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Børge Jessen, 19.6.1907 – 20.3.1993

Christian Berg*

1 Introduction

This document is based on the obituary about Børge Jessen, published by the author in Danish in the Yearbook of the University of Copenhagen for 1994, see (1). The main source of information for this obituary has been the publication [87]¹, the manuscript of a lecture by Jessen in connection with his retirement.

Another obituary in Danish was written by E. Sparre Andersen and presented as a memorial address on March 24, 1994 in the Royal Danish Academy of Sciences and Letters, see (2).

After Jessen passed away the Association of Mathematics Teachers and the Danish Mathematical Society took the initiative to publish a memorial book, see (3). The memorial book contains a very detailed biography in Danish written by Sigurd Elkjær and Trine Uglebjerg as partial fulfilment of the requirements for the Master's Degree. For this book, they also selected and commented on

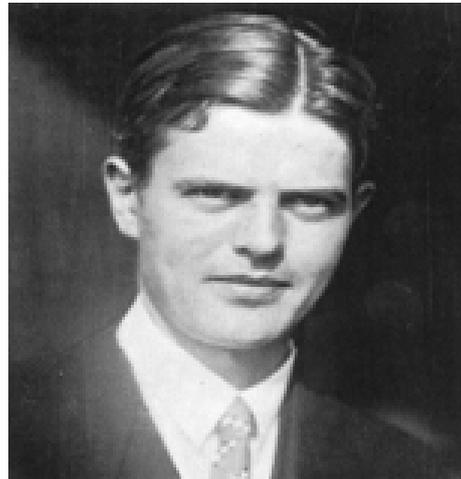


Fig. 1: Børge Jessen (1907-1993)

*Christian Berg Department of Mathematics University of Copenhagen. BERG@math.ku.dk

¹The numbers in brackets refer to the list of Jessen's publications at the end of the document.

several papers by Jessen of pedagogical interest, and they included a list of Jessen's publications. In preparing the present English version of the obituary about Jessen, I am especially grateful to Sigurd Elkjær, who has given me the possibility of including a revised version of this bibliography, which to the best of my knowledge is complete.

Finally, it should be mentioned that the extensive correspondence of Jessen is kept in the Jessen Archive at the Department of Mathematical Sciences at the University of Copenhagen, see <http://www.math.ku.dk/arkivet>. This archive has been very useful in particular concerning Jessen's visits abroad 1933–34. The archive contains copies of letters from Jessen to the family during the trip.

I wish to end this introduction by thanking Sigurd Elkjær and my colleagues Bent Fuglede, Christian U. Jensen, Jesper Lützen, Tage Gutmann Madsen and Anders Thorup for many improvements of the present text.

- (1) C. Berg, *Børge Jessen*. In: Årbog 1994, Københavns Universitet, pp 809–813 (ISBN 87-87848-783).
- (2) E. Sparre Andersen, *Børge Jessen*. In: Annual Report 1993-94 of the Royal Danish Academy of Sciences and Letters, pp 228–238 (ISSN 0368-7201. ISBN 87-7304-256-0).
- (3) *Børge Jessen. Matematiker. Pædagog*. Matematiklærerforeningen i samarbejde med Dansk Matematisk Forening, 1995 (ISBN 87-90098-06-4).

2 Biography

Børge Christian Jessen was born in Copenhagen, as the youngest of the five children of Hans and Christine Jessen. His father was a business manager.

Børge Jessen's interest in mathematics began early in school, stimulated especially by Mrs. Teilmann in the secondary school and Dr. Julius Pál. The latter was his teacher during his first year at Sankt Jørgens Gymnasium. Pál came from Győr in Hungary, and he later became Harald Bohr's assistant. Jessen finished the Gymnasium in 1925 and the same year he began studying mathematics, physics, chemistry and astronomy at the University of Copenhagen. Following the advice of Pál he had already in the Gymnasium studied the mathematical analysis treatise in 4 volumes of H. Bohr and J. Møllerup, so he skipped most of the introductory lectures and just followed the courses by J. Hjelmslev in geometry and by Jakob Nielsen in rational mechanics.

Jessen's first acquaintance with contemporary mathematics was the lectures of Jakob Nielsen on the topology of closed surfaces. He also followed courses by Harald Bohr on number theory and complex analysis. It is clear that these two mathematicians of international reputation influenced his career. Jakob Nielsen had an unusual ability to exposit complicated matters in a clear and efficient way,

so he was able to cover a lot of material. For Harald Bohr it was not so important to cover a lot, but rather to transfer deep understanding and enthusiasm to the students. Jessen stated that every lesson with Bohr was enjoyable, and that Bohr was a master in using coloured chalk to illustrate the lectures. Jessen himself became a master of exposition, and he combined the qualities of these two teachers in a brilliant way.

