

ALMOST SYMPLECTIC N -LINEAR CONNECTIONS IN THE BUNDLE OF ACCELERATIONS

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Abstract

In the present paper we treat an almost symplectic d -structure in the bundle of accelerations $E = Osc^2M$, defined as an alternate, nondegenerate d -tensor field of type $(0, 2)$, we determine all almost symplectic N -linear connections in the bundle of accelerations, we study the group of transformations of these connections and its invariants.

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1 Introduction

Motivated by concrete problems in variational calculus, the differential geometry of higher order Lagrange spaces is introduced and studied by R.Miron and Gh.Atanasiu in [6] – [9].

The various applications of the Lagrange geometry of order k in physics and mechanics are considerable, [2], [12].

The study of higher order Lagrange spaces is grounded on the k - osculator bundle notion. The bundle space of accelerations correspond in this study to $k = 2$, [1], [5].

The almost symplectic geometry of Finsler spaces was studied by many geometers. (See e.g.[11], [2], [3]).

In the present paper we shall treat an almost symplectic d -structure, defined as an alternate, nondegenerate d -tensor field of type $(0, 2)$.

We first introduce the notion of almost symplectic d -structure, define the notion of almost symplectic N -linear connection and study the properties of

these notions (§2). And, the structure of the set of all almost symplectic N -linear connections preserving a nonlinear connection is discussed (§3), and finally the group of their transformations gives us the various important invariants (§4).

As to the terminology and notations we use those from [10], which are essentially based on M.Matsumoto [4].

2 The notion of almost symplectic d-structure in the bundle of accelerations

Let M be a real $n = 2n'$ -dimensional C^∞ -manifold and (Osc^2M, π, M) its 2-osculator bundle, or the bundle of accelerations. The local coordinates on the total space $E = Osc^2M$ are denoted by $(x^i, y^{(1)i}, y^{(2)i})$. If N is a nonlinear connection on E with the coefficients $N_{(1)j}^i, N_{(2)j}^i$, then let be $D\Gamma(N) = (L_{jk}^i, C_{(1)jk}^i, C_{(2)jk}^i)$ an N -linear connection D on $E = Osc^2M$.

Definition 2.1 *A d-tensor field $a_{ij}(x, y^{(1)}, y^{(2)})$ of type $(0, 2)$ on a differentiable manifold $E = Osc^2M$ is called an almost symplectic d-structure on E , if it is alternate and nondegenerate:*

$$(2.1) \quad a_{ij}(x, y^{(1)}, y^{(2)}) = -a_{ji}(x, y^{(1)}, y^{(2)}),$$

$$(2.2) \quad \det(a_{ij}(x, y^{(1)}, y^{(2)})) \neq 0, \forall y^{(1)} \neq 0, \forall y^{(2)} \neq 0,$$

with the restriction on $TM = Osc^1M$, also nondegenerate.

Given an almost symplectic d-structure a_{ij} , we associate Obata's operators:

$$(2.3) \quad \Phi_{sj}^{ir} = \frac{1}{2}(\delta_s^i \delta_j^r - a_{sj} a^{ir}), \Phi_{sj}^{*ir} = \frac{1}{2}(\delta_s^i \delta_j^r + a_{sj} a^{ir}),$$

where (a^{ij}) is the inverse matrix of (a_{ij}) :

$$(2.4) \quad a_{ij} a^{jk} = \delta_i^k.$$

a^{ij} is also alternate. Obata's operators have the same properties as ones associated with the Finsler space [10].

Definition 2.2 An N -linear connection $D\Gamma(N) = (L_{jk}^i, C_{(\alpha)jk}^i), (\alpha = 1, 2)$, with the property:

$$(2.5) \quad a_{ij|k} = 0, \quad a_{ij} \Big|_k^{(\alpha)} = 0, \quad (\alpha = 1, 2),$$

is said to be an almost symplectic N -linear connection, or compatible with the almost symplectic d -structure (2.1), (2.2).

