J. Nonlinear Sci. Appl. 4 (2011), no. 1, 60–69

The Journal of Nonlinear Sciences and Applications
http://www.tjnsa.com

APPROXIMATELY PARTIAL TERNARY QUADRATIC DERIVATIONS ON BANACH TERNARY ALGEBRAS

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Dedicated to Themistocles M. Rassias on the occasion of his sixtieth birthday

Abstract. Let $A_1, A_2, ..., A_n$ be normed ternary algebras over the complex field $\mathbb{C}$ and let B be a Banach ternary algebra over $\mathbb{C}$. A mapping δ_k from $A_1 \times \cdots \times A_n$ into B is called a *k-th partial ternary quadratic derivation* if there exists a mapping $g_k : A_k \to B$ such that

$$\begin{aligned}\delta_k(x_1, \cdots, [a_k b_k c_k], \cdots, x_n) &= [g_k(a_k) g_k(b_k) \delta_k(x_1, \cdots, c_k, \cdots, x_n)] \\ &+ [g_k(a_k) \delta_k(x_1, \cdots, b_k, \cdots, x_n) g_k(c_k)] \\ &+ [\delta_k(x_1, \cdots, a_k, \cdots, x_n) g_k(b_k) g_k(c_k)]\end{aligned}$$

and

$$\begin{aligned}&\delta_k(x_1, \cdots, a_k + b_k, \cdots, x_n) + \delta_k(x_1, \cdots, a_k - b_k, \cdots, x_n) \\ &= 2\delta_k(x_1, \cdots, a_k, \cdots, x_n) + 2\delta_k(x_1, \cdots, b_k, \cdots, x_n)\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ $(i \neq k)$. We prove the Hyers-Ulam-Rassias stability of the partial ternary quadratic derivations in Banach ternary algebras.

1. Introduction

The stability of functional equations was first introduced by Ulam [41] in 1940. More precisely, he proposed the following problem: given a group G_1, a metric group (G_2, d), and a positive number ϵ, does there exists a $\delta > 0$ such that if a function $f : G_1 \to G_2$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$

Date: Received: September 28, 2010; Revised: November 22, 2010.

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2000 *Mathematics Subject Classification.* Primary 46K05, 39B82, 39B52, 47B47.
Key words and phrases. Hyers-Ulam-Rassias stability; Banach ternary algebra; Partial ternary quadratic derivation.

for all $x, y \in G_1$, then there exists a homomorphism $T : G_1 \to G_2$ such that $d(f(x), T(x)) < \epsilon$ for all $x \in G_1$? As mentioned above, when this problem has a solution, we say that the homomorphisms from G_1 to G_2 are stable. In 1941, Hyers [31] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1978, Rassias [39] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Rassias [39] is called the Hyers-Ulam-Rassias stability. The following is according to Rassias' theorem.

Theorem 1.1. *Let $f : E \to E'$ be a mapping from a norm vector space E into a Banach space E' subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to E'$ such that for all $x \in E$,

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2^p}\|x\|^p.$$

This new concept is known as Hyers-Ulam-Rassias stability of functional equations (see [27],[28],[30],[32],[35],[36] and [38]). The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.1}$$

is related to symmetric bi-additive function. It is natural that this equation is called a quadratic functional equation. In particular, every solution of the quadratic equation (1.1) is said to be a quadratic function. It is well known that a function f between real vector spaces is quadratic if and only if there exists a unique symmetric bi-additive function B such that $f(x) = B(x, x)$ for all x (see [34]). The bi-additive function B is given by

$$B(x, y) = \frac{1}{4}(f(x+y) - f(x-y))$$

Hyers-Ulam-Rassias stability problem for the quadratic functional equation (1.1) was proved by Skof for functions $f : A \longrightarrow B$, where A is normed space and B Banach space (see [4],[6],[29] and [40]). For more detailed definitions of such terminologies, we can refer to [1],[7]–[14],[16],[19]–[22] and [24].

