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On the co-completeness of the category of Hausdorff uniform spaces

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Abstract. A construction of colimits in the category of Hausdorff uniform spaces is carried out by means of a family of pseudometrics. Since this is not a topological category, its co-completeness can not be ensured by the standard procedures. A careful revision of the usual arguments is then required to this end.

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1. Uniformities in terms of pseudometrics

For many purposes, the appropriate generalization of the concept of a metric space is that of a *Hausdorff* uniform space, and not merely that of a uniform space. The latter could rather be viewed as a generalization of the notion of a pseudometric space. We focus our attention in this paper on the category $Unif_{\mathcal{H}}$ of Hausdorff uniform spaces and uniformly continuous maps, to establish the existence of colimits. Although all topological categories are known to be co-complete, and the category $Unif$ of uniform spaces and uniformly continuous maps is such while $Unif_{\mathcal{H}}$ is not, the construction of colimits in this latter category deviates somewhat from the standard procedures followed for the former. First we examine some basic and preliminary facts.

A collection $\mathcal{D}$ of pseudometrics on a set X (with values in the real line $\mathbb{R}$) defines a uniformity on X by means of the subbasis of entourages $\{V_d^\varepsilon : d \in$

$\mathcal{D}, \varepsilon > 0\}$, where $V_d^\varepsilon = \{(x,y) \in X \times X \ : \ d(x,y) < \varepsilon\}$. This uniformity is known to be the coarsest on X such that each $d \in \mathcal{D}$ is uniformly continuous for the product uniformity on $X \times X$ and the usual additive uniformity on $\mathbb{R}$.

1.1. Definition. Let $(X,\mathcal{U})$ be a uniform space. The collection $\mathcal{C}_X$ of all finite uniformly continuous pseudometrics on $X \times X$ is called the *total caliber* of $(X,\mathcal{U})$.

The following two well known results can be found in various textbooks on General Topology. For instance, in [6] pages 183 and 188.

1.2. Theorem. *Let $(X,\mathcal{U})$ be a uniform space and $d\colon X \times X \longrightarrow \mathbb{R}$ be a finite pseudometric. Then d is uniformly continuous if and only if $V_d^\varepsilon \in \mathcal{U}$, for every $\varepsilon > 0$.*

1.3. Theorem. *Every uniformity on X is generated by its total caliber.*

The next preliminary result is due to R. De Castro, see [4].

1.4. Theorem. *Let $(X,\mathcal{U})$, $(Y,\mathcal{V})$ be uniform spaces with respective total calibers $\mathcal{C}_X$ and $\mathcal{C}_Y$, and let $f\colon X \longrightarrow Y$. The following assertions are equivalent:*

(i) *f is uniformly continuous.*
(ii) *There exists a unique function $\ell\colon \mathcal{C}_Y \longrightarrow \mathcal{C}_X$, $d \longmapsto d_\ell$, such that $d(f(x),f(y)) = d_\ell(x,y)$ for all $x,y \in X$.*
(iii) *There exists a function $k\colon \mathcal{C}_Y \longrightarrow \mathcal{C}_X$, $d \longmapsto d_k$, such that $d(f(x),f(y)) \leq d_k(x,y)$ for all $x,y \in X$.*

Proof. (i) $\Longrightarrow$ (ii) Assume that f is uniformly continuous and let d be a pseudometric in $\mathcal{C}_Y$. Then the map $d'\colon X \times X \longrightarrow \mathbb{R}$ defined by $(x,y) \longmapsto d'(x,y) = d(f(x),f(y))$ is readily seen to be a pseudometric, and since f is uniformly continuous, for a given $V_d^\varepsilon \in \mathcal{V}$ there exists $U \in \mathcal{U}$ such that $(x,y) \in U$ implies $(f(x),f(y)) \in V_d^\varepsilon$. Thus $U \subseteq V_{d'}^\varepsilon$, and then $V_{d'}^\varepsilon \in \mathcal{U}$. Theorem 1.2 then implies that d' is uniformly continuous, and consequently an element of the total caliber $\mathcal{C}_X$. The conclusion in item (ii) is then obtained by taking $d_\ell = d'$ for each $d \in \mathcal{C}_Y$.

(ii) $\Longrightarrow$ (iii) If ℓ is as specified in (ii), the choice $k = \ell$ also satisfies (iii).