Early during his years of study Jessen became interested in functions of a real variable, and he studied monographs by Lebesgue, Baire and de la Vallée Poussin in the “Borel Collection”. He emphasized the book by Carathéodory: “Vorlesungen über reelle Funktionen” as a masterpiece, which he had studied in every detail.

It was decisive for Jessen’s mathematical development that Harald Bohr invited him to help with the presentation of an investigation about the value distribution of the Riemann zeta-function. The collaboration led to the joint work [8] [English translation of the title: “On probability distributions by addition of convex curves”]. It appeared in 1929 during Jessen’s fourth year of study, and he had then already independently published 5 small papers in Danish, see [1]–[5]. After four years of study he obtained the degree of “Magister” on June 22, 1929. This was the final degree ending university studies based on a report called “Magister thesis” and an examination. Jessen’s Magister thesis dealt with the theory of almost periodic functions, a subject which Bohr had developed in three long papers published in the years 1924–26. In connection with this work, Jessen realized the importance of a theory of integration for functions of infinitely many variables. Jessen included a preliminary treatment of this topic in the thesis.

The Carlsberg Foundation was established in 1876 as the main shareholder in the Carlsberg Breweries and with the purpose of supporting art and sciences. The founder of the breweries, I.C. Jacobsen (1811–1887), had the vision that the brewing process should be based on the highest scientific level, and he decided in his will that the board of the Carlsberg Foundation should be elected by the members of the Royal Danish Academy of Sciences and Letters among its members. For more than a century the Carlsberg Foundation has been a very important source of support of basic science including mathematics.

Supported by the Carlsberg Foundation, Jessen left for a two months’ stay in Szeged in the fall of 1929. There, he met with F. Riesz, Haar and Fejér, and following the advice of Riesz, he examined strong convergence in connection with his work on integration of functions of infinitely many variables. After his stay in Hungary Jessen spent the winter semester in Göttingen attending lectures by Hilbert and Landau. This stay at the world centre of mathematics broadened his knowledge, and it also brought him into contact with many young mathematicians, who a few years later were forced to leave because of the influence of the Nazis. Several of them came to Copenhagen in 1933 including Busemann, Feller, Neugebauer and Fenchel, see [92, 93]. The number of academic positions in mathematics was very limited, so the newcomers were supported by different

foundations in the beginning. In 1936 Busemann went to USA and after a year Feller moved to Stockholm and from there to USA in 1939. Neugebauer edited *Zentralblatt* from Copenhagen in the years 1934–38, but resigned from the job when Levi-Civita was dismissed from the editorial board and it was decided that work of German mathematicians was not allowed to be reviewed by emigrants. Neugebauer left for USA in 1939. Fenchel remained in Denmark for the rest of his life and had a lasting influence on Danish mathematics.

Jessen's stay in Göttingen was interrupted by a trip to Copenhagen, where he defended his doctoral thesis on May 1st, 1930. Because of this, he had to shorten the planned stay in Paris, and he described it as almost a vacation. He met Lebesgue and had discussions with him.

Following the tradition at the time, Jessen's doctoral thesis was published in Danish, see [9]. He developed his results further, and a comprehensive paper [29] was finally published in *Acta Mathematica* in 1934.

In the fall of 1930 Jessen was appointed docent at the Royal Veterinary and Agricultural School in Copenhagen, and he taught there until 1935 except for a leave of absence from April 1933 to June 1934 supported by the Rockefeller Foundation. He spent the spring and summer of 1933 in Cambridge, England, where he met Hardy, Besicovitch, L. C. Young, and G. Birkhoff. Jessen spent the rest of the period at the Institute for Advanced Study in Princeton. During that stay he collaborated with Bochner and Wintner, see [30],[38], and he visited Harvard, Yale and Brown Universities. During the whole stay abroad Børge Jessen was accompanied by his wife Ellen (1903–1979), a mathematician by education and the daughter of Professor P. O. Pedersen, rector of the Polytechnical School, now the Technical University of Denmark.

The young couple corresponded frequently with their family members in Denmark, and these letters give a vivid insight in their activities, social as well as mathematical. They took lessons in English, went frequently to cinemas and theaters and enjoyed the social life at parties in Princeton, thus meeting for example Einstein. Jessen enjoyed the stay so much that he jokingly proposed in a letter that he would like to arrive in Copenhagen by the morning train on September 1st—the start of the semester—and then walk directly from the train to the School to deliver his first lecture. It remained, however, a joke: The couple was back in Denmark in August so that he could attend the Scandinavian Congress of Mathematicians in Stockholm in mid-August 1934.