A ternary (associative) algebra $(A, [\ ])$ is a linear space A over a scalar field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ equipped with a linear mapping, the so-called ternary product, $[\,] : A \times A \times A \to A$ such that $[[abc]de] = [a[bcd]e] = [ab[cde]]$ for all $a, b, c, d, e \in A$. This notion is a natural generalization of the binary case. Indeed if $(A, \odot)$ is a usual (binary) algebra then $[abc] := (a \odot b) \odot c$ induced a ternary product making A in to a ternary algebra which will be called trivial. By a Banach ternary algebra we mean a ternary algebra equipped with a complete norm $\|.\|$ such that $\|[abc]\| \leq \|a\|\|b\|\|c\|$ for all $a, b, c \in A$. It seems that approximate derivations were first investigated by Jun and Park [33]. Recently, the stability of derivations has been investigated by some authors; see [2, 5, 23, 26] and references therein. For more detailed definitions of such terminologies, we can refer to [3],[15],[17],[18],[25] and [37].

2. **Main results**

Let $A_1, A_2, ..., A_n$ be normed ternary algebras over the complex field $\mathbb{C}$ and let B be a Banach ternary algebra over $\mathbb{C}$. A mapping δ_k from $A_1 \times \cdots \times A_n$ into B is called a *k-th partial ternary quadratic derivation* if there exists a mapping $g_k : A_k \to B$ such that

$$\begin{aligned}\delta_k(x_1, \cdots, [a_k b_k c_k], \cdots, x_n) &= [g_k(a_k) g_k(b_k) \delta_k(x_1, \cdots, c_k, \cdots, x_n)] \\ &+ [g_k(a_k) \delta_k(x_1, \cdots, b_k, \cdots, x_n) g_k(c_k)] \\ &+ [\delta_k(x_1, \cdots, a_k, \cdots, x_n) g_k(b_k) g_k(c_k)]\end{aligned}$$

and

$$\begin{aligned}&\delta_k(x_1, \cdots, a_k + b_k, \cdots, x_n) + \delta_k(x_1, \cdots, a_k - b_k, \cdots, x_n) \\ &= 2\delta_k(x_1, \cdots, a_k, \cdots, x_n) + 2\delta_k(x_1, \cdots, b_k, \cdots, x_n)\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ $(i \neq k)$. We denote that $0_k, 0_B$ are zero elements of A_k, B, respectively.

Theorem 2.1. *Let $p \geq 0$ be given with $p < 2$ and let θ be nonnegative real numbers. Let $F_k : A_1 \times \cdots \times A_n \to B$ be a mapping with $F_k(x_1, \cdots, 0_k, \cdots, x_n) = 0_B$. Suppose that there exists a quadratic mapping $g_k : A_k \to B$ such that*

$$\begin{aligned}&\|F_k(x_1, \cdots, a_k + b_k, \cdots, x_n) + F_k(x_1, \cdots, a_k - b_k, \cdots, x_n) \\ &-2F_k(x_1, \cdots, a_k, \cdots, x_n) - 2F_k(x_1, \cdots, b_k, \cdots, x_n)\| \\ &\leq \theta(\|a_k\|^p + \|b_k\|^p)\end{aligned} \tag{2.1}$$

$$\begin{aligned}&\|F_k(x_1, \cdots, [a_k b_k c_k], \cdots, x_n) - [g_k(a_k) g_k(b_k) F_k(x_1, \cdots, c_k, \cdots, x_n)] \\ &-[g_k(a_k) F_k(x_1, \cdots, b_k, \cdots, x_n) g_k(c_k)] - [F_k(x_1, \cdots, a_k, \cdots, x_n) g_k(b_k) g_k(c_k)]\| \\ &\leq \theta(\|a_k\|^p + \|b_k\|^p + \|c_k\|^p)\end{aligned} \tag{2.2}$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. Then there exists a unique k-th partial ternary quadratic derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ such that

$$\|F_k(x_1, \cdots, x_n) - \delta_k(x_1, \cdots, x_n)\| \leq \frac{2\theta}{4 - 2^p}\|x_k\|^p \tag{2.3}$$

holds for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$.

