Mathematics education research has found these kind of affective responses often negative and inhibiting in nature, resulting in disturbance of students' mathematics learning, problem solving, or performances. For example, significant and constant gender-related differences are measured in students' perceptions of their mathematical abilities as well as in their self-affect, such as anxiety or pride and shame, and performances or achievement behaviours (e.g., Fennema & Hart, 1994; McLeod, 1992). The arousal and role of students' affective responses are seen here to be closely connected with their personal and situational self-perceptions, efforts, goals, and self-regulation in the social and contextual mathematics learning environment (cf., Pekrun, Goetz, Titz & Perry, 2002; Skaalvik, 1997). More specifically, these central aspects of learning are considered here as the qualities, functioning and development of students' self-systems and self-system processes in learning and doing mathematics. In addition to mathematical knowledge systems, understanding and skills, students' personal *self-systems* involve their self-beliefs and self-knowledge systems, mathematical beliefs and belief systems, related affective responses, and the related behavioural patterns in mathematical situations. By students' *self-system processes* it is referred to the functioning of their mathematical self-systems in unique social mathematical situations, with their active self-regulation and personal agency to varying extent as included. Moreover, the aspects of personal self-systems and self-system processes represent different degrees of abstraction in students' mental processes or cognition. In this way, the varying levels of consciousness or *self-awareness* in these systems and processes are also applied here as an unconstrained path from cognition to affect and vice versa.

Affective arousals in social mathematics learning situations

The arousal and development of students' highly influential affective responses (self-affects, e.g., anxiety, fear) to mathematics are intertwined with their situational or learned habitual beliefs, perceptions, and appraisals of the self in mathematics learning contexts and social environments. These constitute central occasions for the dynamic interplay of students' cognition and affect in learning mathematics. In this, essential arguments are given for such unique situational and constantly ongoing self-system processes as *self-appraisals* and *self-judgments*. Personal, situational and social environmental features and conditions create a context for a significant self-evaluative situation to emerge and, thence, for the evoking of essential personal self-beliefs and self-appraisals with mathematics. The related unique evaluations and judgments of the self in a mathematical situation are accompanied by affective arousals and constructive or directive processes with affect and behaviours, implying important *affective self-states* for doing and learning mathematics (cf., Lewis, 1999). Especially important are students' perceptions and appraisals of their personal capability, agency and control with respect to mathematics and mathematics learning.

Students' appraisals or judgments are influenced not only by personal but also by unique contextual and socio-cultural features of mathematics and its learning. Influential self-appraisals mediate not only the effects of students' past personal mathematical history (e.g., personal beliefs), but also those of the fundamental socio-cultural and contextual features of mathematics learning on their affective responses to mathematics (cf. Malmivuori, 1996). In this *individual-environmental interaction*, the characteristics of an actual learning context, or unexpected, new, or rapidly changing occurrences in this context, represent more direct environmental influences on students' self-appraisals and on such self-affects as anxiety or test anxiety. Less direct environmental influences on students' self-appraisals and affect are again linked to particular kinds of socio-cultural beliefs about mathematics and mathematics learning or about performance situations that are reflected by students as well as by the larger social environment (e.g., perceptions of the difficulty of mathematics, attributions for mathematical successes or failures).

In referring to the *constantly operating mind* and general flow of affective mental processes and states, we indicate that different appraisals and processing activities can coexist at different levels of consciousness or self-awareness, and cause several (continuously flowing or changing and perhaps conflicting) affective experiences or self-states that are more or less influential in students' mathematics learning processes (Malmivuori, 2001). The scene of the related mental activity can be called a *student's contextual consciousness* that is conditioned by internal personality aspects as well as by various external features of mathematics learning situations and contexts. That is the primary personal and unique situational scene for the individual-environmental interaction to occur and develop in doing and learning. Within this scene, affective responses do not only arouse, tone or disturb students' learning or performing processes but also serve them as a significant source of information about their own mental content and ongoing processing activities, of their action conditions, and of their self-states with respect to mathematics learning.

Self-regulatory features of affect

Self-regulation processes represent the central combining feature of self-system processes with affect. In addition to self-appraisals and self-judgments, these metalevel mental processes involve students' self-directive constructions, self-control and self-regulatory actions. They represent the other significant aspect of the dynamic affective-cognitive interplay that are then accompanied by and/or directed towards affective responses and states. The most common approach to this interplay can be referred to as *affective regulation* that illustrates the property of affective experiences to form a kind of affective feedback system that dominates the cognitive evaluation system or behaviours at a relatively low level of control without clear notions of self-regulatory mental activity (cf., Leventhal, 1982; Taylor et al., 1997). It includes preventive effects of affect such as mental blockages, simplification of mental processings and hindering of the maintenance of higher order metalevel processes, or again, intensification of mental processes and change of content of thoughts caused by promotive positive affective responses (McLeod, 1988).

Affective responses also give rise to, accelerate, or sustain additional interpretations, personal meanings, and beliefs with several evaluation processes going on at the same time at different levels of consciousness. They further establish a set of *additional behavioural goals* related to or independent of students' specific goals or objectives with ongoing original learning intentions or behaviours, and cause differing and possibly conflicting action tendencies (Evans, 2000; Leventhal, 1982). In this way affective responses have important organizing, motivating, and adaptive functions, and directly induce or regulate also other affective responses (e.g., interest attenuating fear and sadness or shame attenuating joy; cf., DeBellis & Goldin, 1997; Goldin, 2000; Taylor et al., 1997). Integration of this kind of whole level affective-cognitive dynamics or self-system processes has a major impact on the organization of personality with important individual differences in affective development, as well as in the development of self-system or self-regulatory personal processes in general. The dynamic view connects this integration to students' self-conscious monitoring, assessment and judgments of their own affective arousals, responses and self-states, to their self-conscious decisions and choices directed toward these responses or states and the causes or effects of these, and to their conscious control over their own affective responses. Students' affective arousals and responses thus become objects of their conscious evaluations and regulation and their unique situational mental processes have significant power in affecting the arousals, experiences and effects of their affective responses in learning or doing mathematics. The dynamic view refers to these kinds of self-system processes as *active regulation of affective responses*.

