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A NOTE ON TYPED COMBINATORS AND TYPED LAMBDA TERMS

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Abstract

The combinatory completeness of the typed theory of combinators and the equality in expressive power of the typed theory of combinators and typed λ-calculus are proved.

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1. Introduction

Schönfinkel and Church introduced, in the 20's and 30's, two different formal systems: the theory of combinators and the λ-calculus, respectively. The basic notion of both of these systems is the notion of function. Although they dealt with the syntax of untyped theories, almost all of their work is in accordance with the indroduction of types in these theories.

In this paper we shall introduce the formal system of typed combinators and typed λ-terms. The formalization of the untyped theories can be found in [1], [2] and [3]. Then, we shall prove some results similar to the already known results in untyped theories: the combinatory completeness of the typed theory of combinators and the equality in expressive power of the two typed systems.

2. Typed λ-terms

Let P be the language of implicational propositional logic. The formulae of P will be called *types*.
The *alphabet* to the typed theory is determined by:

- for every α a denumerable set of variables of type α

$$Var_\alpha = \{x_\alpha, y_\alpha, z_\alpha, x^1_\alpha ...\},$$

- for every type α a denumerable set of constants of type α

$$Const_\alpha = \{c_\alpha, d_\alpha, e_\alpha, c^1_\alpha ...\},$$

- the operation of application,

- the operation of λ-abstraction,

- the left and right parenthesis.

Now, we can be give a recursive definition of typed $\lambda - terms$.

Definition 1.

1. *Each variable $x_\alpha, y_\beta, ...$ and each constant $c_\alpha, d_\beta, ...$ is a term of type $\alpha, \beta, ...$*

2. *If $t_{\alpha\to\beta}$ and s_α are terms of type $\alpha \to \beta$ and α, respectively, then $(ts)_\beta$ is a term of type β.*

3. *If t_β is a term of type β, then $(\lambda x_\alpha.t)_{\alpha\to\beta}$ is a term of type $\alpha \to \beta$.*

We use $t_\alpha, s_\beta, ...$ as schematic letters for typed λ-terms. The type of a λ-term will be denoted by its subscript as in t_α, or by $tp(t) = \alpha$.

Substitution and equality of typed λ-terms are defined in [2]. There is not much difference between the equality of typed and untyped λ-terms. The consequence of type introduction is: if $t = s$, then $tp(t) = tp(s)$.

3. Typed combinators

The alphabet of the typed theory of combinators does not have the operation of λ-abstraction, but instead:

– for every type, α, β and γ there is a set of basic combinators

$$0_{\alpha,\beta,\gamma} = \{I_{\alpha\to\alpha}, K_{\alpha\to(\beta\to\alpha)}, S_{\alpha\to(\beta\to\gamma)\to(\alpha\to\beta\to(\alpha\to\gamma))}, B_{\alpha\to\beta\to(\gamma\to\alpha\to(\gamma\to\beta))},$$
$$C_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))}, W^{-1}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))}, W_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)}\}.$$

Now, we can give a recursive definition of *typed combinatory terms* (c-terms).

Definition 2.

1. *Every variable* $x_\alpha, y_\beta, \ldots$ *and every constant* $c_\alpha, d_\beta, \ldots$ *is a c-term of type* $\alpha, \beta, \ldots$

2. *Every basic combinator* $I_{\alpha\to\alpha}, \ldots, W_{\eta\to(\eta\to\theta)\to(\eta\to\theta)}$ *is a c-term of type* $\alpha \to \alpha, \ldots, \eta \to (\eta \to \theta) \to (\eta \to \theta)$.

3. *If* $X_{\alpha\to\beta}$ *is a c-term of type* $\alpha \to \beta$ *and* Y_α *is a c-term of type* α, *then* $(XY)_\beta$ *is a c-term of type* β.

We use $X_\alpha, Y_\alpha, Z_\alpha, X^1_\alpha, \ldots$ as schematic letters for typed c-terms. The type of a c-term will be denoted like the type of a λ-term. A *combinator* is a c-term formed only by the application of basic combinators.