(iii) $\Longrightarrow$ (i) Assume $k\colon \mathcal{C}_Y \longrightarrow \mathcal{C}_X$ is as specified in item (iii) in the statement of the theorem. Given $V_d^\varepsilon \in \mathcal{V}$, take $V_{d_k}^\varepsilon \in \mathcal{U}$. Since $(x,y) \in V_{d_k}^\varepsilon$ implies $(f(x),f(y)) \in V_d^\varepsilon$, then f is uniformly continuous. ☑

2. A construction

2.1. Definition. Given a uniform space $(Z, \mathcal{W})$, an arbitrary set X and a surjective map $h\colon Z \longrightarrow X$, the *quotient uniformity* on X by h is the uniformity $\{U \subseteq X \times X \ : \ (h \times h)^{-1}(U) \in \mathcal{W}\}$.

2.2. Definition. Let $(X_i)_{i\in I}$ be a family of disjoint uniform spaces, and for each $i \in I$, call $\mathcal{U}_i$ the uniformity of X_i. The *sum uniformity* on $Z = \bigcup_{i\in I} X_i$ is the uniformity $\{V \subseteq Z \times Z : \text{for each } i \in I, \ \ V \cap (X_i \times X_i) \in \mathcal{U}_i\}$.

2.3. Remark. As a point of reference and comparison, recall that the colimit of the direct system $\big((X_\alpha, \mathcal{U}_\alpha)_{\alpha\in\Lambda}, (f_{\alpha\beta})_{(\alpha,\beta)\in\Lambda_1}\big)$ in Unif, where Λ is a directed set and Λ_1 is the set $\{(\alpha, \beta) \in \Lambda \times \Lambda\colon \alpha \le \beta\}$, is defined in a similar manner as to that in the category Set. That is, as the uniform space $(Z/R, \mathcal{U})$, where Z is the disjoint union of the spaces X_α equipped with the sum uniformity, R is the equivalence relation given by $x_\alpha \, R \, x_\beta \iff \exists\gamma \ge \alpha, \beta$ such that $f_{\alpha\gamma}(x_\alpha) = f_{\beta\gamma}(x_\beta)$, and $\mathcal{U}$ is the quotient uniformity on Z/R by the canonical map $\phi : Z \longrightarrow Z/R, \ x_\alpha \longmapsto \overline{x}_\alpha$. Nevertheless, in the case of the category of Hausdorff uniform spaces the construction has to be modified somewhat, as shown below.

Denote by $\mathit{Unif}_{\mathcal{H}}$ the category of Hausdorff uniform spaces and uniformly continuous maps, and by $\big((X_\alpha, \mathcal{U}_\alpha)_{\alpha\in\Lambda}, (f_{\alpha\beta})_{(\alpha,\beta)\in\Lambda_1}\big)$, a direct system in $\mathit{Unif}_{\mathcal{H}}$. For each $\alpha \in \Lambda$, let $\mathcal{C}_\alpha$ denote the total caliber of the uniform space $(X_\alpha, \mathcal{U}_\alpha)$, and for a given uniformly continuous function $f_{\alpha\beta}\colon X_\alpha \longrightarrow X_\beta$, let the unique map $\ell\colon \mathcal{C}_\beta \longrightarrow \mathcal{C}_\alpha$ such that $d_\beta(f_{\alpha\beta}(x_\alpha), \ f_{\alpha\beta}(y_\alpha)) = (d_\beta)_\ell(x_\alpha, \ y_\alpha)$ be denoted by $\ell_{\alpha\beta}$.

2.4. Theorem. *The direct system $\big((X_\alpha, \mathcal{U}_\alpha)_{\alpha\in\Lambda}, (f_{\alpha\beta})_{(\alpha,\beta)\in\Lambda_1}\big)$ in $\mathit{Unif}_{\mathcal{H}}$ has a colimit.*

Proof. The construction is carried on in the next four steps.