The essential difference between these two forms of the interplay of affect and cognition is linked here with the varying degrees of students' consciousness or *states of self-awareness* in the functioning of their self-system processes. Thereby, affective regulation represents lower level or more automatic self-regulatory processes with weak self-control beliefs or personal agency and lower states of self-awareness, while active regulation of affective responses relates to enhanced self-control beliefs and high personal agency with efficiently integrated self-regulatory processes and promoted self-awareness. This variation in the qualities of students' self-system processes determine the role that affective responses play in their personal learning processes and performances. It is the key feature of students' contextual consciousness in any mathematics learning situation. With respect to the individual-environment interaction we may characterize active regulation of affective responses as individually and situationally directed personal processes with affect. The interaction between environmental features and students' mathematical affective responses is then less direct and more flexible or independent of the instant environmental conditions and specific social features of school mathematics learning, but also of their own stable or habitual self-systems (i.e., self-beliefs, mathematical belief systems, affective responses, behavioural patterns). Instead, affective regulation can be considered as basically retaining functioning, in which the interaction between environmental features and students' affective responses is rather direct. Arousal, repetition and effects of similar strong and often hindrance affective responses (cf., global affect; DeBellis & Goldin, 1997) depend then mainly on the qualities of students' stable self-systems and/or on the particular contextual and socio-cultural features of mathematics learning.

Theoretical applications

The offered dynamic viewpoint is designed to deal with the complexity of affect-cognition interplay in social learning situations. It supports the idea of personally and situationally unique affective constructions and also considers these constructions in interaction with social environment (cf. socio-constructive views by Op 't Eynde, local affect by Goldin). Examination of the functioning of powerful processes of personal learning and affect (i.e., significant self-system processes) in situations offers better opportunities for understanding not only the importance of students´ self-identity or self-referential information (Goldin, Op 't Eynde) but also their personal involvement and self-regulatory features with their affect. The emphasis on self-reflection and self-regulatory processes in the model also relate to the important ideas of meta-affect presented by Goldin. Furthermore, linking affective aspects to mental, behavioural, and control or regulatory processes at different levels of abstraction and personal functioning will connect affect more closely to cognition, and also such concepts as embodied cognition and affect (Brown & Reid) can be fitted to the model. On the other hand, the role and impact of important affective responses are seen here to vary along with the qualities and functioning of personal self-systems in mathematical situations. In this, a basic qualitative distinction is made between students' *fully functioning self-system processes* and personally powerful learning or doing of mathematics and, in turn, their *defectively operating self-system processes* and learning with self-defending, habitual or retaining, and externally directed performance behaviours, often filled with negative affect.

A SOCIO-CONSTRUCTIVIST PERSPECTIVE ON THE STUDY OF AFFECT IN MATHEMATICS EDUCATION

Peter Op 't Eynde [3rd theoretical framework]

The study of the role of affect in mathematics education typically is not only determined by the way affect is defined but, more generally, also by the researcher's view on learning and instruction. One's view on mathematics learning determines the key aptitudes and processes to be investigated and how this is done. More specifically, it clarifies which role affective aptitudes and processes might have in learning and sets the stage for the affective processes looked for, how they are conceptualised and how they should be studied. Therefore, in introducing our perspective on the study of the role of affect in mathematics education, more specifically on the study of students' beliefs and emotions, we first need to explicate our view on mathematics learning in general.

Learning, engagement, and identity

From a socio-constructivist perspective learning is conceived as a fundamentally social activity. Learning is getting acquainted with the language, rules and practices that govern the activities in a certain community, in our case the mathematics education community. By engaging in the practices of this community people discover meaning, come to know. Meaning, then, becomes jointly constructed in the sense that it is neither handed down ready-made nor constructed by individuals on their own. Well established meanings might be implied in practices characterizing a specific community for many years, but it is through engaging in such a practice anew that the individual experiences meaning and renegotiates the currently accepted meanings. Greeno, Collins, and Resnick (1996, p. 26) clarify that

> "The view of learning as becoming more adept at participating in distributed cognitive systems focuses on engagement that maintains the person's interpersonal relations and identity in communities in which the person participates"

In this way, students' learning in the classroom is characterized by an actualisation of their identity through the interactions with the teacher, the books, the peers, they engage in. On the one hand, these interactions are determined by the class and school context they are situated in and as such the *social context* is constitutive for students' identities. But, on the other hand, students bring with them to the classroom the experiences of numerous other practices in other communities they have participated or are participating in. Continuously challenged to integrate them in one self, this wide spectrum of past experiences determines the specific way students find themselves in the class context and its practices, discover meaning, and renegotiate or *construct* new meanings through *their* way of engaging in the class activities.

