A PROPERTY OF FIBONACCI WORDS

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Let A be a finite, nonempty, set or *alphabet* and A^* the free monoid generated by A. The elements of A are called *letters* and those of A^* *words*. We suppose that the cardinality $|A|$ is ≥ 2. The sequence $\{f_n\}$, $n \geq 1$, of words of Fibonacci is, inductively, defined as:

$$f_1 = a \ , \ f_2 = b \ , \ f_{n+1} = f_n f_{n-1} \ , a,b \in A \ , \ a \neq b \ , n \geq 2 \ .$$

In the combinatorial theory of free monoids the sequence of words of Fibonacci plays a very important role since the words of Fibonacci have remarkable combinatorial properties some of which have been stressed by Knuth, Morris and Pratt [1] in relation with problems of "string matching" and, more recently, by Duval [2] in the study of "periodicity" of words. A survey on properties of Fibonacci words can be found in [3].

By making use of a result which states that for $n \geq 3$ the Fibonacci words $\{f_n\}$ have a palindrome left factor of length $|f_n| - 2$, we have proved in [4] that for all $n \geq 4$, f_n *is the product of two, uniquely determined, palindrome words of lengths* $F(n-1)-2$ *and* $F(n-2)+2$, where $F(n) = |f_n|$ is the n-th term of the Fibonacci numerical sequence.

These two properties of the Fibonacci words are of great interest since one can show (cf. [4]) that for $n > 4$, *the Fibonacci sequence* $\{f_n\}$ *is the unique sequence of words satisfying the previous properties and the additional requirements that the words contain at least two different letters and that begin with a same letter* (the letter "b" in our case).

References

[1] .D. Knuth, J. Morris and V. Pratt, Fast pattern matching in strings, SIAM J. Comput. 6(1977) 323-350.

[2] .J.P. Duval, Contribution à la combinatoire du monoide libre, Thèse d'Etat, Université de Rouen, 1980.

[3]. J.Berstel, Mots de Fibonacci,Séminaire d'Informatique Théorique Année 1980/81 , Institut de Programmation,Université Paris VI

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