Proposition 2.1 Let $D\Gamma(N)$ be an almost symplectic N -linear connection. Then Obata's operators are h - and v_α -covariant constants ($\alpha = 1, 2$):

$$(2.6) \quad \Phi_{sj|l}^{ir} = 0, \quad \Phi_{sj}^{ir} \Big|_l^{(\alpha)} = 0, \quad \Phi_{sj|l}^{*ir} = 0, \quad \Phi_{sj}^{*ir} \Big|_l^{(\alpha)} = 0, \quad (\alpha = 1, 2),$$

and the Ricci identities, applied to a d -tensor field a_{ij} , give:

$$(2.7) \quad a_{im}R_j^m{}_{kh} + a_{mj}R_i^m{}_{kh} = 0,$$

$$(2.8) \quad a_{im}P_{(\alpha)j}^m{}_{kh} + a_{mj}P_{(\alpha)i}^m{}_{kh} = 0, \quad (\alpha = 1, 2)$$

$$(2.9) \quad a_{im}S_{(21)j}^m{}_{kh} + a_{mj}S_{(21)i}^m{}_{kh} = 0,$$

$$(2.10) \quad a_{im}S_{(\alpha\alpha)j}^m{}_{kh} + a_{mj}S_{(\alpha\alpha)i}^m{}_{kh} = 0, \quad (\alpha = 1, 2),$$

where $R_h^i{}_{jk}$, $P_{(\alpha)h}^i{}_{jk}$, $S_{(21)h}^i{}_{jk}$, $S_{(\alpha\alpha)h}^i{}_{jk}$ are curvature d -tensor fields.

From (2.6), (2.7), ..., (2.10) we easily get

Theorem 2.1 The curvature d -tensor fields $R_m^r{}_{pq}$, $P_{(\alpha)m}^r{}_{pq}$, $S_{(21)m}^r{}_{pq}$, $S_{(\alpha\alpha)m}^r{}_{pq}$, ($\alpha = 1, 2$) of an almost symplectic N -linear connection have the following properties:

$$(2.11) \quad \Phi_{sj}^{*ir}R_r^s{}_{kl} = 0, \quad \Phi_{sj}^{*ir}R_r^s{}_{kl|l_1\dots l_p} = 0,$$

$$\Phi_{sj}^{*ir}R_r^s{}_{kl} \Big|_{l_1\dots l_p}^{(\alpha)} = 0, \quad (\alpha = 1, 2; \quad p = 1, 2, \dots),$$

$$(2.12) \quad \Phi_{sj}^{*ir}P_{(\alpha)r}^s{}_{kl} = 0, \quad \Phi_{sj}^{*ir}P_{(\alpha)r}^s{}_{kl|l_1\dots l_p} = 0,$$

$$\Phi_{sj}^{*ir}P_{(\alpha)r}^s{}_{kl} \Big|_{l_1\dots l_p}^{(\beta)} = 0, \quad (\alpha = 1, 2; \quad p = 1, 2, \dots; \quad \beta = 1, 2),$$

$$(2.13) \quad \Phi_{sj}^{*ir}S_{(21)r}^s{}_{kl} = 0, \quad \Phi_{sj}^{*ir}S_{(21)r}^s{}_{kl|l_1\dots l_p} = 0,$$

$$\begin{aligned}
 & \Phi^{*ir}_{sj} S_{(21)r}^s \Big|_{l_1 \dots l_p}^{(\alpha)} = 0, \quad (\alpha = 1, 2; p = 1, 2, \dots), \\
 (2.14) \quad & \Phi^{*ir}_{sj} S_{(\alpha\alpha)r}^s \Big|_{kl} = 0, \quad \Phi^{*ir}_{sj} S_{(\alpha\alpha)r}^s \Big|_{kl|l_1 \dots l_p} = 0, \\
 & \Phi^{*ir}_{sj} S_{(\alpha\alpha)r}^s \Big|_{l_1 \dots l_p}^{(\beta)} = 0, \quad (\alpha = 1, 2; p = 1, 2, \dots, \beta = 1, 2).
 \end{aligned}$$

3 The set of almost symplectic N -linear connections

Let $D \overset{0}{\Gamma}(N) = (L_{jk}^i, C_{(\alpha)jk}^i)$, $(\alpha = 1, 2)$ be a fixed N -linear connection on E . Then any N -linear connection on $E : D\Gamma(N) = (L_{jk}^i, C_{(\alpha)jk}^i)$, $(\alpha = 1, 2)$ can be expressed in the form

$$(3.1) \quad L_{jk}^i = L_{jk}^i \overset{0}{-} B_{jk}^i, \quad C_{(\alpha)jk}^i = C_{(\alpha)jk}^i \overset{0}{-} D_{(\alpha)jk}^i, \quad (\alpha = 1, 2),$$

where $B_{jk}^i, D_{(\alpha)jk}^i$, $(\alpha = 1, 2)$ are components of the difference d -tensor fields of $D\Gamma(N)$ from $D \overset{0}{\Gamma}(N)$ [4].