The way students engage in classroom activities is function of the interplay between their identity and the specific classroom context. Their motivation to participate in a specific way in certain classroom activities is grounded in the way they find them"selves" in that context. However, their self, their identity, is only partially transparent to them. Who they are, what they value in this context, what they find worthwhile acting upon, is seldom known a priori, it emerges in the situation. It is through their experienced motivations and emotions that subjects recognize the value a situation bears for them. More specifically, students' emotional reactions toward mathematics are the outcome of consciously or subconsciously activated personal e*valua*tive cognitions or appraisals of mathematics, the self, and mathematics learning situations (Malmivuori, 2001). Students' beliefs about mathematics and the mathematics classroom, and especially their self-beliefs related to math (e.g., their expectancy and value beliefs) have been shown to be influential factors determining the interpretation and appraisal processes constituting their affective responses and emotions (see Mandler, 1989; McLeod, 1992). Students' mathematics-related belief system as well as students' mathematical knowledge can be identified as the central mental structures underlying students' understanding of and functioning in the mathematics classroom (see De Corte, Op 't Eynde, & Verschaffel, 2002). An *understanding* that never is only cognitive in nature but always function of cognitive-affective linkages due to the value-loaded character of some of the underlying cognitions.

Students' emotions: A situated and integrated approach

Taking into account the embeddedness of students' knowledge as well as beliefs in the social context (see e.g., de Abreu, Bishop, & Pompeu, 1997) the interpretation and appraisal processes that ground students' emotions in the classroom (e.g., anger, fear, etc.) are fundamentally constituted by the social-historical context in which they are situated. Harré (1986) points out that emotions can differ depending on the social context they are embedded in and this as well in terms of the different kinds of emotions that are experienced, as in the specific characteristics of what at first sight appear to be the same emotions. In line with Paris and Turner's (1994) characterization of situated motivation, one can claim then that every emotion is situated in its instructional context by virtue of four characteristics. First, emotions are based on students' cognitive interpretations and appraisals of specific *situations*. Second, students construct interpretations and appraisals based on the knowledge they have and the beliefs they hold, and thus they vary by factors such as age, personal history and home culture. Third, emotions are contextualised because individuals create unique appraisals of events in different situations. Fourth, emotions are unstable because situations and also the person-in-the-situation continuously develop.

There is, however, much more to emotions than the appraisal processes that determine them and their cultural situatedness. Taking seriously the accumulated findings from emotion research, what is needed is an emotion theory that explains (see Scherer, 2000):

- *both* the phenomenological distinctiveness and the intricate interweaving of cognition and emotion
- *both* the dynamic nature of emotional processes and the existence of steady states that can be labelled with discrete terms (e.g., anger, happy, proud)
- *both* the psychobiological nature of emotion and its cultural constitution

A component systems approach (Mascolo, Harkins, & Harakal, 2000) presents a promising and integrative conceptualisation of emotions that reconciles these dichotomies by bringing them to a synthesis. Three main principles are at the basis of this approach. Firstly, it emphasizes the emotion *process* characterizing emotions as an emotional episode within which appraisal-affect-action systems coact. Emotional experiences are perceived as emerging on-line in a specific context through the interactions between 5 distinct systems: (1) the cognitive system (appraisal); (2) the autonomic nervous system (affect); (3) the monitor system (affect); (4) the motor system (action); (5) the motivational system (action). The mutual feedback processes between these systems in a specific context constitute the experienced emotions explaining their dynamic nature. Secondly, a component system approach points to the non-chaotic nature of these feedback processes clarifying that emotions self-organize in real time as well as in ontogenesis. Framed by the specific socio-historical context emotional experiences tend to self-organize into a finite number of stable patterns, i.e. basic emotions. However, different patterns of component systems interactions will be produced even within each 'basic' emotional category. The sensitive dependence of emotional experiences on initial conditions account for numerous variations found within each basic category. Variations that are not trivial and, although many times labelled with the same emotional term, can refer to large differences in the organization of component systems and thus in the nature of the emotion. A final principal characterizing a component systems approach deals with the social nature of emotions. Emotional experiences are always situated in the immediate and broader social-historical context. This does not imply, however, a denial of the relevant biogenetic and organismic processes. On the contrary, socio-cultural systems always coact with biogenetic and organismic systems in every emotional experience and they all together influence an individual's emotional development.

Investigating the role of emotions in mathematics learning

Combining a socio-constructivist perspective on learning and a component systems approach of emotions to study the role of emotions in mathematics learning necessarily implies: (1) holding a conception of emotions as consisting of multiple component systems that mutually regulate each other in a specific context, i.e. the mathematics classroom, and (2) holding a conception of learning as an engagement in the practices of a specific community that maintains the person's interpersonal relations and identity in a particular socio-historical context. To our opinion, the integration of both perspectives provides a comprehensive and promising theoretical framework for the study of the role of *emotions* in classroom *learning*, involving a clear shift in the methodologies and instruments used to investigate these phenomena.

<u>Studying the student-in-the-classroom.</u> The situatedness of emotions or emotional experiences, and of classroom learning in general, forces research from this perspective to take place in the classroom. A study of the role of students' emotions in classroom learning has to document how students engage and reorganize their ways of participating in classroom practices and clarify the role of emotions in this process. This approach stresses intentionality and emotionality, next to intellectuality, and takes activity and meaning as its basic currency. Emotions are not treated as objects that can be studied as independent and detachable from the specific individual and context. On the contrary, emotions are perceived as an act of participating in certain practices and contexts. To study, for example, joy then implies an analysis of joyful acts as they occur in the concrete world of contexts and activities, in our case, in the context of the mathematics classroom.

<u>Taking an actor's perspective.</u> This focus on the meaning structure of emotional activities and of learning activities in general, implies a shift for researchers from an observer's perspective to an actor's perspective (Cobb & Bowers, 1999). What matters is not so much students' activities and the classroom environment and practices as observed by the researcher, but the meaning students (and teachers) give to it and upon which they act.