Børge and Ellen had a son, Lars in 1934, but tragically the boy became ill of meningitis at the age of two and was disabled.

On June 1st, 1935 Jessen was appointed professor of geometry at the Polytechnical School, and in 1942 he succeeded J. Hjelmslev as professor at the University of Copenhagen, and here he remained until his retirement in 1977. Actually, Hjelmslev had decided to retire by the fall of 1940, and Jessen was the only applicant for the job as a successor. However, before the appointment had been carried out by the Faculty of Sciences, Denmark had been occupied on April 9, 1940 by

German armed forces. Jessen withdrew his application, and Hjelmslev was persuaded to continue, which he did until the end of August 1942. Again, Jessen was the only applicant for the position, probably because the other potential Danish applicants knew that they could not compete with him.

The invitation to collaborate with Harald Bohr was the beginning of a close friendship having a great impact both personally and mathematically as witnessed by the memorial address published after Bohr's premature death in 1951, see [75] and the obituary [77].

Jessen edited and commented on the collected papers of Bohr in collaboration with E. Følner. This work is still available as a publication in 3 volumes from the Danish Mathematical Society. The quarter of a century from Jessen's time of study until Bohr passed away, was marked by strong research activity. In this time, Jessen published more than 70 papers of which 21 had co-authors. With Bohr alone, he published 11 papers. The years to follow were loaded with administrative duties, to which I will return below, but from the mid-sixties on, Jessen resumed doing research, in particular in geometry.

Jessen's scientific production is broad in scope, and his publications can be divided into four groups.

1. A number of papers deal with the value distribution of the Riemann zeta-function. As already mentioned, Bohr invited Jessen to help him with a manuscript about convex curves, a preliminary study intended for the former subject. They published two joint papers [11], [20] in 1930 and 1932 in *Acta Mathematica* about the value distribution. During his stay in the USA, Jessen collaborated with A. Wintner, and they wrote an important paper [38] about infinite convolutions with applications to the value distribution of the zeta-function. This joint paper contains the famous Jessen-Wintner purity law stating that an infinite convolution of discrete probability distributions is always of pure type: i.e. either discrete, continuous singular or absolutely continuous. Jessen's last contribution in this area was the long paper [70] in *Acta* written in collaboration with V. Borchsenius.
2. The theory of integration for functions of infinitely many variables was partly motivated by its applications to the theory of almost periodic functions. Jessen made significant contributions to the distribution of zeros of analytic almost periodic functions. In [17] he gave a formula for the distribution of the zeros, generalizing the famous formula of Jensen. A decisive step was the introduction of a convex function, which Jessen called the "Jensen function" of the almost periodic function. In 1938 he found the complete solution of a problem going back to Lagrange, about the variation of argument for trigonometric polynomials. Several prominent mathematicians had studied the problem earlier. Jessen's result was announced as a note in *Comptes Rendus de l'Académie des Sciences, Paris* in 1938, see [47]. Jessen invited Hans Tornehave (1915–1998) to take part in the elaboration of the manuscript. This led to a long collaboration culminating in the joint paper [60], published in *Acta Mathematica* in 1945. The paper also

contained a characterization of those convex functions which can occur as the Jensen function of an analytic almost periodic function.

3. Within the area of measures and integrals, I have already mentioned the paper in Acta about the integration theory for functions of infinitely many variables. In 1935 Jessen, Marcinkiewicz and Zygmund published a paper [37] about differentiability of multiple integrals. This paper is cited quite often, and at the conference on “Le développement des mathématiques entre 1900 et 1950” in Luxembourg in 1992, this paper was listed among the most important ones published in the year 1935. In the years 1934–1947, Jessen published a series of papers about abstract measure and integration theory in Danish in *Matematisk Tidsskrift B*, and in 1947 they were issued as a monograph by the Danish Mathematical Society, see [II] at the end of the list of references. These papers gave a short and precise introduction to the abstract theory of measure and integration, and although they were in Danish they were widely read, and they certainly enhanced the knowledge about and teaching of this subject among Danish mathematicians. Jessen invited Erik Sparre Andersen to a collaboration about deeper problems in the area, and this resulted in three often cited joint papers [62], [68], [69] from the years 1946–48.