Proof. In (2.1), putting $a_k = b_k = x_k$, we have

$$\|F_k(x_1, \cdots, 2x_k, \cdots, x_n) - 4F_k(x_1, \cdots, x_k, \cdots, x_n)\| \leq 2\theta\|x_k\|^p, \tag{2.4}$$

that is,

$$\|F_k(x_1, \cdots, x_k, \cdots, x_n) - \frac{1}{4}F_k(x_1, \cdots, 2x_k, \cdots, x_n)\| \leq \frac{\theta}{2}\|x_k\|^p \tag{2.5}$$

for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$. One can use induction on m to show that

$$\begin{aligned} &\|F_k(x_1, \cdots, x_k, \cdots, x_n) - \frac{1}{2^{2m}} F_k(x_1, \cdots, 2^m x_k, \cdots, x_n)\| \\ &\leq \frac{\theta}{2} \sum_{i=0}^{m-1} 2^{i(p-2)} \|x_k\|^p \end{aligned} \tag{2.6}$$

for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$ and all non-negative integers m. Hence

$$\begin{aligned} &\|\frac{1}{2^{2j}} F_k(x_1, \cdots, 2^j x_k, \cdots, x_n) - \frac{1}{2^{2(m+j)}} F_k(x_1, \cdots, 2^{(m+j)} x_k, \cdots, x_n)\| \\ &\leq \frac{\theta}{2} \sum_{i=j}^{m+j-1} 2^{i(p-2)} \|x_k\|^p \end{aligned} \tag{2.7}$$

for all non-negative integers m and j with $m \geq j$ and all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$. It follows from $p < 2$ that the sequence $\{\frac{1}{2^{2m}} F_k(x_1, \cdots, 2^m x_k, \cdots, x_n)\}$ is Cauchy. Due to the completeness of B, this sequence is convergent. So one can define the mapping $\delta_k : A_1 \times \cdots \times A_n \to B$ given by

$$\delta_k(x_1, \cdots, x_n) = \lim_{m \to \infty} \frac{1}{2^{2m}} F_k(x_1, \cdots, 2^m x_k, \cdots, x_n) \tag{2.8}$$

for all $x_i \in A_i$ $(i = 1, \cdots, n)$. In (2.1), replacing a_k, b_k with $2^m a_k, 2^m b_k$, respectively, we obtain that

$$\begin{aligned} &\|\frac{1}{2^{2m}} F_k(x_1, \cdots, 2^m(a_k + b_k), \cdots, x_n) + \frac{1}{2^{2m}} F_k(x_1, \cdots, 2^m(a_k - b_k), \cdots, x_n) \\ &- \frac{2}{2^{2m}} F_k(x_1, \cdots, 2^m a_k, \cdots, x_n) - \frac{2}{2^{2m}} F_k(x_1, \cdots, 2^m b_k, \cdots, x_n)\| \\ &\leq \theta \cdot 2^{m(p-2)} (\|a_k\|^p + \|b_k\|^p) \end{aligned}$$

which tends to zero as $m \to \infty$. Thus we obtain

$$\begin{aligned} &\delta_k(x_1, \cdots, a_k + b_k, \cdots, x_n) + \delta_k(x_1, \cdots, a_k - b_k, \cdots, x_n) \\ &= 2\delta_k(x_1, \cdots, a_k, \cdots, x_n) + 2\delta_k(x_1, \cdots, b_k, \cdots, x_n) \end{aligned} \tag{2.9}$$

for all $a_k, b_k \in A_k$ and all $x_i \in A_i (i \neq k)$. Hence δ_k is quadratic with respect to the k-th variable. It follows from (2.7) that

$$\|F_k(x_1, \cdots, x_k, \cdots, x_n) - \delta_k(x_1, \cdots, x_k, \cdots, x_n)\| \leq \frac{2\theta}{4 - 2^p} \|x_k\|^p$$

for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$.
Replacing a_k, b_k, c_k with $2^m a_k, 2^m b_k, 2^m c_k$, respectively, in (2.2), we obtain

$$\begin{aligned} &\|F_k(x_1, \cdots, 2^{3m}[a_k b_k c_k], \cdots, x_n) - [2^{2m} g_k(a_k) 2^{2m} g_k(b_k) F_k(x_1, \cdots, 2^m c_k, \cdots, x_n)] \\ &-[2^{2m} g_k(a_k) F_k(x_1, \cdots, 2^m b_k, \cdots, x_n) 2^{2m} g_k(c_k)] \\ &-[F_k(x_1, \cdots, 2^m a_k, \cdots, x_n) 2^{2m} g_k(b_k) 2^{2m} g_k(c_k)]\| \\ &\leq 2^{mp}.\theta(\|a_k\|^p + \|b_k\|^p + \|c_k\|^p). \end{aligned}$$