The language of typed c-terms differs from the language of typed λ-terms because the last one has in its formalism the operation of function-construction (abstraction). What is lacking in this definition ? We did not get the "instruction" how to construct a new c-term (function) from a given one. This happened because the operation of function- construction (abstraction) is lacking in the alphabet of c- term. This is the main difference between the language of typed λ-terms and the language of typed c-terms: the λ-abstraction, λx is a part of the formalism, while nothing similar, exists in Definition 2. So, the first thing to do is to define an *abstraction* $\mid x_\alpha \mid$ in terms of the existing language (not adding anything to the primitive alphabet).

Definition 3. *For a c-term X_β and a variable x_α the c-term $\mid x_\alpha \mid X_\beta$ of the type $\alpha \to \beta$ is defined inductively:*

(i) $\mid x_\alpha \mid x_\alpha \equiv I_{\alpha\to\alpha}$;

(ii) $\mid x_\alpha \mid X_\alpha \equiv K_{\beta\to(\alpha\to\beta)} X_\beta$ *if x_α does not occur in X_β;*

(iii) $\mid x_\alpha \mid X_{\alpha\to\beta} \equiv X_{\alpha\to\beta}$ *if x_α does not occur in $X_{\alpha\to\beta}$;*

(iv) $\mid x_\alpha \mid X_{\beta\to\gamma} Y_\beta \equiv S_{\alpha\to(\beta\to\gamma)\to\alpha\to\beta\to(\alpha\to\gamma))}(\mid x_\alpha \mid X_{\beta\to\gamma})(\mid x_\alpha \mid Y_\beta)$
if we cannot apply (ii) or (iii).

To complete the introduction of this typed formal system we need the notion of equality, so-called *weak equality.*

Definition 4. *The weak equality is given with the following axiom-schemes:*

(1I) $I_{\alpha\to\alpha} X_\alpha = X_\alpha$,

(2k) $K_{\alpha\to(\beta\to\alpha)} X_\alpha Y_\beta = X_\alpha$,

(3S) $S_{\alpha\to(\beta\to\gamma)\to(\alpha\to\beta\to(\alpha\to\gamma))} X_{\alpha\to(\beta\to\gamma)} Y_{\alpha\to\beta} Z_\alpha = X_{\alpha\to(\beta\to\gamma)} Z_\alpha (Y_{\alpha\to\beta} Z_\alpha)$,

(4B) $B_{\alpha\to\beta\to(\gamma\to\alpha\to(\gamma\to\beta))} X_{\alpha\to\beta} Y_{\gamma\to\alpha} Z_\gamma = X_{\alpha\to\beta}(Y_{\gamma\to\alpha} Z_\gamma)$,

(5C) $C_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))} X_{\alpha\to(\beta\to\gamma)} Y_\beta Z_\alpha = X_{\alpha\to(\beta\to\gamma)} Z_\alpha Y_\beta$,

(6W^{-1}) $W^{-1}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))} X_{\alpha\to\beta} Y_\alpha Z_\alpha = X_{\alpha\to\beta} Y_\alpha$,

(7W) $W_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)} X_{\alpha\to(\alpha\to\beta)} Y_\alpha = X_{\alpha\to(\alpha\to\beta)} Y_\alpha Y_\alpha$,

(8) $X_\alpha = X_\alpha$,

and deductive rules:

(9) $\frac{X_\alpha = Y_\alpha}{Y_\alpha = X_\alpha}$,

(10) $\frac{X_\alpha = Y_\alpha \quad Y_\alpha = Z_\alpha}{X_\alpha = Z_\alpha}$,

(11) $\frac{X_{\alpha\to\beta} = X^1_{\alpha\to\beta} \quad Y_\alpha = Y^1_\alpha}{X_{\alpha\to\beta}Y_\alpha = X^1_{\alpha\to\beta}Y^1_\beta}$.