(1) Let $Z = \dot{\bigcup}_{\alpha\in\Lambda} X_\alpha$ be the disjoint union of the family $(X_\alpha)_{\alpha\in\Lambda}$ and let $\mathcal{C} = \varprojlim \mathcal{C}_\alpha$ be the projective limit of the system $\mathcal{C}_\alpha$ with maps $\ell_{\alpha\beta}$; that is, let

$$\mathcal{C} = \left\{ \mathbf{d} = (d_\alpha)_{\alpha\in\Lambda} \in \prod_{\alpha\in\Lambda} \mathcal{C}_\alpha \colon (d_\beta)_{\ell_{\alpha\beta}} = d_\alpha \text{ for each } (\alpha, \beta) \in \Lambda_1 \right\}.$$

Consider on Z a collection of subbasic entourages $\mathcal{B} = \{W^\varepsilon_{\mathbf{d}} \ : \ \varepsilon > 0, \ \mathbf{d} \in \mathcal{C}\}$, where

$$W^\varepsilon_{\mathbf{d}} = \{(x_\alpha, y_\beta) \in Z \times Z \ : \ \exists\gamma \in \Lambda \text{ s. t. } \gamma \ge \alpha, \beta \ \ \& \ \ (f_{\alpha\gamma}(x_\alpha), f_{\beta\gamma}(y_\beta)) \in V^\varepsilon_{d_\gamma}\}.$$

For each $\varepsilon > 0$ and $\mathbf{d} \in \mathcal{C}$ we have $\triangle_Z \subseteq W^\varepsilon_{\mathbf{d}}$, $W^{\varepsilon/2}_{\mathbf{d}} \circ W^{\varepsilon/2}_{\mathbf{d}} \subseteq W^\varepsilon_{\mathbf{d}}$ and $(W^\varepsilon_{\mathbf{d}})^{-1} = W^\varepsilon_{\mathbf{d}}$. Hence, $\mathcal{B}$ is a subbasis for a uniformity $\mathcal{W}$ on Z such that all the canonical injections $j_\alpha\colon X_\alpha \longrightarrow Z$ are uniformly continuous. Furthermore, the uniformity $\mathcal{W}$ on Z is coarser than the sum uniformity. In fact, $V_{d_\alpha}^{\varepsilon} \subseteq W_{\mathbf{d}}^{\varepsilon} \cap (X_\alpha \times X_\alpha)$, and $V_{d_\alpha}^{\varepsilon} \in \mathcal{U}_\alpha$ for each $\alpha \in \Lambda$.

(2) Define on Z the equivalence relation

$$x_\alpha \, R \, y_\beta \Longleftrightarrow (\forall \mathbf{d} \in \mathcal{C})(\forall \varepsilon > 0)\big((x_\alpha, y_\beta) \in W_{\mathbf{d}}^{\varepsilon}\big).$$

(3) Let $X = Z/R$ and $\phi\colon Z \longrightarrow X$, $x_\alpha \longmapsto \overline{x}_\alpha$, be the quotient map. The collection $\{(\phi \times \phi)\,(W_{\mathbf{d}}^{\varepsilon}) \ : \ W_{\mathbf{d}}^{\varepsilon} \in \mathcal{B}\}$ is a subbasis of entourages for a uniformity $\mathcal{U}$ on X. It is apparent that the map ϕ is uniformly continuous with respect to the uniformities $\mathcal{W}$ and $\mathcal{U}$ above.

The uniformity $\mathcal{U}$ is in general strictly coarser than the quotient uniformity on X by ϕ. In fact, if $W \in \mathcal{W}$ then $\bigcap_{j \in J} W_{\mathbf{d}_j}^{\varepsilon} \subseteq W$ for some collection in $\mathcal{C}$ indexed by a finite set J and some $\varepsilon > 0$. But $(\phi \times \phi)\big(\bigcap_{j\in J} W_{\mathbf{d}_j}^{\varepsilon}\big) \subseteq \bigcap_{j \in J} (\phi \times \phi)\big(W_{\mathbf{d}_j}^{\varepsilon}\big)$, and it may well happen that these two sets are different. Then, it can not be guaranteed that $(\phi \times \phi)(W)$ contains a finite intersection of sets of the form $(\phi \times \phi)\big(W_{\mathbf{d}}^{\varepsilon}\big)$.