Measuring the different component systems. To grasp this dynamic interplay between the student and the class context that fundamentally determines his emotional experiences and learning behaviour in general, a variety of research methods has to be used. Interviews, observations and discourse analysis seem to be more appropriate methods for revealing the meanings that students give to situations and how they are constituted through interactions in class, than, for example, questionnaires. On-line questionnaires, experience sampling methods, video-based stimulated recall interviews, are examples of appropriate techniques in view of reaching the intended goals as far as the continuous flow of interpretation and appraisal processes is involved (see e.g., Prawatt & Anderson, 1994). However, an emotional experience is constituted by the mutual interactions between different component systems of which the appraisal system is but one. The use of facial coding systems and registration systems of physiological parameters (see e.g., DeBellis, 1996) that grasp the evolutions in respectively the action system (e.g., the motor systems) and the affective system should complement the information about the appraisal process to get a more solid and comprehensive picture of the emotional experience.

From an isolated to a multidimensional approach. Analysing the emotional dimension of students' activities in the classroom can not take place in isolation from the study of cognitive and conative processes. Although different in nature, we have shown above that there are close interactions between these processes. On the one hand, the emotional experience itself consists of multiple interactions between affective, cognitive (appraisal) and conative (motivational) processes. On the other hand, and highly related, within learning activities students' emotional experiences are intricately linked to the learning goals strived for and the cognitive and metacognitive strategies used.

A multilevel approach for a deeper understanding. The analysis of the emotional experiences of an individual student in the classroom can reveal how he continuously interprets and appraises the situation and acts upon it. A meta-level analysis of the appraisal processes and the actual learning activities can disclose some of the beliefs and knowledge underlying these emotions and actions, leading to a deeper understanding. However, to fully understand the nature of these beliefs and the consequences of the actions, an analysis of the norms and practices that characterize the classroom the student is a member of, is also necessary. One might even take it one step further and study the rules and values that are dominant in the school community and the society as a whole. A "multilevel" approach that incorporates these three planes of analysis, corresponding to personal, interpersonal, and community processes will probably result in the most complete understanding of the emotional experiences and learning activities studied (see Op 't Eynde, De Corte, & Verschaffel, 2001; Rogoff, 1995).

EMOTIONAL ORIENTATIONS AND SOMATIC MARKERS: IMPLICATIONS FOR MATHEMATICS EDUCATION

Laurinda Brown and David A Reid[2] [4th theoretical framework]

Our interest in emotional orientations and somatic markers is related to our interest in how teachers and students make decisions in mathematics classrooms.

Experienced teachers deal with situations where there are many different possible responses all the time. How do they decide what to do? In the first years of teaching there is little past experience on which to draw and student teachers report an emotional roller-coaster ride. We are concerned with finding ways of working with students so that they are not taken over by strongly negative or strongly positive emotions - becoming incapable of acting as teachers. How can they learn what to do when they do not know what to do and their actions can conflict with their beliefs? They need to develop complex decision-making strategies where there is not one simple right answer of what to do.

Students learning mathematics face similar challenges. Many come from experiences of mathematics that have led them to expect that mathematics is a safe domain of predictable rules and procedures. At some point they encounter teaching approaches and subject matter where their past experiences are insufficient, and they experience the stress of not knowing what to do. Their teachers hope that they will become capable of dealing with complex mathematics situations without being taken over by emotions that leave them unable to act. How can students of mathematics learn what to do when they do not know what to do and their actions can conflict with their beliefs? They also need to develop complex decision-making strategies where there is not one simple right answer of what to do.

Somatic markers

Damasio (1996) has studied the making of such decisions through the neurological characteristics of people who no longer seem able to make them. He has put forward the somatic marker hypothesis to explain what he has observed. The term "somatic marker" is used for the juxtaposition of image, emotion and bodily feeling we have that informs our decision-making:

> Because the feeling is about the body, I gave the phenomenon the technical term *somatic* state ("soma" is Greek for body); and because it "marks" an image, I called it a *marker*. Note again that I use *somatic* in the most general sense (that which pertains to the body) and I include both visceral and nonvisceral sensation when I refer to somatic markers (Damasio, 1996, p.173).

We would suggest that your somatic markers come into play when you judge some actions to be likely actions of a teacher and others to be unlikely. In their work on teachers' complex decision-making, Brown and Coles (2000) state:

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[2] This paper is a collaborative work. The authorship is equally shared.

> Somatic markers act to simplify the decision as to which behaviour to try. Negative somatic markers mean that the behaviours do not even come to mind as possibilities for action. A positive somatic marker means that the behaviour becomes one of a number available for use (p.168).

Somatic markers can be based on “primary emotions”, in which case we make decisions on the basis of inborn reactions. For example, one might move to avoid a snake on a path before even recognising it as a snake. The fear of snakes is part of the inborn structure of the human brain, and is “primary” in that sense. But we are also capable of feeling an emotion without the stimulus being present. If one opens a laundry hamper and discovers a snake inside, one might learn through this experience to feel fear in similar situations, whenever one encounters a laundry hamper, and so one might change ones behaviours in the future. The fear of laundry hampers is an example of what Damasio calls a “secondary emotion” triggered by the feelings we have associated with an event. A somatic marker has been created: a linkage of thought, emotion and feeling that inclines one to do, or in this case not to do, an action.

> Somatic markers are thus acquired though experience, under the control of an internal preference system and under the influence of an external set of circumstances which include not only entities and events with which the organism must interact, but also social conventions and ethical rules (Damasio 1996 p. 179).

