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A generalization of Banach's contraction principle for nonlinear contraction in a partial metric space

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This paper is dedicated to Professor Lj. B. Ćirić

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Abstract

We establish a fixed point theorem for nonlinear contraction in a complete partial metric space. Our result generalizes the Banach type fixed point theorem in a partial metric space in the sense of Matthews.

Keywords: Partial metric space, Banach principle, Fixed Point Theory.
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1. Introduction and Preliminaries

In 1994, Matthews [22] introduced the notion of a partial metric space in such a way that each object doesn't necessarily have to have a zero distance from itself. Also, Matthews [22] studied the Banach's contraction principle in such space. After then, many authors studied many fixed point results in partial metric spaces (see [1–5,17,19–25,27]).
In this section, we give the necessarily definitions and lemmas for the partial metric spaces.

Definition 1.1. [22] A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p_1) $x = y \Longleftrightarrow p(x, x) = p(x, y) = p(y, y)$,

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(p_2) $p(x,x) \leq p(x,y)$,

(p_3) $p(x,y) = p(y,x)$,

(p_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

Each partial metric p on X generates a T_0 topology τ_p on X. The set $\{B_p(x,\varepsilon) : x \in X,\ \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$ forms the base of τ_p

If p is a partial metric on X, then the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.1}$$

is a metric on X.

Definition 1.2. [22] Let (X,p) be a partial metric space. Then:

1. A sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.
2. A sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$.
3. A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$

The following lemma plays a major role in proving our main results.

Lemma 1.3. *[22] Let (X,p) be a partial metric space.*

1. *$\{x_n\}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).*
2. *A partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m).$$

Lemma 1.4. *[20] Let $x_n \to z$ as $n \to +\infty$ in a partial metric space (X,p) where $p(z,z) = 0$, then $\lim_{n\to+\infty} p(x_n,y) = p(z,y)$ for every $y \in X$.*

Ćirić is one of the pioneer workers in the field of fixed point theory. Ćirić established and studied many fixed point theorems for mappings satisfying different contractive conditions in complete metric spaces, for example see [8]-[16]. Then after, many authors studied many fixed point theorems by using the different types of Ćirić contractions, for example see [6, 7, 27].

In this paper, we establish some fixed point results for strong Ćirić type quasi contractions in the setting of a complete partial metric space. Also, we introduce an example to support the useability of our results.

2. The Main Result

We start our work by giving a fixed point theorem for nonlinear contraction in a partial metric space.

Theorem 2.1. *Let (X,p) be a complete partial metric space and $T : X \to X$ be a mapping satisfying*

$$\begin{aligned} p(Tx,Ty) \quad \leq \quad & \max\{p(x,y), p(x,Tx), p(y,Ty), \frac{1}{2}[p(x,Ty) + p(Tx,y)]\} \\ & -\psi(p(x,y), p(x,Tx)), \quad \forall\, x,y \in X, \end{aligned} \tag{2.1}$$

where $\psi : [0,\infty) \times [0,+\infty) \to [0,\infty)$ is a continuous mapping such that $\psi(t,s) = 0$ if and only if $t = s = 0$. Then T has a unique fixed point.

Proof. Let x_0 be an arbitrary point in X. We choose $x_1 \in X$ such that $x_1 = Tx_0$. By continuing in the same way, we construct a sequence (x_n) in X such that

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, 3, \cdots .$$

If there exists $n \in \mathbf{N}$ such that $p(x_n, x_{n+1}) = 0$, then by (p_1) and (p_2) we have $x_n = x_{n+1} = Tx_n$. Hence x_n is a fixed point of T. Now, we assume that $p(x_n, x_{n+1}) \neq 0$ for all $n \geq 0$. Thus, by (2.1), we have

$$\begin{aligned} & p(x_{n+1}, x_{n+2}) \\ = \; & p(Tx_n, Tx_{n+1}) \\ \leq \; & \max\{p(x_n, x_{n+1}), p(x_n, Tx_n), p(x_{n+1}, Tx_{n+1}), \frac{1}{2}[p(x_n, Tx_{n+1}) + p(Tx_n, x_{n+1})]\} \\ & -\psi(p(x_n, x_{n+1}), p(x_n, Tx_n)) \\ = \; & \max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2}), \frac{1}{2}[p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})]\} \\ & -\psi(p(x_n, x_{n+1}), p(x_n, x_{n+1})). \end{aligned} \tag{2.2}$$