We claim that $(X,\mathcal{U})$ is a Hausdorff space. Suppose that $(\overline{x}_\alpha, \overline{y}_\beta) \in (\phi \times \phi)\,(W_{\mathbf{d}}^{\varepsilon/3})$ for all $\varepsilon > 0$ and all $\mathbf{d} \in \mathcal{C}$. Now, since for each $\varepsilon > 0$ and each $\mathbf{d} \in \mathcal{C}$ there exists $(u_\theta, v_\xi) \in W_{\mathbf{d}}^{\varepsilon/3}$ such that $\overline{x}_\alpha = \overline{u}_\theta$, $\overline{y}_\beta = \overline{v}_\xi$, so that $(x_\alpha, u_\theta) \in W_{\mathbf{d}}^{\varepsilon/3}$, $(y_\beta, v_\xi) \in W_{\mathbf{d}}^{\varepsilon/3}$, then for every $\varepsilon > 0$ and every $\mathbf{d} \in \mathcal{C}$ we have that $(x_\alpha,\, y_\beta) \in W_{\mathbf{d}}^{\varepsilon}$, and therefore $\overline{x}_\alpha = \overline{y}_\beta$.

(4) $(X,\mathcal{U})$ is the sought colimit. In fact, define for each $\alpha \in \Lambda$, $\tau_\alpha\colon X_\alpha \longrightarrow X$, $x_\alpha \longmapsto \overline{x}_\alpha$. The map τ_α is a morphism in $Unif_{\mathcal{H}}$, because $\tau_\alpha = \phi \circ j_\alpha$. But since $x_\alpha \, R \, f_{\alpha\beta}(x_\alpha)$, it follows that $\tau_\beta \circ f_{\alpha\beta} = \tau_\alpha$ if $\alpha \leq \beta$.

Given a second inductive cone for the given system, say for instance $\big((Y,\mathcal{V}), (\sigma_\alpha\colon X_\alpha \longrightarrow Y)_{\alpha\in\Lambda}\big)$, we claim that $\psi\colon X \longrightarrow Y$, $\overline{x}_\alpha \longmapsto \sigma_\alpha(x_\alpha)$, is a morphism in $Unif_{\mathcal{H}}$ (necessarily unique) such that for each $\alpha \in \Lambda$, $\psi \circ \tau_\alpha = \sigma_\alpha$. Indeed:

(a) ψ is a well defined map. To see this, let $\mathcal{C}_Y$ be the total caliber of $(Y,\mathcal{V})$. The uniform continuity of σ_α and Theorem 1.4 above imply that for each $\alpha \in \Lambda$ there exists $q_\alpha : \mathcal{C}_Y \longrightarrow \mathcal{C}_\alpha$, $d \longmapsto d_{q_\alpha}$, such that for each $d \in \mathcal{C}_Y$ and for each $x_\alpha,\ y_\alpha \in X_\alpha$, $d(\sigma_\alpha(x_\alpha), \sigma_\alpha(y_\alpha)) = d_{q_\alpha}(x_\alpha, y_\alpha)$. Hence, it follows that for each $d \in \mathcal{C}_Y$, each $(\alpha,\beta) \in \Lambda_1$ and each $(x_\alpha,\, y_\alpha) \in X_\alpha \times X_\alpha$,

$$\begin{aligned} d_{q_\alpha}(x_\alpha, y_\alpha) &= d(\sigma_\alpha(x_\alpha), \sigma_\alpha(y_\alpha)) = d(\sigma_\beta(f_{\alpha\beta}(x_\alpha)), \sigma_\beta(f_{\alpha\beta}(y_\alpha))) \\ &= d_{q_\beta}(f_{\alpha\beta}(x_\alpha),\ f_{\alpha\beta}(y_\alpha)) = (d_{q_\beta})_{\ell_{\alpha\beta}}(x_\alpha, y_\alpha). \end{aligned}$$

Thus, for every $d \in \mathcal{C}_Y$, $d_{q_\alpha} = d_{\ell_{\alpha\beta} \circ q_\beta}$; that is, $q_\alpha = \ell_{\alpha\beta} \circ q_\beta$ for each $(\alpha,\beta) \in \Lambda_1$.

Taking into account that $\mathcal{C} = \overleftarrow{\lim}\, \mathcal{C}_\alpha$, there exists a unique map $\varphi\colon \mathcal{C}_Y \longrightarrow \mathcal{C}$, $d \longmapsto \varphi(d) = \big(d_{q_\alpha}\big)_{\alpha\in\Lambda}$, such that, for each $\alpha \in \Lambda$, $\pi_\alpha \circ \varphi = q_\alpha$.