We believe that Damasio’s notion of ‘somatic markers’ helps us to describe the development of teachers and students engaged in mathematical activity in classrooms. You have a constellation of “teacherly” somatic markers that are active in teaching situations and a constellation of “mathematical” somatic markers that are active in mathematical situations. While another teacher or another mathematician would make different decisions than you would, at the same time you can recognise similarities in the choices your somatic markers would guide you towards.

Emotional Orientations

What we have called a constellation of somatic markers can be seen as what Maturana (1988a) would call by the name "emotional orientation". An emotional orientation is what characterises someone's actions as appropriate to a context, like teaching. Maturana would call teaching a "domain of explanation", characterised by a community whose members can recognise in others behaviours appropriate to the community, although they probably could not give specific criteria for doing so. Many communities are like this, and communities can overlap, contain other communities, or subtly blend into other communities according to the behaviours different members accept as legitimate. For example, algebra is a domain of explanation, just as teaching is:

> …if someone claims to know algebra, that is, to be an algebraist, we demand of him or her to perform in the domain of what we consider algebra to be, and if according to us she or he performs adequately in that domain, we accept the claim. (Maturana, 1988b, p. 3)

In responding to a student's question a teacher can act in many ways, but as a teacher you might recognise that not all possible responses are appropriate coming from a teacher. As we noted before, a teacher's choice must be based on something other than conscious reflection as there is no time for conscious reflection here. Instead something like a somatic markers, or a constellation of somatic markers, must be at work, and in the larger context of all her or his teaching the somatic markers that guide a teacher implicitly define a preference for certain behaviour s/he, and we, would see as appropriate for a teacher.

> …whether an observer operates in one domain of explanations or in another depends on his or her preference (emotion of acceptance) for the basic premises that constitute the domain in which he or she operates. Accordingly, games, science, religions, political doctrines, philosophical systems, and ideologies in general are different domains of operational coherences in the praxis of living of the observer that he or she lives as different domains of explanations or as different domains of actions (and therefore of cognition), according to his or her operational preferences. (1988a, pp. 33-34)

Maturana suggests the phrase "emotional orientation" to name the set of criteria (which we read as somatic markers) appropriate to a domain of explanations. He uses "emotional" because it is a bodily predisposition rather than conscious reflection that is operating when we perceive some behaviours as appropriate and others as not.

As different kinds of explanations are appropriate to different domains of explanation we have different emotional orientations appropriate to each domain. We observe the actions of others as being appropriate to a particular domain according to our own emotional orientation for that domain. So a mathematical emotional orientation allows us to recognise the activity of others as being mathematical, and hence to identify those people as mathematicians. A teacherly emotional orientation allows us to recognise the activity of others as being teacherly, and hence to identify those people as teachers. We have many emotional orientations as we belong to many communities, characterised by different (probably overlapping) constellations of somatic markers.

Looking ahead

Our current collaborative research looks at the ways in which somatic markers influence teacher decision-making and students' reasoning, and the degree to which those markers can be observed by us, by colleagues, and perhaps by the teachers and students involved. Because somatic markers are a part of unconscious mental activity they cannot be observed by introspective reflection. In fact, the stories we tell after the fact about our decision making are likely to include inventions to account for the influence of somatic markers of which we are not aware. How then can we research something we cannot observe? The process described above, of examining decision points in a person's actions, seems to hold promise. We can observe changes in behaviour, indicative of unconscious decision making, and consider what markers based on past experience might account for those decisions. Our work with colleagues has indicated that mathematics educators see similar events as suggesting the sort of unconscious decision-making accounted for by Damasio's hypothesis of somatic markers. This leaves us optimistic that it will be possible in our work to observe the effects of somatic markers in a range of contexts, to distinguish positive and negative somatic markers, and to suggest ways in which they form and evolve in mathematics classrooms.

MATHEMATICAL THINKING, VALUES AND THEORETICAL FRAMEWORK

Shlomo Vinner [1st evaluative perspective]

I would like to use my role as a reactor to reflect about certain tendencies in the community of PME. These tendencies are general. Since they are general, they might be relevant also to specific cases. However, I am trying not to relate to specific cases here. I believe that part of a reactor's role in forums like this is to relate to broad and essential aspects of the topic under consideration. So essential that it can be compared with the essential aspects raised by the question "to be or not to be." However, the question is not "to be or not to be." The question is what to be? It is related to the identity and character of the PME community. In fact, Nicolas Balacheff (1996), a former president of PME, already raised this question when he called PME members to question the aims and directions of their activities as PME members.

The way I understand the history of PME (and I am not a historian), it really started with mathematical thinking. We saw our role to describe and to explain mathematical thinking. We did not use the name "Psychology of Mathematical Thinking" because we believed that the name "Psychology of Mathematics Education" has a broader scope. By doing this we opened the door to all kinds of issues related to education. An important issue related to education is the issue of values. In the context of values, questions about the educational goals of learning mathematics, about its merit and about its contribution to the moral development of the students could have been discussed. The overall impression is that it almost did not happen. The reason for it can be the fact that we consider ourselves as experimental researchers and discussions about values are not within the domain of experimental research. We took for granted the current situation in which mathematics is taught to certain extent to everybody. For instance, in this forum we discuss students with special needs. It is really a thoughtful gesture. These people have the rights to have normal and worthy life. However, we do not ask why they should study mathematics and to what extent. So, we did not enter the door that we opened for ourselves by choosing the name "Psychology of Mathematics Education." We remained in the domain of mathematical thinking.