By (p_4), we have

$$p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1}) \leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}).$$

Therefore,

$$\begin{aligned} & \max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2}), \frac{1}{2}[p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})]\} \\ \leq \; & \max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\}. \end{aligned} \tag{2.3}$$

By (2.2) and (2.3), we have

$$p(x_{n+1}, x_{n+2}) \leq \max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\} - \psi(p(x_n, x_{n+1}), p(x_n, x_{n+1})). \tag{2.4}$$

If $\max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\} = p(x_{n+1}, x_{n+2})$, then from (2.15), we have

$$p(x_{n+1}, x_{n+2}) \leq p(x_{n+1}, x_{n+2}) - \psi(p(x_n, x_{n+1}), p(x_n, x_{n+1})) < p(x_{n+1}, x_{n+2}). \tag{2.5}$$

which is a contradiction since $\psi(p(x_n, x_{n+1}), p(x_n, x_{n+1})) = 0$ and so $p(x_n, x_{n+1}) = 0$, that $x_n = x_{n+1}$. Therefore, we have $\max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\} = p(x_n, x_{n+1})$ and hence

$$p(x_{n+1}, x_{n+2}) \leq p(x_n, x_{n+1}) - \psi(p(x_n, x_{n+1}), p(x_n, x_{n+1})) \leq p(x_n, x_{n+1}). \tag{2.6}$$

By (2.6), we have $\{p(x_n, x_{n+1})\}$ is a non-increasing sequence of positive real numbers. Thus, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = r. \tag{2.7}$$

Letting $n \to \infty$ in (2.6) and using (2.7) and the properties of ψ, we have
$r \leq r - \psi(r, r)$. Thus $\psi(r, r) = 0$ and hence $r = 0$. Therefore

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0. \tag{2.8}$$

Our next step is to prove that

$$\lim_{n,m\to\infty} p(x_n, x_m) = 0.$$

Suppose the contrary, that is,

$$\lim_{n,m\to\infty} p(x_n, x_m) \neq 0.$$

Then there exists $\epsilon > 0$ for which we can find two subsequences $\{x_{m(k)}\}, \{x_{n(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which

$$n(k) > m(k) > k, \;\; p(x_{n(k)}, x_{m(k)}) \geq \epsilon. \tag{2.9}$$

This means that

$$p(x_{n(k)-1}, x_{m(k)}) < \epsilon. \tag{2.10}$$

From (2.9) and (2.10), we have

$$\begin{aligned}\epsilon \leq p(x_{n(k)}, x_{m(k)}) &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) - p(x_{n(k)-1}, x_{n(k)-1}) \\ &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) \\ &< \epsilon + p(x_{n(k)}, x_{n(k)-1})\end{aligned}$$

Taking $k \to \infty$ and using (2.8), we get

$$\lim_{k\to\infty} p(x_{n(k)}, x_{m(k)}) = \epsilon \tag{2.11}$$

By (p_3) and (p_4), we have

$$\begin{aligned}& p(x_{n(k)}, x_{m(k)}) \\ \leq\;& p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)+1}, x_{m(k)}) - p(x_{n(k)+1}, x_{n(k)+1}) \\ \leq\;& p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)+1}, x_{m(k)}) \\ \leq\;& p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)+1}, x_{m(k)+1}) + p(x_{m(k)+1}, x_{m(k)}) - p(x_{m(k)+1}, x_{m(k)+1}) \\ \leq\;& p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)+1}, x_{m(k)+1}) + p(x_{m(k)+1}, x_{m(k)}) \\ \leq\;& 2p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)}, x_{m(k)+1}) + p(x_{m(k)+1}, x_{m(k)}) - p(x_{n(k)}, x_{n(k)}) \\ \leq\;& 2p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)}, x_{m(k)+1}) + p(x_{m(k)+1}, x_{m(k)}) \\ \leq\;& 2p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)}, x_{m(k)}) + 2p(x_{m(k)+1}, x_{m(k)}) - p(x_{m(k)}, x_{m(k)}) \\ \leq\;& 2p(x_{n(k)}, x_{n(k)+1}) + p(x_{n(k)}, x_{m(k)}) + 2p(x_{m(k)+1}, x_{m(k)})\end{aligned}$$