Assume that $\overline{x}_\alpha = \overline{y}_\beta$. Given $d \in \mathcal{C}_Y$, take $\mathbf{d} = \varphi(d)$. Then, for every $\varepsilon > 0$ there exists $\gamma \in \Lambda$ with $\gamma \geq \alpha, \beta$ such that $(f_{\alpha\gamma}(x_\alpha), f_{\beta\gamma}(y_\beta)) \in V^{\varepsilon}_{d_{q_\gamma}}$. Hence,

$$(\forall d \in \mathcal{C}_Y)(\forall \varepsilon > 0)\big(\ (\sigma_\alpha(x_\alpha), \sigma_\beta(y_\beta)) = (\sigma_\gamma \times \sigma_\gamma)(f_{\alpha\gamma}(x_\alpha), f_{\beta\gamma}(y_\beta)) \in V^{\varepsilon}_{d}\big).$$

Now, since $(Y, \mathcal{V})$ is *Hausdorff* and $\{V^{\varepsilon}_{d} \ : \ \varepsilon > 0\ ,\ d \in \mathcal{C}_Y\}$ is a basis of entourages of $\mathcal{V}$, it follows that $\psi(\overline{x}_\alpha) = \sigma_\alpha(x_\alpha) = \sigma_\beta(y_\beta) = \psi(\overline{y}_\beta)$. Thus, ψ is a well defined map, as claimed.

(b) ψ is a morphism of $Unif_{\mathcal{H}}$. Let $V^{\varepsilon}_{d} \in \mathcal{V}$. Then

$$\begin{aligned}
&(\psi \times \psi)^{-1}(V^{\varepsilon}_{d}) \\
&= \{(\overline{x}_\alpha, \overline{y}_\beta) \ : \ (\psi \times \psi)(\overline{x}_\alpha, \overline{y}_\beta) \in V^{\varepsilon}_{d}\} \\
&= \{(\phi \times \phi)(x_\alpha, y_\beta) \ : \ (\sigma_\alpha(x_\alpha), \sigma_\beta(y_\beta)) \in V^{\varepsilon}_{d}\} \\
&= \big\{(\phi \times \phi)(x_\alpha, y_\beta) \ : \ (\exists \gamma \geq \alpha, \beta)\big[\big(\sigma_\gamma(f_{\alpha\gamma}(x_\alpha)), \sigma_\gamma(f_{\beta\gamma}(y_\beta))\big) \in V^{\varepsilon}_{d}\big]\big\} \\
&= \big\{(\phi \times \phi)(x_\alpha, y_\beta) \in X \times X \ : \ (\exists \gamma \geq \alpha, \beta)\big[\big(f_{\alpha\gamma}(x_\alpha), f_{\beta\gamma}(y_\beta)\big) \in V^{\varepsilon}_{q_\gamma(d)}\big]\big\} \\
&= \{\phi \times \phi)(x_\alpha, y_\beta) \in X \times X \ : \ (x_\alpha, y_\beta) \in W^{\varepsilon}_{\varphi(d)}\} = (\phi \times \phi)(W^{\varepsilon}_{\varphi(d)}) \in \mathcal{U}. \quad ☑
\end{aligned}$$

$Unif_{\mathcal{H}}$ has a closely related category in which the construction of colimits is straightforward. Let $\mathcal{P}met_{\mathcal{H}}$ be the category whose objects are sets endowed with a family of pseudometrics $(X, (d_i)_{i \in I})$ such that $x = y$ if $d_i(x, y) = 0$ for each $i \in I$, and, given a second object $(Y, (d_j)_{j \in J})$, a morphism is a pair of functions $(f, k) : X \times J \longrightarrow Y \times I,\ \ (x, j) \longmapsto (f(x), j_k)$, such that $d_j(f(x), f(y)) \leq d_{j_k}(x, y)$ for each $x,\ y \in X$.

2.5. Theorem. *A direct system* $\big((X_\alpha, (d_{i_\alpha})_{i_\alpha \in I_\alpha})_{\alpha \in \Lambda},\ \ (f_{\alpha\beta}, k_{\alpha\beta})_{(\alpha,\beta) \in \Lambda_1}\big)$ *in* $\mathcal{P}met_{\mathcal{H}}$ *has a colimit*