But here we discovered that mathematical thinking is determined not only by purely cognitive factors. It is influenced by emotional factors of all kinds, as well as by social and cultural factors. Therefore, if we really want to describe and explain mathematical thinking we should relate also to these factors. Some of them are included in what we call affect in mathematics education. Thus, for instance, if we have to analyse the mathematical thinking of a student who solves a given problem on given a test, we should consider his or her emotional state. Under emotional pressure people might have difficulties to form certain thought processes. It is important to characterize thought processes which people can perform under pressure and others which usually are not expected to occur under pressure. It is reasonable to assume that algorithmic thought processes can be produced under pressure while heuristic processes, in the majority of people, hardly occur under pressure.

By saying that I have made a claim which has a general character. It explains and it can predict. Is it a theory? A grounded theory? A theoretical framework? It does not look like a theory. I will not discuss here the question what is a theory. However, I would like to ask what makes a set of claims to look like a theory. It seems to me that one of its features is terminology - special terms, technical notion, big words used in ways different from the way they are used in ordinary language. Why do we need such theories? Take physics, for instance. The majority of people have an intuitive theory about stable balance of physical bodies. We know that if we tilt a chair it might fall down. But when we introduce the center of mass and its laws as theoretical constructs we will be able to say much more. We will be able to explain and predict many events that we were not able to handle earlier. However, if a theory in the above sense does not explain or predict more than what we know intuitively, then it is quite superfluous. It serves mainly the researchers who invented it and others who develop it at the theoretical level to the extent of giant dimensions, but fail to tie it again to the real world. Such a development is mentioned already by Vygotsky (1927) when he discussed the crisis in Psychology. It seems to be an ongoing crisis.

The need to explain phenomena and events is perhaps imprinted in us by evolution. It gives us an evolutionary advantage. It helps us to survive dangers and disasters. From here, the distance to theory production is very small. However, theories of the above type are not crucial to the survival of mankind. They might be crucial for the survival of some university professors. It is acceptable, let us say, when we consider physicists. But is it acceptable in a group of people who consider themselves Mathematics educators?

SPECIAL STUDENTS FEELING MATHEMATICS

Melissa Rodd [2nd evaluative perspective]

For our purposes, here, students with ‘special needs’ are children who are developing differently from typical children in any way such that adaptations have to be made so that they are able to access the standard curriculum. Children who have sense impairments (e.g. deafness), medical, mobility and developmental conditions (e.g. cystic fibrosis, cerebral palsy, autism, respectively) and children who have suffered extreme adverse social conditions (e.g. abuse) are examples of students with ‘special needs’. Clearly, these students are close to the heart of the conference theme of *Inclusion and Diversity* and their affective responses to mathematics are central to their participation in mathematical activity.

While outlining distinct perspectives, our four theoretical frameworks are linked in several ways – for example, a somatic marker is a way of representing emotion in the body; self-regulation, as a dynamic process, is very strongly influenced by social norms and practices, indeed it is central to developing identity, which is most relevant in the consideration of children with special needs. The subconscious aspects of affect are noted and the question of managing emotion is particularly relevant for children with slow emotional development. All the theories are in some way trying to grapple with the relationship between learning and feeling, understanding that you cannot separate these two aspects of life; cognition and affect are integral parts of, in Damasio's words, "the feeling brain".

Students with special needs are disproportionately emotionally vulnerable in the rough-edged social world of school. Because they are, by definition, at the margins of the assumed normal distribution for some attribute, other children notice difference and may test out their own position in the social order by teasing or bossing the special needs child. And what starts in the playground seeps into the classroom. The social context of the classroom, as Op 't Eynde's theoretical framework emphasises, is central to a positive feeling about mathematics. The nature of interactions with the teacher develop a self-image of being a mathematical person. In the UK, at least, where most children with special needs are educated within the mainstream, special needs students frequently have a non-mathematician 'teaching assistant' to help them. While this arrangement facilitates access, the presence of the other adult dilutes the relationship between the mathematics teacher and the special needs student and thus the intensity of inspiration from the mathematics teacher is diminished, reducing the possibility of a neophyte relationship and a burgeoning mathematical identity.

Malmivuori's theoretical framework is particularly relevant when teaching students with challenging behaviour, as it starts from the individual, while incorporating essential social or contextual features. These students' self-regulatory systems are impaired relative to the norm of the mainstream mathematics classroom. These students get emotionally flooded very easily, a very small environmental impact, can arouse the student beyond their self-control. The frequent result is that the teacher takes firmer and firmer control, thus preventing the development of the crucial self-regulation that other children develop more easily. Indeed, in the context of mathematics, it is important that these students do experience challenge, both in mathematics and in the social space of the classroom. Pages of sums to do in silence at separate desks may be a teacher's solution to quell the difficult behaviour. Yet, a curriculum involving, for example, small groups playing maths games, should develop their interest, self-esteem and positive attitude to the mathematics classroom, which, in turn may help to improve their delayed emotional self-regulation.

Frustration is commonly experienced by students with special needs and Gerald Goldin's framework can be used to explain why their problem-solving capability may be limited and so how this frustration could have arisen: they may not be motivated by *values* that direct towards doing well in school, nor may they have the *belief* that maths is not for them, their *attitude* towards self-improvement may be wanting, and, indeed, their *emotions* may be intense and difficult to control, making successful mathematical progress less likely. The complement to student frustration is student satisfaction and this framework gives a way of connecting different aspects of affect which impinge on student frustration or satisfaction.