Taking $k \to \infty$ in the above inequalities and using (2.8), (2.11), we get that

$$\begin{aligned}& \lim_{k\to\infty} p(x_{n(k)}, x_{m(k)}) = \lim_{k\to\infty} p(x_{n(k)+1}, x_{m(k)}) \\ =\;& \lim_{k\to\infty} p(x_{n(k)}, x_{m(k)+1}) = \lim_{k\to\infty} p(x_{n(k)+1}, x_{m(k)+1}) = \epsilon\end{aligned} \tag{2.12}$$

Now, from (2.1), we have

$$\begin{aligned}& p(x_{m(k)+1}, x_{n(k)+1}) = p(Tx_{m(k)}, Tx_{n(k)}) \\ \leq\;& \max\{p(x_{m(k)}, x_{n(k)}), p(x_{m(k)}, Tx_{m(k)}), p(x_{n(k)}, Tx_{n(k)}), \\ & \frac{1}{2}(p(x_{m(k)}, Tx_{n(k)}) + p(Tx_{m(k)}, x_{n(k)}))\} - \psi(p(x_{m(k)}, x_{n(k)}), p(x_{m(k)}, Tx_{m(k)})) \\ =\;& \max\{p(x_{m(k)}, x_{n(k)}), p(x_{m(k)}, x_{m(k)+1}), p(x_{n(k)}, x_{n(k)+1}), \\ & \frac{1}{2}(p(x_{m(k)}, x_{n(k)+1}) + p(x_{m(k)+1}, x_{n(k)}))\} - \psi(p(x_{m(k)}, x_{n(k)}), p(x_{m(k)}, x_{m(k)+1}))\end{aligned} \tag{2.13}$$

On letting $k \to \infty$ in (2.13) and using (2.8), (2.12) and the properties of ψ, we have

$$\epsilon \leq \epsilon - \psi(\epsilon, \epsilon) < \epsilon$$

which is a contradiction. So, we have

$$\lim_{n,m\to\infty} p(x_n, x_m) = 0.$$

Since $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and finite, we conclude that (x_n) is a Cauchy sequence in (X, p).

By (1.1), we have $p^s(x_n, x_m) \leq 2p(x_n, x_m)$. Therefore

$$\lim_{n,m\to\infty} p^s(x_n, x_m) = 0. \tag{2.14}$$

Thus, by Lemma 1.3, $\{x_n\}$ is a Cauchy sequence in both (X, p^s) and (X, p). Thus, there exists $x \in X$ such that $\lim_{n\to\infty} p^s(x_n, x) = 0$ if and only if

$$p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m) = 0.$$

Now, we prove that x is a fixed point of T. From (2.1), we have

$$\begin{aligned} & p(Tx, x_{n+1}) = p(Tx, Tx_n) \\ \leq \ & \max\{p(x, x_n), p(x, Tx), p(x_n, Tx_n), \frac{1}{2}(p(Tx, x_n) + p(x, Tx_n)\} - \psi(p(x, x_n), p(x, Tx)) \\ = \ & \max\{p(x, x_n), p(x, Tx), p(x_n, x_{n+1}), \frac{1}{2}(p(Tx, x_n) + p(x, x_{n+1})\} - \psi(p(x, x_n), p(x, Tx)). \end{aligned}$$