It is obvious that if the equality of c-terms X and $Y, X = Y$ is deduced from Definition 4, then $tp(X) = tp(Y)$. The combinatory completeness proved for the untyped theory in [3] can be proved for the typed theory also.

Theorem 1. *The basic combinators*

$$I_{\alpha\to\alpha}, B_{\alpha\to\beta\to(\gamma\to\alpha\to(\gamma\to\beta))},$$

$$C_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))},\ W^{-1}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))}\ \textit{and}\ W_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)}$$

can be expressed by the application of the combinators K and S of appropriate types.

Proof.

$$I : I_{\alpha\to\alpha}X_\alpha = X_\alpha$$

$$= K^1X(K^2X)$$

$$tp(K^1) = \alpha \to ((\beta \to \alpha) \to \alpha),\ tp(K^2) = \alpha \to (\beta \to \alpha)$$

$$= S^1K^1K^2X$$

$$\mathrm{tp}(\mathrm{S}^1) = \alpha \to ((\beta \to \alpha) \to \alpha) \to ((\alpha \to (\beta \to \alpha)) \to (\alpha \to \alpha))$$

$$I_{\alpha\to\alpha} \equiv S^1K^1K^2 \text{ (in [Ste] } I \equiv SKK).$$

$$B : B_{\alpha\to\beta\to((\gamma\to\alpha)\to(\gamma\to\beta))}X_{\alpha\to\beta}Y_{\gamma\to\alpha}Z_\gamma = X_{\alpha\to\beta}(Y_{\gamma\to\alpha}Z_\gamma)$$

$$= K^3XZ(YZ)$$

$$\mathrm{tp}(\mathrm{K}^3) = \alpha \to \beta \to (\gamma \to (\alpha \to \beta))$$

$$= S^2(K^3X)YZ$$

$$\mathrm{tp}(\mathrm{S}^2) = \gamma \to (\alpha \to \beta) \to ((\gamma \to \alpha) \to (\gamma \to \beta))$$

$= K^4S^2X(K^3X)YZ$

tp(K^4) $= \gamma \to (\alpha \to \beta) \to ((\gamma \to \alpha) \to (\gamma \to \beta)) \to ((\alpha \to \beta) \to (\gamma \to (\alpha \to \beta) \to ((\gamma \to \alpha) \to (\gamma \to \beta))))$

$= S^3(K^4S^2)K^3XYZ$

tp(S^3) $= (\alpha \to \beta) \to (\gamma \to (\alpha \to \beta) \to ((\gamma \to \alpha) \to (\gamma \to \beta))) \to (\alpha \to \beta \to (\gamma \to (\alpha \to \beta)) \to ((\alpha \to \beta) \to ((\gamma \to \alpha) \to (\gamma \to \beta)))$

$B_{\alpha\to\beta\to(\gamma\to\alpha\to(\gamma\to\beta)))} \equiv S^3(K^4S^2)K^3$ (in [Ste] $B \equiv S(KS)K$).

$C : C_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))}X_{\alpha\to(\beta\to\gamma)}Y_\beta Z_\alpha = XZY$

$= XZ(K^5YZ)$

tp(K^5) $=\beta \to (\alpha \to \beta)$

$= S^4X(K^5Y)Z$

tp(S^4) $= \alpha \to (\beta \to \gamma) \to (\alpha \to \beta \to (\alpha \to \gamma))$

$= B^1(S^4X)K^5YZ$

tp(B^1) $= \alpha \to \beta \to (\alpha \to \gamma) \to (\beta \to (\alpha \to \beta) \to (\beta \to (\alpha \to \gamma)))$

$= B^2B^1S^4XK^5YZ$

tp(B^2) $= \alpha \to \beta \to (\alpha \to \gamma) \to (\beta \to (\alpha \to \beta) \to (\beta \to (\alpha \to \gamma))) \to (\alpha \to (\beta \to \gamma) \to (\alpha \to \beta \to (\alpha \to \gamma)) \to (\alpha \to (\beta \to \gamma) \to (\beta \to (\alpha \to \beta) \to (\beta \to (\alpha \to \gamma)))))$