Then we have

$$\begin{aligned}\|\frac{1}{2^{6m}}&F_k(x_1,\cdots,2^{3m}[a_kb_kc_k],\cdots,x_n)\\&-\frac{1}{2^{6m}}[2^{2m}g_k(a_k)2^{2m}g_k(b_k)F_k(x_1,\cdots,2^mc_k,\cdots,x_n)]\\&-\frac{1}{2^{6m}}[2^{2m}g_k(a_k)F_k(x_1,\cdots,2^mb_k,\cdots,x_n)2^{2m}g_k(c_k)]\\&-\frac{1}{2^{6m}}[F_k(x_1,\cdots,2^ma_k,\cdots,x_n)2^{2m}g_k(b_k)2^{2m}g_k(c_k)]\|\\&\leq 2^{m(p-6)}.\theta(\|a_k\|^p+\|b_k\|^p+\|c_k\|^p).\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$. Passing the limit $m \to \infty$ in above inequality, we obtain

$$\begin{aligned}\delta_k(x_1,\cdots,[a_kb_kc_k],\cdots,x_n) &= [g_k(a_k)g_k(b_k)\delta_k(x_1,\cdots,c_k,\cdots,x_n)]\\&+[g_k(a_k)\delta_k(x_1,\cdots,b_k,\cdots,x_n)g_k(c_k)]\\&+[\delta_k(x_1,\cdots,a_k,\cdots,x_n)g_k(b_k)g_k(c_k)]\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ $(i \neq k)$.
Finally, to prove the uniqueness of δ_k, let $\delta_k^{'} : A_1 \times \cdots \times A_n \to B$ be another k-th partial ternary quadratic derivation satisfying (2.3). Then we have

$$\begin{aligned}&\|\delta_k(x_1,\cdots,x_n)-\delta_k^{'}(x_1,\cdots,x_n)\|\\&=\frac{1}{2^{2m}}\|\delta_k(x_1,\cdots,2^mx_k,\cdots,x_n)-\delta_k^{'}(x_1,\cdots,2^mx_k,\cdots,x_n)\|\\&\leq\frac{1}{2^{2m}}(\|\delta_k(x_1,\cdots,2^mx_k,\cdots,x_n)-F_k(x_1,\cdots,2^mx_k,\cdots,x_n)\|\\&+\|F_k(x_1,\cdots,2^mx_k,\cdots,x_n)-\delta_k^{'}(x_1,\cdots,2^mx_k,\cdots,x_n)\|)\\&\leq\theta\sum_{i=m}^{\infty}2^{i(p-2)}\|x_k\|^p.\end{aligned}$$

which tends to zero as $m \to \infty$ for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$. So we can conclude that $\delta_k(x_1, \cdots, x_n) = \delta_k^{'}(x_1, \cdots, x_n)$. This proves the uniqueness of δ. □

Theorem 2.2. *Let $p > 2$ be and let θ be nonnegative real numbers. Let $F_k : A_1 \times \cdots \times A_n \to B$ be a mapping with $F_k(x_1, \cdots, 0_k, \cdots, x_n) = 0_B$. Suppose that there exists a quadratic mapping $g_k : A_k \to B$ such that satisfying (2.1) and (2.2) for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. Then there exists a unique k-th partial ternary quadratic derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ such that*

$$\|F_k(x_1,\cdots,x_n)-\delta_k(x_1,\cdots,x_n)\|\leq\frac{2\theta}{2^p-4}\|x_k\|^p \tag{2.10}$$

holds for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$.

Proof. In (2.1), putting $a_k = b_k = \frac{x_k}{2}$, we have

$$\|F_k(x_1,\cdots,x_k,\cdots,x_n)-4F_k(x_1,\cdots,\tfrac{x_k}{2},\cdots,x_n)\|\leq\tfrac{2\theta}{2^p}\|x_k\|^p, \tag{2.11}$$

for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$. One can use induction on m to show that

$$\|F_k(x_1, \cdots, x_k, \cdots, x_n) - 2^{2m} F_k(x_1, \cdots, \frac{x_k}{2^m}, \cdots, x_n)\| \leq \frac{2\theta}{2^p} \sum_{i=0}^{m-1} 2^{i(2-p)} \|x_k\|^p \tag{2.12}$$

for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$ and all non-negative integers m. Hence