In order that $D\Gamma(N)$ is almost symplectic, that is (2.5) holds for $D\Gamma(N)$, it is necessary and sufficient that $B_{jk}^i, D_{(\alpha)jk}^i$, $(\alpha = 1, 2)$ satisfy

$$(3.2) \quad a_{ij|k}^0 + a_{sj} B_{ik}^s + a_{is} B_{jk}^s = 0, \quad a_{ij} \Big|_k^{(\alpha)} + a_{sj} D_{(\alpha)ik}^s + a_{is} D_{(\alpha)jk}^s = 0, \quad (\alpha = 1, 2),$$

which is equivalent to

$$(3.3) \quad \Phi^{*ir}_{sj} B_{rk}^s = -\frac{1}{2} a^{im} a_{mj|k}^0, \quad \Phi^{*ir}_{sj} D_{(\alpha)rk}^s = -\frac{1}{2} a^{im} a_{mj} \Big|_k^{(\alpha)}, \quad (\alpha = 1, 2),$$

where $\overset{0}{|}$ and $\Big|^{(\alpha)}$, $(\alpha = 1, 2)$ denote the h - and v_α -covariant derivatives $(\alpha = 1, 2)$, with rapport to $D \overset{0}{\Gamma}(N)$.

Proposition 3.1 *Let $D \overset{0}{\Gamma}(N)$ be a fixed N -linear connection on E . Then the set of all almost symplectic N -linear connections $D\Gamma(N)$ is given by (3.1), where $B_{jk}^i, D_{(\alpha)jk}^i$, $(\alpha = 1, 2)$ are arbitrary d -tensor fields satisfying*

(3.3).

Epecially, if $D \overset{0}{\Gamma}(N)$ is almost symplectic, then (3.3) becomes:

$$(3.4) \quad \Phi^{*ir} B_{rk}^s = 0, \quad \Phi^{*ir} D_{(\alpha)rk}^s = 0, \quad (\alpha = 1, 2).$$

From Theorem 5.4.3[12], however, the system (3.3) has solutions in $B_{jk}^i, D_{(\alpha)jk}^i, (\alpha = 1, 2)$. Substituting in (3.1) from the general solution we have:

Theorem 3.1 *Let $D \overset{0}{\Gamma}(N)$ be a fixed N -linear connection. The set of all almost symplectic N -linear connections $D\Gamma(N)$ is given by*

$$(3.5) \quad \begin{cases} L_{jk}^i = L_{jk}^i \overset{0}{+} \frac{1}{2} a^{im} a_{mj|k} \overset{0}{+} \Phi_{sj}^{ir} X_{rk}^s, \\ C_{(\alpha)jk}^i = C_{(\alpha)jk}^i \overset{0}{+} \frac{1}{2} a^{im} a_{mj|k} \overset{(\alpha)}{0}{+} \Phi_{sj}^{ir} Y_{(\alpha)rk}^s, \quad (\alpha = 1, 2), \end{cases}$$

where $X_{jk}^i, Y_{(\alpha)jk}^i, (\alpha = 1, 2)$ are arbitrary d -tensor fields, and $\overset{0}{|}, \overset{(\alpha)}{|}$, denote the h - and v_α -covariant derivatives $(\alpha = 1, 2)$ with respect to $D \overset{0}{\Gamma}(N)$.

