

A NONCOMMUTATIVE CONVEXITY IN C^* -BIMODULES

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Abstract. Let \mathcal{A} and \mathcal{B} be C^* -algebras. We consider a noncommutative convexity in Hilbert \mathcal{A} - \mathcal{B} -bimodules, called \mathcal{A} - \mathcal{B} -convexity, as a generalization of C^* -convexity in C^* -algebras. We show that if \mathcal{X} is a Hilbert \mathcal{A} - \mathcal{B} -bimodule, then $\mathcal{M}_n(\mathcal{X})$ is a Hilbert $\mathcal{M}_n(\mathcal{A})$ - $\mathcal{M}_n(\mathcal{B})$ -bimodule and apply it to show that the closed unit ball of every Hilbert \mathcal{A} - \mathcal{B} -bimodule is \mathcal{A} - \mathcal{B} -convex. Some properties of this kind of convexity and various examples have been given.

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2010 Mathematics Subject Classification: Primary 46L89; Secondary 52A01, 46L08.

Keywords: Matrix convex set; C^* -algebra; Hilbert C^* -bimodule; noncommutative convexity.

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Surveys in Mathematics and its Applications **12** (2017), 7 – 21
<http://www.utgjiu.ro/math/sma>