$= B^2B^1S^4X(K^6K^5X)YZ$

tp(K^6) $= \beta \to (\alpha \to \beta) \to (\alpha \to (\beta \to \gamma) \to (\beta \to (\alpha \to \beta)))$

$= S^5(B^2B^1S^4)(K^6K^5)XYZ$

tp(S^5) $= \alpha \to (\beta \to \gamma) \to (\beta \to (\alpha \to \beta) \to (\beta \to (\alpha \to \gamma))) \to (\alpha \to (\beta \to \gamma) \to (\beta \to (\alpha \to \beta)) \to (\alpha \to (\beta \to \gamma) \to (\beta \to (\alpha \to \gamma))))$

C $_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))} \equiv S^5(B^2B^1S^4)(K^6K^5)$ (in [Ste] $C \equiv S(BBS)(KK)$).

$W^{-1}: W^{-1}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))} X_{\alpha\to\beta} Y_\alpha Z_\alpha = X_{\alpha\to\beta} Y_\alpha$

$= K^7(XY)Z$

tp(K^7) $= \beta \to (\alpha \to \beta)$

$= B^3 K^7 XYZ$

tp(B^3) $= \beta \to (\alpha \to \beta) \to (\alpha \to \beta \to (\alpha \to (\alpha \to \beta)))$

$W^{-1}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))} \equiv B^3 K^7$

$W: W_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)} X_{\alpha\to(\alpha\to\beta)} Y_\alpha = XYY$

$= XY(I^1 Y)$

tp(I^1) $= \alpha \to \alpha$

$= S^6 X I^1 Y$

tp(S^6) $= \alpha \to (\alpha \to \beta) \to (\alpha \to \alpha \to (\alpha \to \beta))$

$= S^6 X K^8 I^1 XY$

tp(K^3) $= \alpha \to \alpha \to (\alpha \to (\alpha \to \beta) \to (\alpha \to \alpha))$

$= S^7 S^6 (K^8 I^1) XY$

tp(S^7) $= \alpha \to (\alpha \to \beta) \to (\alpha \to \alpha \to (\alpha \to \beta)) \to (\alpha \to (\alpha \to \beta) \to (\alpha \to \alpha) \to (\alpha \to (\alpha \to \beta) \to (\alpha \to \beta)))$

$W_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)} \equiv S^7 S^6 (K^8 I^1)$.□

4. The equivalence of the typed λ-calculus with the typed theory of combinators

We shall introduce two transformations. If t is a typed λ-term, then t^c is a typed c-term such that:

(i) $x^c \equiv x$

(ii) $(su)^c \equiv s^c u^c$

(iii) $(\lambda x_\alpha.s)^c \equiv\mid x_\alpha \mid s^c.$

Conversely, if X is a typed c-term, then X^λ is a typed λ-term such that:

(i) $x_\alpha^\lambda \equiv x_\alpha$

(ii) $I^\lambda_{\alpha\to\alpha} \equiv \lambda y_\alpha.y$

(iii) $K^\lambda_{\alpha\to(\beta\to\alpha)} \equiv \lambda x_\alpha y_\beta.x$

(iv) $S^\lambda_{\alpha\to(\beta\to\gamma)\to(\alpha\to\beta\to(\alpha\to\gamma))} \equiv \lambda x_{\alpha\to(\beta\to\gamma)} y_{\alpha\to\beta} z_\alpha.xz(yz)$

(v) $B^\lambda_{\alpha\to\beta\to(\gamma\to\alpha\to(\gamma\to\beta))} \equiv \lambda x_{\alpha\to\beta} y_{\gamma\to\alpha} z_\gamma.x(yz)$

(vi) $C^\lambda_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))} \equiv \lambda x_{\alpha\to(\beta\to\gamma)} y_\beta z_\alpha.xzy$

(vii) $W^{-1}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))} \equiv \lambda x_{\alpha\to\beta} y_\alpha z_\alpha.xy$

(viii) $W^\lambda_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)} \equiv \lambda x_{\alpha\to(\alpha\to\beta)} y_\alpha.xyy$

(ix) $(YZ)^\lambda \equiv Y^\lambda Z^\lambda.$

Now, we can prove the equality in expressive power of these two typed systems. This statement is similar to the one for the untyped theories in [3].