As the particular case $X_{jk}^i = Y_{(\alpha)jk}^i = 0, (\alpha = 1, 2)$ in Theorem 3.1 we have

Theorem 3.2 [15] *Let $D \overset{0}{\Gamma}(N)$ be a given N -linear connection. Then the following N -linear connection $D\tilde{\Gamma}(N)$ is almost symplectic:*

$$(3.6) \quad \begin{cases} \tilde{L}_{jk}^i = L_{jk}^i \overset{0}{+} \frac{1}{2} a^{im} a_{mj|k} \overset{0}{+}, \\ \tilde{C}_{(\alpha)jk}^i = C_{(\alpha)jk}^i \overset{0}{+} \frac{1}{2} a^{im} a_{mj|k} \overset{(\alpha)}{0}{+}, \quad (\alpha = 1, 2), \end{cases}$$

where $\overset{0}{|}$ and $\overset{(\alpha)}{|}$ denote the h - and v_α -covariant derivatives $(\alpha = 1, 2)$ with respect to $D \overset{0}{\Gamma}(N)$.

If we take an almost symplectic N -linear connection (e.g. $D\tilde{\Gamma}(N)$) as $D \overset{0}{\Gamma}(N)$ in Theorem 3.1, we have:

Theorem 3.3 [15] *The set of all almost symplectic N -linear connections $D\Gamma(N)$ is given by:*

$$(3.7) \quad \begin{cases} L_{jk}^i = \bar{L}_{jk}^i + \Phi_{sj}^{ir} X_{rk}^s, \\ C_{(\alpha)jk}^i = \bar{C}_{(\alpha)jk}^i + \Phi_{sj}^{ir} Y_{(\alpha)rk}^s, \quad (\alpha = 1, 2), \end{cases}$$

where $X_{jk}^i, Y_{(\alpha)jk}^i, (\alpha = 1, 2)$ are arbitrary d -tensor fields.

4 The group of transformations of almost symplectic N -linear connections.

Let us consider the transformations $D\Gamma(N) \rightarrow D\bar{\Gamma}(N)$ of almost symplectic N -linear connections [13], which preserve the nonlinear connection N . Owing to Theorem 3.3 they are given by:

$$(4.1) \quad \begin{cases} \bar{L}_{jk}^i = L_{jk}^i + \Phi_{sj}^{ir} X_{rk}^s, \\ \bar{C}_{(\alpha)jk}^i = C_{(\alpha)jk}^i + \Phi_{sj}^{ir} Y_{(\alpha)rk}^s, \quad (\alpha = 1, 2), \end{cases}$$

where $X_{jk}^i, Y_{(\alpha)jk}^i, (\alpha = 1, 2)$ are arbitrary given d -tensor fields.

Evidently we have:

Theorem 4.1 *The set of all transformations (4.1) and the mapping product form an abelian group \mathcal{G}_{as} , which is isomorphic to the additive group of the triads of d -tensor fields $(\Phi_{sj}^{ir} X_{rk}^s, \Phi_{sj}^{ir} Y_{(1)rk}^s, \Phi_{sj}^{ir} Y_{(2)rk}^s)$.*

We determine the invariants of the group \mathcal{G}_{as} . The torsion d -tensor fields $T_{(0)jk}^i, S_{(\alpha)jk}^i, R_{(0\alpha)jk}^i, P_{(\alpha\beta)jk}^i, Q_{(\alpha\beta)jk}^i, (\alpha = 1, 2; \beta = 1, 2)$ are given in §3.4 [12]. We denote with:

$$(4.2) \quad t_{(\alpha)jk}^i = \mathcal{A}_{jk} \left\{ \frac{\delta N_{(\alpha)j}}{\delta y^{(\alpha)k}} \right\}, \quad (\alpha = 1, 2),$$

where $\mathcal{A}_{jk}\{\dots\}$ denotes the alternate summation: $\mathcal{A}_{jk}\{A_{jk}\} = A_{jk} - A_{kj}$.

Since $R_{(0\alpha)jk}^i, t_{(\alpha)jk}^i, (\alpha = 1, 2), Q_{(21)jk}^i, P_{(12)jk}^i, P_{(21)jk}^i$ depend on N only, they are invariants of \mathcal{G}_{as} .