The neurologically-based theory of somatic markers is also helpful in understanding learners' responses and also in recognising that mere telling students not to panic, for example, has very little effect! The framework outlined by Brown and Reid explains how a critical mass of somatic markers can lead to a charged emotional orientation towards mathematics. Furthermore they implicate the unconscious both in learners' attitudes and their competencies. It shows us that teaching involves working to re-position students' somatic markers. It also shows that all fine words about awareness and self-regulation are challenged by learners' sub-conscious embodied orientations.

From the perspective of championing special needs students, these frameworks don't yet incorporate an explanation of how learning styles have an affective dimension. And teaching students with specific needs demands acknowledgement of their specific learning styles otherwise frustration and possibly anger or panic arise. So while there are obvious teaching methods for sense-impaired students (e.g. high use of visuals for the hearing-impaired), other tacks are required for other special needs in order to give them every chance to succeed. Examples: students on the autistic spectrum, who are impaired in their grasp of social situations, may be more comfortable accessing mathematics via pattern and logic rather than via a (social) context; attention-deficit hyper-active students respond to kinaesthetic activities, as their need to move can be channelled into embodying mathematical relationships.

Education in the training of the emotional mind is another issue to consider: techniques for meditation and cultivation of positive attitudes have existed for millennia. The one-pointed thinking experienced in mathematical concentration is a means by which the over-easily aroused emotional mind may be soothed; routinisation of mathematical concepts helps fluency and mathematical intimacy as well as building self-esteem.

Affective issues are clearly central to devising effective teaching methods for children with special needs. And in attending to learners with urgent and distinct needs we may well find that our raised awareness of individuality and of culture will provoke better learning environments for all students.

METHODOLOGICAL QUESTIONS IN RESEARCHING AFFECT

Jeff Evans [3rd evaluative perspective]

Here I consider the four theoretical presentations from the point of view of a set of methodological questions:

1. What is the role of theory in the study of affect in the particular approach to mathematics education research? What are the objects of study in this approach?

2. To what extent is affect understood as a social (rather than simply individual) experience? How do we need to take account in research of the social context of experiencing emotion?
3. What sorts of research questions are generated by a particular approach to affect?
4. What research strategies are preferred in this approach, and what sorts of data are appropriate? What methods of operationalising key concepts are preferred?

The exponents of all four theoretical approaches indicate their aim to establish a basis for *description* for the affective area, and all aim to *explain* the relationship of the affective to mathematical thinking and problem solving. Brown and Reid are the most explicit about wanting to provide a basis for *intervening* to help 'students of mathematics learn what to do when they do not know what to do', though all of the other contributions also mention or imply interests in students' development / change.

Goldin analyses the affective as one of several 'mutually-interacting' *systems of representation* that 'encode essential information' He presents four components of the affective domain – emotion, attitudes, beliefs and values – as a tetrahedron, thereby resisting the tendency (e.g. in McLeod, 1992) to rank them as to ‘intensity’ or ‘stability’ over time – although Goldin's *local / global* dimension resembles the latter.

Malmivuori's basic framework seems highly compatible with Goldin's; here, her ongoing, interacting processes relate to *self-systems*. Her main objects of study are students' *self-perceptions*, and their related *self-affects*, the latter connected with 'experiences of self-esteem, self-worth and /or personal control with respect to mathematics'. Malmivuori's *self-systems* are *self-regulating*, from which flows at least part of their 'dynamic' quality; put another way, *metacognition* is central to self-regulation. Goldin takes this one step further, by emphasising the importance of *meta-affect*, the 'monitoring of affect both through cognition and affect'.

Op ‘t Eynde presents two levels of analysis of affect: a *component-system approach*, which is also explicitly *situated* in the social-historical context. This framework aims to explain a number of key issues: the interweaving of cognition and emotion, along with their apparent distinctiveness; the dynamic nature of emotional processes, and the existence of 'steady states that can be labelled (e.g. anger, happiness, pride)’; and the physiological nature of emotion and its sociocultural constitution.

Brown and Reid bring together the concepts of *somatic markers* (Damasio) and *emotional orientations* (Maturana), that can be considered as constellations of somatic markers: the latter 'characterises someone's actions as appropriate to a context, like teaching [or doing mathematics]'. Teaching can be considered a *domain*, which is characterised by a *community*. This linking of emotional orientations to communities allows bringing the social into their analysis of emotions.

The other exponents also emphasise the individual-social relationship. Malmivuori points to the 'individual - environmental interaction', and its effects on students' self-appraisals and on self-affects. Goldin emphasises the role of affect as a *language for* *communication* among human beings; here, he refers presumably to physiological and behavioural aspects of the emotion component of affect. This idea also has reverberations with the claim that emotional expression functions as 'sign(s) in the network of social relations' (Burkitt, 1997, p.45, quoted in Evans, 2000, p.113). Op ‘t Eynde's bringing together of 'psychobiological' and cultural addresses the same issue.

The sorts of research questions proposed here include:

- How to understand the persistence of belief systems that are counter-productive to the goals of mathematics education? (Goldin)
- How to understand students' personal involvement and self-regulatory processes with their affect? (Malmivuori)
- How can one research the occurrence of somatic markers? Can one distinguish positive and negative somatic markers? How do somatic markers influence teacher decision-making and students' reasoning processes? How are somatic markers themselves formed? (Brown & Reid)
- How to develop a theory of emotion that addresses the three 'key issues' above? How does an individual student in a classroom continuously interpret and appraise the situation, and act upon it? Does this allow the uncovering of the beliefs and knowledge underlying these emotions and actions? And how are the rules and values that are dominant in the school community and society as a whole implicated? (Op ‘t Eynde).