Theorem 2.

(i) For c-terms X, $(X^\lambda)^c \equiv X$ *and* $(\mid x_\alpha \mid X)^\lambda = \lambda x_\alpha.X^\lambda.$

(ii) *For* λ *-terms* t, $(t^c)^\lambda = t$.

(iii) *For c-terms* X *and* Y, $X = Y$ *iff* $X^\lambda = Y^\lambda$.

(iv) *For* λ*-terms* t *and* s, $t = s$ *iff* $t^c = s^c$.

Proof. We emphasize that part of the proof which deals with the combinators B, C, W^{-1} and W, because it differs from the elementary proof with I, K and S. For the basic combinators I, K and S we have, by Definition 3:

$$I : (I^\lambda_{\alpha\to\alpha})^c \equiv (\lambda x_\alpha . x)^c \equiv \mid x_\alpha \mid x \equiv I_{\alpha\to\alpha}$$

$$K : (K^\lambda_{\alpha\to(\beta\to\alpha)})^c \equiv (\lambda x_\alpha y_\beta . x) \equiv \mid x_\alpha \mid\mid y_\beta \mid x$$
$$\equiv \mid x_\alpha \mid Kx$$

$$tpK = \alpha \to (\beta \to \alpha)$$

$$\equiv K_{\alpha\to(\beta\to\alpha)}$$

$$S : (S^\lambda_{\alpha\to(\beta\to\gamma)\to(\alpha\to\beta)\to(\alpha\to\gamma)})^\lambda \equiv (\lambda x_{\alpha\to(\beta\to\gamma)} y_{\alpha\to\beta} z_\alpha . xz(yz))^c$$
$$\equiv \mid x_{\alpha\to(\beta\to\gamma)} \mid\mid z_\alpha \mid xz(yz)$$

$$\equiv \mid x_{\alpha\to(\beta\to\gamma)} \mid\mid y_{\alpha\to\beta} \mid S(\mid z_\alpha \mid xz)(\mid z_\alpha \mid yz)$$

$$\equiv \mid x_{\alpha\to(\beta\to\gamma)} \mid\mid y_{\alpha\to\beta} \mid Sxy$$

$$\equiv S.$$

For the proof with other combinators, besides Definition 3, we use Theorem 1.

$$B : (B^\lambda_{\alpha\to\beta\to(\gamma\to\alpha\to\gamma\to\beta))})^c \equiv (\lambda x_{\alpha\to\beta} y_{\gamma\to\alpha} z_\gamma . x(yz))^c$$
$$\equiv \mid x_{\alpha\to\beta} \mid\mid y_{\gamma\to\alpha} \mid\mid z_\gamma \mid x(yz)$$

$$\equiv \mid x_{\alpha\to\beta} \mid\mid y_{\gamma\to\alpha} \mid\mid y_{\gamma\to\alpha} \mid S^2(\mid z_\gamma \mid x)(\mid z_\gamma \mid yz)$$

$$\equiv \mid x_{\alpha\to\beta} \mid\mid y_{\gamma\to\alpha} \mid S^2(K^3x)y$$

$\equiv| x_{\alpha\to\beta} \mid S^2(K^3x)$

$\equiv S^3(| x_{\alpha\to\beta} \mid S^2)(| x_{\alpha\to\beta} \mid K^3x)$

$\equiv S^3(K^4S^2)K^3$

$\equiv B.$

$C : (C^{\lambda}_{\alpha\to(\beta\to\gamma)\to(\beta\to(\alpha\to\gamma))})^c \equiv (\lambda x_{\alpha\to(\beta\to\gamma)}y_\beta z_\alpha.xzy)^c$
$\equiv| x_{\alpha\to(\beta\to\gamma)} \|\, y_\beta \|\, z_\alpha \mid xzy$