We make some notations:

$$(4.3) \quad \begin{cases} t_{(\alpha)^*ijk} = \mathcal{S}_{ijk}\{a_{im}t_{(\alpha)jk}^m\}, \quad T_{(0)^*ijk} = \mathcal{S}_{ijk}\{a_{im}T_{(0)jk}^m\}, \\ R_{(0\alpha)^*ijk} = \mathcal{S}_{ijk}\{a_{im}R_{(0\alpha)jk}^m\}, \quad S_{(\alpha)^*ijk} = \mathcal{S}_{ijk}\{a_{im}S_{(\alpha)jk}^m\}, \quad (\alpha = 1, 2) \end{cases}$$

where $\mathcal{S}_{ijk}\{\dots\}$ denotes the cyclic summation: $\mathcal{S}_{ijk}\{A_{ijk}\} = A_{ijk} + A_{jki} +$

A_{kij} , and

$$(4.4) \quad \left\{ \begin{array}{l} {}^1 k_{(\alpha\alpha)ijk} = a_{km} T_{(0)}^m{}_{ij} + \mathcal{A}_{ij} \{ a_{im} P_{(\alpha\alpha)}^m{}_{jk} \}, \\ {}^2 k_{(\alpha)ijk} = a_{im} S_{(\alpha)}^m{}_{jk} + \mathcal{A}_{jk} \{ a_{km} C_{(\alpha)ij}^m \}, \\ {}^3 k_{(\alpha\beta)ijk} = \mathcal{A}_{jk} \{ a_{km} P_{(\alpha\beta)}^m{}_{ij} \}, \quad {}^4 k_{(\alpha)ijk} = \mathcal{A}_{ij} \{ a_{im} C_{(\alpha)jk}^m \}, \\ \alpha = 1, 2; \beta = 1, 2. \end{array} \right.$$

It is noted that $t_{(\alpha)}^*{}_{ijk}$, $R_{(0\alpha)}^*{}_{ijk}$, $T_{(0)}^*{}_{ijk}$, $S_{(\alpha)}^*{}_{ijk}$ are alternate, and ${}^1 k_{(\alpha\alpha)ijk}$, ${}^4 k_{(\alpha)ijk}$ are alternate with respect to i, j and ${}^2 k_{(\alpha)ijk}$, ${}^3 k_{(\alpha\beta)ijk}$ are alternate with respect to j, k .

Theorem 4.2 *The d -tensor fields $t_{(\alpha)}^i{}_{jk}$, $R_{(0\alpha)}^i{}_{jk}$, $t_{(\alpha)}^*{}_{ijk}$, $R_{(0\alpha)}^*{}_{ijk}$, $T_{(0)}^*{}_{ijk}$, $S_{(\alpha)}^*{}_{ijk}$, ${}^1 k_{(\alpha\alpha)ijk}$, ${}^2 k_{(\alpha)ijk}$, ${}^3 k_{(\alpha\beta)ijk}$, ${}^4 k_{(\alpha)ijk}$, ($\alpha = 1, 2; \beta = 1, 2$), are invariants of the group \mathcal{G}_{as} .*

Proof. By a transformation of almost symplectic N -linear connections (4.1) we find $\bar{T}_{(0)}^i{}_{jk} = T_{(0)}^i{}_{jk} + \frac{1}{2} \mathcal{A}_{jk} \{ X_{jk}^i - a_{sj} a^{ir} X_{rk}^s \}$.
By direct calculations we have $\bar{T}_{(0)}^*{}_{ijk} = T_{(0)}^*{}_{ijk}$ etc.

Proposition 4.1 *Between the invariants in Theorem 4.2 there exist the following relations:*

$$(4.5) \quad \left\{ \begin{array}{l} \mathcal{S}_{ijk} \{ {}^1 k_{(11)ijk} \} = 2T_{(0)}^*{}_{ijk} + t_{(1)}^*{}_{ijk}, \quad \mathcal{S}_{ijk} \{ {}^2 k_{(\alpha)ijk} \} = 2S_{(\alpha)}^*{}_{ijk}, \\ \mathcal{S}_{ijk} \{ {}^3 k_{(11)ijk} \} = T_{(0)}^*{}_{ijk} + t_{(1)}^*{}_{ijk}, \quad \mathcal{S}_{ijk} \{ {}^4 k_{(\alpha)ijk} \} = S_{(\alpha)}^*{}_{ijk}, \\ {}^2 k_{(\alpha)ijk} + {}^4 k_{(\alpha)jki} = S_{(\alpha)}^*{}_{ijk}, \quad (\alpha = 1, 2). \end{array} \right.$$

Theorem 4.3 *Let N be a nonlinear connection on $E = Osc^2M$.*

(1) *The invariant $T_{(0)}^*{}_{ijk}$ (resp. $S_{(\alpha)}^*{}_{ijk}$, ($\alpha = 1, 2$)) vanishes if and only if there exists an almost symplectic N -linear connection $D\Gamma(N)$ with $T_{(0)}^i{}_{jk} =$*

0 (resp. $S_{(\alpha)}^i{}_{jk} = 0$, $(\alpha = 1, 2)$).