Proof. Let $Z = \dot{\bigcup}_{\alpha \in \Lambda} X_\alpha$ be the disjoint union of the family $(X_\alpha)_{\alpha \in \Lambda}$. Let I be the limit of the inverse system $\big((I_\alpha)_{\alpha \in \Lambda},\ \ (k_{\alpha\beta})_{(\alpha,\beta) \in \Lambda_1}\big)$ in the category Set. For each $i \in I$ consider the pseudometric $\delta_i \colon Z \times Z \longrightarrow \overline{\mathbb{R}},\ \ (x_\alpha, y_\beta) \longmapsto \inf_{\gamma \in \Lambda} \big\{d_{i_\gamma}(f_{\alpha\gamma}(x_\alpha), f_{\beta\gamma}(y_\beta)) \colon \alpha, \beta \leq \gamma\big\}$. Let R be the equivalence relation on Z defined by $x_\alpha\, R\, y_\beta$ if $\delta_i(x_\alpha, y_\beta) = 0$ for each $i \in I$.

If $X = Z/R$ then $\overline{\delta}_i \colon X \times X \longrightarrow \overline{\mathbb{R}},\ \ (\overline{x}_\alpha, \overline{y}_\beta) \longmapsto \delta_i(x_\alpha, y_\beta)$ is a well defined pseudometric on X for each $i \in I$, as it can be easily verified. Consequently, $(X, (\overline{\delta}_i)_{i \in I})$ is an object of $\mathcal{P}met_{\mathcal{H}}$.

For each $\alpha \in \Lambda$ define $\tau_\alpha \colon X_\alpha \longrightarrow X$ by $x_\alpha \longmapsto \overline{x}_\alpha$. Then $(\tau_\alpha, \pi_\alpha)$ is a morphism in $\mathcal{P}met_{\mathcal{H}}$, and $\big((X, (\overline{\delta}_i)_{i \in I}),\ \ ((\tau_\alpha, \pi_\alpha) \colon X_\alpha \times I \longrightarrow X \times I_\alpha)_{\alpha \in \Lambda}\big)$ is an inductive cone which is the colimit of the given direct system. The separation property of the objects in $\mathcal{P}met_{\mathcal{H}}$ is necessary to establish the universal property of this cone. ☑

Let $\mathcal{C}alib$ be the category whose objects $(X, \mathcal{C}_X)$ are sets endowed with a family of pseudometrics coinciding with the total caliber of the underlying uniformity. Given a second object $(Y, \mathcal{C}_Y)$, a morphism is a pair of functions $(f, \ell) : X \times \mathcal{C}_Y \longrightarrow Y \times \mathcal{C}_X, \quad (x, d) \longmapsto (f(x), d_\ell)$, such that $d(f(x), f(y)) = d_\ell(x, y)$ for each $x,\ y \in X$.

2.6. Proposition. *The category $Unif$ is equivalent to $\mathcal{C}alib$.*

Proof. Let $F\colon Unif \longrightarrow \mathcal{C}alib$ be the functor defined by $F(X, \mathcal{U}) = (X, \mathcal{C}_X)$, where $\mathcal{C}_X$ is the total caliber of the uniformity $\mathcal{U}$, and by $F(f) = (f, \ell)\colon X \times \mathcal{C}_Y \longrightarrow Y \times \mathcal{C}_X$, where ℓ is as in Theorem 1.4., if $f\colon X \longrightarrow Y$ is a uniformly continuous map. Then F is readily seen to be full, faithful and isomorphism–dense. Hence, the asserted equivalence of categories (see [2]) follows. ☑

References

[1] Jiří Adámek et al., *Abstract and Concrete Categories. The Joy of Cats*, Wiley & Sons, 1989.

[2] Serafín Bautista, *Co-completez de Categorías de Espacios Uniformes y Procesos de Lacalización, Tesis de Maestría*, Programa de Posgrado en Matemáticas, Universidad Nacional, Bogotá, 1995.

[3] Nicolas Bourbaki, *General Topology*, Addison-Wesley, 1966.

[4] Rodrigo De Castro, *Localización en Campos de Espacios Uniformes, Tesis de Maestría*, Programa de Posgrado en Matemáticas, Universidad Nacional, Bogotá, 1985.

[5] Rodrigo De Castro & Januario Varela, *Localization in bundles of uniform spaces*, Rev. Colombiana Mat. **24** (1990), 103–112.

[6] John Kelly, *General Topology*, Van Nostrand, 1955.

[7] Saunders Mac Lane, *Categories for the Working Mathematician*, Springer-Verlag, 1971.

[8] Stephen Willard, *General Topology*, Addison-Wesley, 1970.

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