Because of the emphasis on a theoretical focus and the space constraints, there is little discussion on other methodological and 'methods' issues. Op ‘t Eynde's response to the problem of how to take account in research of the social level emphasises studying the student in the classroom, and 'taking the actor's perspective', both ethnographic approaches. His suggestions for 'revealing the meanings that students give to situations and how they are constituted through interactions in class' are interviews, observations, and discourse analysis; for grasping 'the continuous flow of interpretation and appraisal processes', he recommends on-line questionnaires, 'experience sampling methods', and video-based simulated recall interviews.

There may be scope for making more explicit the overlaps and communalities in the approaches described. It may be fruitful to compare in more detail the 'systems' of Goldin, Op ‘t Eynde, and Malmivuori. For example, we could ask how Goldin's 'mathematical self-identity' relates to Malmivuori's self-affect; how his 'mathematical intimacy' relates to Op ‘t Eynde's 'engagement' in a mathematical community; how his 'mathematical integrity' relates to Cobb et al.'s (1989) 'socio-mathematical norms'.

The absence of reference to psychoanalytic perspectives is noticeable in the chosen frameworks. True, most, if not all, of the approaches mention 'unconscious' processes, but this is generally used more in the sense of 'non-conscious'. The test is whether the idea or memory has been repressed into the Unconscious, or just 'forgotten' in the subconscious; in the former case, defence mechanisms may become apparent, when certain topics are raised (Evans, 2000, pp.140-145).

SOME CLOSING (OPENING) REMARKS

George Philippou & Rosetta Zan

The effort to encompass in a single paper multiple theoretical frameworks for affect in mathematical education constitutes in itself a gigantic task. Even so, what appears in the above short presentations seems to cover the domain reasonably well. The four presenters have summarized, each from a different perspective, the most recent developments in the field; the reactions summarize how these theoretical frameworks could serve the needs of special students, and provide an appraisal of the methodological questions involved.

One is impressed by the evolution of research in affect in mathematics education. Looking back to the long history of studies in this area, it can be useful to underline the renewal of interest that in the 80's was stimulated by problem solving research. This new trend is represented by the pioneer book 'Affect and mathematical problem solving' (Adams & McLeod, 1989), in which several papers contain words such as emotions, beliefs, and attitudes in the real context of a mathematical activity. These constructs were used to better *interpret* students' mathematical behaviour that a purely cognitive approach was not capable to *explain*.

This need of interpretation instead of explanation is strictly linked to the shift from a normative (positivist) paradigm to an interpretative one, which seems necessary if we want to take into account the complexity of human behaviour, and the fact that human beings act intentionally. According to the interpretative paradigm researchers search for understanding students' intentional actions in the context of mathematical activities, and not for explaining behaviour with general rules based on a cause-effect approach. But how can we understand and interpret human actions without considering affect?

The most recent research in the field of affect in mathematics education aims at developing theoretical frameworks, also in order to increase the coherence between observing instruments and the theory itself: the presented contributions perfectly reflect this aim. As a way to open the stage for the discussion to follow, we would like to point out some unifying elements in the presentations, out of the many that the reader can easily locate and propose to discuss them.

First, one gets the impression of a more or less a consensus among the contributors about the ingredients (components) of "affect" (emotions, attitudes, beliefs and values) as well as concerning the meaning of each variable. On first sight, accepting the "tetrahedral model" refutes the view that there is "considerable diversity in the theoretical frameworks" (Hannula) and the statement of the absence of "a precise, shared language" (Goldin). Even though the ubiquity is only resolved at rather general level, one could consider this as a departure point toward more precise and operational definitions of the constructs, to analyse specifically where and why variable conceptualisations stem.

Of the other important unifying elements of the presentations that might be amplified, we would specify the following: Goldin redefines the relationship among cognition and affect; he views affect as one of several internal representation systems that "functions symbolically so as to encode essential information" that is often "complex self-referential information". How does this perspective relate to Op 't Eynde's demand to clear up "the distinctiveness and the intricate interweaving of cognition and emotion"?

Goldin has also drawn attention on affect as a means of communication; how does this relate to Op 't Eynde's socio-constructivist model, in which meaning is constructed through one's engagement in a social setting? Further, Goldin elaborates on meta-affect and one might wonder how does this construct relate to metacognition, motivation, and self-regulated learning that are extensively discussed and analysed by Malmivuori.

As Goldin describes the "messages" conveyed by eye contact, facial expressions, gestures, voice intonation, etc., that are mediated tacitly among individuals, as useful in understanding students affective state. We wonder again how all these signals concern direct implication of somatic markers in mathematics education. We are sure that Brown & Reid, the proponents of the somatic framework, would like to elaborate further in July. Some specific examples on their part might probably make the difference.

Though self-system processes have been broadly discussed by Malmivuori, within the individuals-environmental interaction, we are of the opinion that some mention of specific obstacles and limitations in pursuing research on self-esteem self-concept and particularly on self-efficacy construct (see e.g. Bandura, 1997) might be warranted. In future discussions we would expect some elaboration on the meaning and function of motivation, which has been mentioned in passive in preceding pages, and certainly the connection of this construct to other affective variables.

Discussing about theoretical frameworks for affect is necessary if we want to improve the quality of our research. But it is also necessary not to forget the very nature of our interest in affect as researchers in mathematics education. Vinner's reaction points out the risk of having theories that don't help to *explain* phenomena, or, using an interpretative approach, to *understand* individuals' intentional actions. So the question is: How do these frameworks help us in interpreting mathematical behaviour? We will face this question in the discussion, proposing the four presenters the same episode to analyse, in order to compare their theories in practice.

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