Letting $n \to \infty$ in the above inequality, and using Lemma (1.4) we obtain

$$p(x, Tx) \leq p(x, Tx) - \psi(0, p(x, Tx)).$$

Hence $\psi(0, p(x, Tx)) = 0$. Thus $p(x, Tx) = 0$. By (p_1) and (p_2), we have $Tx = x$. Therefore x is a fixed point of T. To prove the uniqueness of the fixed point. Suppose that y is another fixed point of T. From (2.1), we have

$$p(x, y) = p(Tx, Ty) \leq \max\{p(x, y), p(x, x), p(y, y)\} - \psi(p(x, y), p(x, x)).$$

Thus, we have $\psi(p(x, y), p(x, x)) = 0$. Hence $p(x, y) = p(x, x) = 0$. By (p_2), we have $p(y, y) = 0$. Therefore by (p_1), we get that $x = y$. □

By taking $\psi : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ via $\psi(s, t) = (1 - r)\max\{s, t\}$ where $r \in [0, 1)$ in Theorem (2.1), we have the following result:

Corollary 2.2. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a mapping satisfying*

$$p(Tx, Ty) \leq r \max\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}[p(x, Ty) + p(Tx, y)]\}$$

for all $x, y \in X$. If $r \in [0, 1)$, then T has a unique fixed point.

As a special case of Corollary (2.2), we have the following result of Matthews.

Corollary 2.3. *[22] Let (X, p) be a complete partial metric space and $T : X \to X$ be a mapping satisfying $p(Tx, Ty) \leq rp(x, y)$ for all $x, y \in X$. If $r \in [0, 1)$, then T has a unique fixed point.*

As a direct result of Theorem 2.1, we have the following result.

Corollary 2.4. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a mapping satisfying*

$$\begin{aligned} p(Tx, Ty) \ \leq \ & \max\{p(x, Tx), p(y, Ty), \frac{1}{2}[p(x, Ty) + p(Tx, y)]\} \\ & -\psi(p(x, y), p(x, Tx)), \quad \forall\, x, y \in X, \end{aligned} \tag{2.15}$$

where $\psi : [0, \infty) \times [0, +\infty) \to [0, \infty)$ is a continuous mapping such that $\psi(t, s) = 0$ if and only if $t = s = 0$. Then T has a unique fixed point.

Now, we introduced an example to support the useability of our results.

Example 2.5. Let $X = [0, +\infty)$. Define the partial metric space on X by $p(x, y) = \max\{x, y\}$. Also, define the mapping $T : X \to X$ by $T(x) = \frac{x^2}{1+x}$ and the function $\psi : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ by $\psi(s, t) = \frac{s+t}{2+s+t}$. Then

1. (X, p) is a complete partial metric space.
2. T satisfies (2.15) of Corollary 2.4.
3. If we replace p by p^s in (2.15) of Corollary 2.4, then T does not satisfy (2.15) of Corollary 2.4.

Proof. For (1) see Ref. [1]. To prove (2), suppose $y \leq x$. Then

$$p(Tx, Ty) = \max\left\{\frac{x^2}{1+x}, \frac{y^2}{1+y}\right\} = \frac{x^2}{1+x},$$

$$\max\{p(x, Tx), p(y, Ty)\} = \max\{x, y\} = x$$

and

$$\psi(p(x, y), p(x, Tx)) = \psi(x, x) = \frac{2x}{2+2x}.$$

Since

$$\frac{x^2}{1+x} \leq x - \frac{2x}{2+2x} = \frac{x^2}{1+x},$$

we have T satisfies (2.15) of Corollary 2.4.

To prove (3), notice that

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y) = 2\max\{x, y\} - x - y = |x - y|.$$

Now, take $x = 1$ and $y = 0$. Then

$$p^s(T1, T0) = p^s\left(\frac{1}{2}, 0\right) = \frac{1}{2},$$

$$\max\{p^s(1, T1), p^s(0, T0)\} = \max\left\{p^s\left(1, \frac{1}{2}\right), p^s(0, 0)\right\} = \frac{1}{2}$$

and

$$\psi(p^s(1, 0), p^s(1, T1)) = \psi\left(1, \frac{1}{2}\right) = \frac{3}{7}.$$

Since $\frac{1}{2}$ is not less or equal $\frac{1}{2} - \frac{3}{7}$, we get that (3) does hold for $x = 1$ and $y = 0$. □

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