$\equiv| x_{\alpha\to(\beta\to\gamma)} \|\, y_\beta \mid S^4(| z_\alpha \mid xz)(| z_\alpha \mid y)$

$\equiv| x_{\alpha\to(\beta\to\gamma)} \|\, y_\beta \mid S^4x(K^5y)$

$\equiv| x_{\alpha\to(\beta\to\gamma)} \|\, y_\beta \mid B^1(S^4x)K^5y$

$\equiv| x_{\alpha\to(\beta\to\gamma)} \|\, y_\beta \mid B^2B^1S^4xK^5y$

$\equiv| x_{\alpha\to(\beta\to\gamma)} \mid B^2B^1S^4xK^5$

$S^5(| x_{\alpha\to(\beta\to\gamma)} \mid B^2B^1S^4x)(| x_{\alpha\to(\beta\to\gamma)} \mid K^5)$

$\equiv S^5(B^2B^1S^4)(K^6K^5)$

$\equiv C.$

W $^{-1} : (W^{-1\lambda}_{\alpha\to\beta\to(\alpha\to(\alpha\to\beta))})^c \equiv (\lambda x_{\alpha\to\beta}y_\alpha z_\alpha.xy)^c$
$\equiv| x_{\alpha\to\beta} \|\, y_\alpha \|\, z_\alpha \mid xy$

$\equiv| x_{\alpha\to\beta} \|\, y_\alpha \mid K^7(xy)$

$\equiv| x_{\alpha\to\beta} \|\, y_\alpha \mid B^3K^7xy$

$\equiv| x_{\alpha\to\beta} \mid B^3K^7x$

$\equiv B^3K^7$

$\equiv W^{-1}.$

$W : (W^{\lambda}_{\alpha\to(\alpha\to\beta)\to(\alpha\to\beta)})^c \equiv (\lambda x_{\alpha\to(\alpha\to\beta)}y_\alpha.xyy)^c$

$\equiv| x_{\alpha\to(\alpha\to\beta)} \|\, y_\alpha \mid xyy$

$\equiv| x_{\alpha\to(\alpha\to\beta)} \mid S^6(| y_\alpha \mid xy)(| y_\alpha \mid y)$

$$\equiv \mid x_{\alpha\to(\alpha\to\beta)} \mid S^6 x I^1$$

$$\equiv S^7(\mid x_{\alpha\to(\alpha\to\beta)} \mid S^6 x)(\mid x_{\alpha\to(\alpha\to\beta)} \mid I^1)$$

$$\equiv S^7 S^6 (K^8 I^1) \qquad \text{(iii) and (ii)}$$

$$\equiv W.$$

Types of $S^2, S^3, S^4, S^5, S^6, S^7, K^3, K^4, K^5, K^6, K^7, K^8$ and I^1 are given in the proof of Theorem 1.

References

[1] BBarendregt, H.P., The Type Free Lambda Calculus, in Handbook of Mathematical Logic (ed. J. Barwise), North-Holland, Amsterdam, 1977. (1091-1132).

[2] HHindley, J.R., Seldin, J.P., Introduction to Combinators and λ-calculus, Cambridge Univ. Press, Cambridge 1986.

[3] SStenlund, S., Combinators, λ-terms and Proof Theory, Reidel, Dordrecht, 1972.

REZIME

O KOMBINATORIMA SA TIPOVIMA I TIPIZIRANIM LAMBDA TERMIMA

U radu je dokazana funkcionalna potpunost teorije kombinatora sa tipovima. Pokazano je kako se funkcionalna apstrakcija može definisati u formalizmu kombinatora sa tipovima.
Uspostavljanjem dve transformacije izmedju klase tipiziranih λ-terma i klase kombinatora sa tipovima dokazano je da su dva sistema iste izražajne moći.

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