$$\|2^{2j} F_k(x_1, \cdots, \frac{x_k}{2^j}, \cdots, x_n) - 2^{2(m+j)} F_k(x_1, \cdots, \frac{x_k}{2^{(m+j)}}, \cdots, x_n)\| \leq \frac{2\theta}{2^p} \sum_{i=j}^{m+j-1} 2^{i(2-p)} \|x_k\|^p \tag{2.13}$$

for all non-negative integers m and j with $m \geq j$ and all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$. It follows from $p > 2$ that the sequence $\{2^{2m} F_k(x_1, \cdots, \frac{x_k}{2^m}, \cdots, x_n)\}$ is Cauchy. Due to the completeness of B, this sequence is convergent. So one can define the mapping $\delta_k : A_1 \times \cdots \times A_n \to B$ given by

$$\delta_k(x_1, \cdots, x_n) = \lim_{m\to\infty} 2^{2m} F_k(x_1, \cdots, \frac{x_k}{2^m}, \cdots, x_n) \tag{2.14}$$

for all $x_i \in A_i$ $(i = 1, \cdots, n)$. In (2.1), replacing a_k, b_k with $\frac{a_k}{2^m}, \frac{b_k}{2^m}$, respectively, we obtain that

$$\begin{aligned} &\|2^{2m} F_k(x_1, \cdots, \frac{a_k + b_k}{2^m}, \cdots, x_n) + 2^{2m} F_k(x_1, \cdots, \frac{a_k - b_k}{2^m}, \cdots, x_n) \\ &-2 \cdot 2^{2m} F_k(x_1, \cdots, \frac{a_k}{2^m}, \cdots, x_n) - 2 \cdot 2^{2m} F_k(x_1, \cdots, \frac{b_k}{2^m}, \cdots, x_n)\| \\ &\leq \theta \cdot 2^{m(2-p)} (\|a_k\|^p + \|b_k\|^p) \end{aligned}$$

which tends to zero as $m \to \infty$. Thus we obtain

$$\begin{aligned} &\delta_k(x_1, \cdots, a_k + b_k, \cdots, x_n) + \delta_k(x_1, \cdots, a_k - b_k, \cdots, x_n) \\ &= 2\delta_k(x_1, \cdots, a_k, \cdots, x_n) + 2\delta_k(x_1, \cdots, b_k, \cdots, x_n) \end{aligned} \tag{2.15}$$

for all $a_k, b_k \in A_k$ and all $x_i \in A_i (i \neq k)$. Hence δ_k is quadratic with respect to the k-th variable. It follows from (2.12) that

$$\|F_k(x_1, \cdots, x_k, \cdots, x_n) - \delta_k(x_1, \cdots, x_k, \cdots, x_n)\| \leq \frac{2\theta}{2^p - 4} \|x_k\|^p$$

for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$.
Replacing a_k, b_k, c_k with $\frac{a_k}{2^m}, \frac{b_k}{2^m}, \frac{c_k}{2^m}$, respectively, in (2.2), we obtain

$$\begin{aligned} &\|F_k(x_1, \cdots, \frac{[a_k b_k c_k]}{2^{3m}}, \cdots, x_n) - [\frac{g_k(a_k)}{2^{2m}} \frac{g_k(b_k)}{2^{2m}} F_k(x_1, \cdots, \frac{c_k}{2^m}, \cdots, x_n)] \\ &-[\frac{g_k(a_k)}{2^{2m}} F_k(x_1, \cdots, \frac{b_k}{2^m}, \cdots, x_n) \frac{g_k(c_k)}{2^{2m}}] \\ &-[F_k(x_1, \cdots, \frac{a_k}{2^m}, \cdots, x_n) \frac{g_k(b_k)}{2^{2m}} \frac{g_k(c_k)}{2^{2m}}]\| \\ &\leq \frac{\theta}{2^{mp}} (\|a_k\|^p + \|b_k\|^p + \|c_k\|^p). \end{aligned}$$