(2) The invariants $T_{(0)}^*{}_{ijk}$ and $S_{(\alpha)}^*{}_{ijk}$ vanish if and only if there exists an almost symplectic N -linear connection $D\Gamma(N)$ with $T_{(0)}^i{}_{jk} = S_{(\alpha)}^i{}_{jk} = 0$, $(\alpha = 1, 2)$.

Proof. If we put $X_{jk}^i = \lambda T_{(0)}^i{}_{jk}$ in (4.1), where λ is a real number, we have $\bar{T}_{(0)}^i{}_{jk} = T_{(0)}^i{}_{jk} + \mathcal{A}_{jk}\{\Phi_{sj}^{ir} X_{rk}^s\} = (1 + \frac{3}{2}\lambda)T_{(0)}^i{}_{jk} - \frac{\lambda}{2}a^{ir}T_{(0)}^*{}_{rjk}$.

Taking $\lambda = -\frac{2}{3}$, $T_{(0)}^*{}_{ijk} = 0$ implies $\bar{T}_{(0)}^i{}_{jk} = 0$. The converse is evident. The statement about $S_{(\alpha)}^*{}_{ijk}$, $(\alpha = 1, 2)$ is proved in the same way. (2) follows from the independence of two procedures in (1).

Paying attention to $a_{ij} \mid_k = \frac{\delta a_{ij}}{\delta y^{(\alpha)k}} - C_{(\alpha)ik}^h a_{hj} - C_{(\alpha)jk}^h a_{ih} = \frac{\delta a_{ij}}{\delta y^{(\alpha)k}} - k_{(\alpha)ijk}^4 = 0$, $(\alpha = 1, 2)$, Proposition 4.1 tells us the condition that a_{ij} be a usual almost symplectic d -structure:

Theorem 4.4 An almost symplectic d -structure a_{ij} does not depend on $y^{(\alpha)}$,

$(\alpha = 1, 2)$, if and only $k_{(\alpha)ijk}^4 = 0$, $(\alpha = 1, 2)$, which is equivalent to $k_{(\alpha)ijk}^2 = 0$, $(\alpha = 1, 2)$. In this case it holds $S_{(\alpha)}^*{}_{ijk}$, $(\alpha = 1, 2)$.

Proposition 4.2 (1) If $k_{(11)ijk}^1 = 0$, then $2T_{(0)}^*{}_{ijk} = -t_{(1)}^*{}_{ijk}$.

(2) Assume that $k_{(11)ijk}^1 + a_{km}R_{(01)}^m{}_{ij} = 0$, $k_{(\alpha)ijk}^2 + k_{(11)ijk}^3 = 0$, $(\alpha = 1, 2)$ and $S_{(\alpha)}^*{}_{ijk}$, $(\alpha = 1, 2)$. Then, $R_{(01)}^*{}_{ijk} = 0$ is equivalent to $T_{(0)}^*{}_{ijk} = 0$.

(3) Assume that $k_{(\alpha)ijk}^4 + \lambda k_{(11)ijk}^1 + a_{km}R_{(01)}^m{}_{ij} = 0$, $\lambda k_{(\alpha)ijk}^2 + k_{(11)ijk}^3 = 0$ and $S_{(\alpha)}^*{}_{ijk} = 0$, $(\alpha = 1, 2)$, where $\lambda \neq \pm 1$ is a real number. Then, $T_{(0)}^*{}_{ijk} + \lambda R_{(01)}^*{}_{ijk} = 0$ is equivalent to $T_{(0)}^*{}_{ijk} = 0$.

Proof. Forming the cyclic summations of each of the assumed formulas, the proof follows from Proposition 4.1.

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