Then we have

$$\begin{aligned}\|2^{6m}F_k(x_1,\cdots,\frac{[a_kb_kc_k]}{2^{3m}},\cdots,x_n)-2^{6m}[\frac{g_k(a_k)}{2^{2m}}\frac{g_k(b_k)}{2^{2m}}F_k(x_1,\cdots,\frac{c_k}{2^m},\cdots,x_n)]\\ -2^{6m}[\frac{g_k(a_k)}{2^{2m}}F_k(x_1,\cdots,\frac{b_k}{2^m},\cdots,x_n)\frac{g_k(c_k)}{2^{2m}}]\\ -2^{6m}[F_k(x_1,\cdots,\frac{a_k}{2^m},\cdots,x_n)\frac{g_k(b_k)}{2^{2m}}\frac{g_k(c_k)}{2^{2m}}]\|\\ \leq 2^{m(6-p)}.\theta(\|a_k\|^p+\|b_k\|^p+\|c_k\|^p).\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$. Passing the limit $m \to \infty$ in above inequality, we obtain

$$\begin{aligned}\delta_k(x_1,\cdots,[a_kb_kc_k],\cdots,x_n)=[g_k(a_k)g_k(b_k)\delta_k(x_1,\cdots,c_k,\cdots,x_n)]\\ +[g_k(a_k)\delta_k(x_1,\cdots,b_k,\cdots,x_n)g_k(c_k)]\\ +[\delta_k(x_1,\cdots,a_k,\cdots,x_n)g_k(b_k)g_k(c_k)]\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ $(i \neq k)$.
Finally, to prove the uniqueness of δ_k, let $\delta_k^{'} : A_1 \times \cdots \times A_n \to B$ be another k-th partial ternary quadratic derivation satisfying (2.10). Then we have

$$\begin{aligned}&\|\delta_k(x_1,\cdots,x_n)-\delta_k^{'}(x_1,\cdots,x_n)\|\\ &=2^{2m}\|\delta_k(x_1,\cdots,\frac{x_k}{2^m},\cdots,x_n)-\delta_k^{'}(x_1,\cdots,\frac{x_k}{2^m},\cdots,x_n)\|\\ &\leq 2^{2m}(\|\delta_k(x_1,\cdots,\frac{x_k}{2^m},\cdots,x_n)-F_k(x_1,\cdots,\frac{x_k}{2^m},\cdots,x_n)\|\\ &+\|F_k(x_1,\cdots,\frac{x_k}{2^m},\cdots,x_n)-\delta_k^{'}(x_1,\cdots,\frac{x_k}{2^m},\cdots,x_n)\|)\\ &\leq \theta\sum_{i=m}^{\infty}2^{i(2-p)}\|x_k\|^p.\end{aligned}$$

which tends to zero as $m \to \infty$ for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$. So we can conclude that $\delta_k(x_1, \cdots, x_n) = \delta_k^{'}(x_1, \cdots, x_n)$. This proves the uniqueness of δ. □

By Theorems 2.1 and 2.2 we solve the following Hyers-Ulam-Rassias stability problem.

Theorem 2.3. *Let ϵ be nonnegative real numbers and let $F_k : A_1\times\cdots\times A_n \to B$ be a mapping with $F_k(x_1, \cdots, 0_k, \cdots, x_n) = 0_B$. Assume that there exist a quadratic mapping $g_k : A_k \to B$ such that*

$$\begin{aligned}\|F_k(x_1,\cdots,a_k+b_k,\cdots,x_n)+F_k(x_1,\cdots,a_k-b_k,\cdots,x_n)\\ -2F_k(x_1,\cdots,a_k,\cdots,x_n)-2F_k(x_1,\cdots,b_k,\cdots,x_n)\|\leq\epsilon\end{aligned} \quad (2.16)$$

$$\begin{aligned}\|F_k(x_1,\cdots,[a_kb_kc_k],\cdots,x_n)-[g_k(a_k)g_k(b_k)F_k(x_1,\cdots,c_k,\cdots,x_n)]\\ -[g_k(a_k)F_k(x_1,\cdots,b_k,\cdots,x_n)g_k(c_k)]\\ -[F_k(x_1,\cdots,a_k,\cdots,x_n)g_k(b_k)g_k(c_k)]\|\leq\epsilon\end{aligned} \quad (2.17)$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. *Then there exists a unique* k*-th partial ternary quadratic derivation* $\delta_k : A_1 \times \cdots \times A_n \to B$ *such that*

$$\|F_k(x_1, \cdots, x_n) - \delta_k(x_1, \cdots, x_n)\| \leq \frac{\epsilon}{3} \tag{2.18}$$

holds for all $x_i \in A_i$ $(i = 1, 2, \cdots, n)$.

Proof. In Theorem 2.1, by putting $p := 0$ and $\theta := \frac{\epsilon}{2}$, we obtain the conclusion of the theorem. □

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