4. Jessen’s first publications are about elementary geometry, and he continued to have a special interest in geometrical problems. In 1938 he published an important paper [48] with Fenchel about convex bodies. To an arbitrary convex body in euclidean n -space they associated $n - 1$ measures on the unit sphere in n -space, called the surface measures of the body, and each of them characterizes the convex body up to a translation. By using the abstract concept of a measure they were able to unify and generalize known results for smooth bodies and convex polytopes. In 1967 Jessen proposed that the present author’s master’s thesis should treat this subject, where some problems were still open, and this led to my first publication.

Jessen’s last papers dealt with questions inspired by J. P. Sydler’s proof of the sufficiency of Dehn’s conditions for scissors equivalence of polyhedra. Jessen had considered the same problem in the 1930s and he was able to simplify the long proof of Sydler, see [83]. Jessen’s proof depended on algebraic results which were obtained in collaboration with J. Karpf and A. Thorup in 1968, see [84]. In [85], Jessen extended the result to polyhedra in four dimensions. In a long paper [89] from 1978 about algebra of polytopes in affine spaces Jessen and Thorup found a characterization of the translation-equivalence of polytopes in any dimension, thereby showing that necessary conditions found by Hadwiger are also sufficient.

As a teacher at the University of Copenhagen, Jessen has had a lasting influence on several generations of mathematicians by his teaching, his guidance for the master’s thesis, and as an inspirer for research. I have already mentioned his collaboration which led to joint papers, but in addition, some of his future

colleagues, such as Bent Fuglede, Christian U. Jensen, and I had the privilege of writing a master's thesis guided by him.

I shall now mention my own years of study 1963–68. In 1960 the curriculum in mathematics was changed, and Jessen taught the second year analysis course. It comprised the basic knowledge about Fourier series, Lebesgue integration and complex analysis, but Jessen also liked to include basic results like Weierstrass' approximation theorem, Peano's curve and Weierstrass' nowhere differentiable continuous function. The lectures were very well prepared, and the blackboard was used in an efficient way, from the upper left corner to the lower right corner. Complicated drawings were often made beforehand and hidden behind another blackboard, and already written formulas were modified with coloured chalk to save writing. It was an esthetic experience to attend Jessen's lectures. The essential points were made clear, and after the lecture one had the impression that everything was easy and elegant. Jessen enjoyed including small anecdotes about the mathematicians behind a theorem, and he often circulated books from the library during the lectures, so we could look at a portrait of a famous mathematician or at a classical mathematical paper. When lecturing about Jensen's inequality in convexity, he said jokingly that there were two mathematicians with the name of Jensen—the famous telephone engineer behind the formula, and himself when his name was misspelled. About Wintner he told that his name was used for a physical unit: “One Wintner meant 100 cigarettes a day”.

Due to the increased number of students in the 1960s a new system involving teaching assistants was introduced with success. Two teaching assistants per class were chosen among the second year students and appointed as “fellow instructors”. They had to learn the material at the same time as they helped their fellow students. Saturday morning Jessen met with the fellow instructors and prepared them for the material to be covered in the lectures and exercises for the following week. I had the privilege of being such a fellow instructor and learned a lot because Jessen could not refrain from telling us many extra things. He also told us that he used to throw away the solutions of the exercises when the year was over, so that he had to rediscover the solutions every year. In this way, he was reminded about the difficulties of the exercises.

At the final exam Jessen was always very favorable towards the students. He liked to cite Bohr for having said that one should always give students a final grade, that is a little higher than what they could expect. In that way they would always remember their years of study with joy.

In addition to his teaching for the first or second year students, Jessen also gave courses for the students preparing for the Master's Degree. The lectures I followed were supported by handwritten notes of high quality distributed to the students. He liked to take up classical subjects such as conformal mapping, the Gamma function and the prime number theorem, where he could set a goal of obtaining a key result at the end. The lecture notes from 1965 called “Elementary

Algebra” began with a citation from Goethe: “Selbst zu erfinden ist schön, doch glücklich von anderen Gefundenes, fröhlich erkannt und geschätzt, nennst du das weniger dein?”.

By the end of the 1960s, Jessen undertook the task of preparing a new set of lecture notes in analysis in Danish for the first year students. He did that in collaboration with a young colleague, Hans Rischel. These notes were a masterpiece, but after a decade they had to be given up, because they required too much of the new students, arriving from gymnasium with less and less knowledge year after year.

In his last semester of teaching, the spring term of 1977, Jessen lectured about almost periodic functions. He handed out new handwritten notes for these lectures, although he undoubtedly had several old versions in his drawers. I followed the lectures and enjoyed every hour. For the last lecture of the course, which was also his last lecture as a teacher, all his colleagues were present, and he finished by explaining how the theory could be extended from functions of a real variable to functions on groups.

As early as 1948, Jessen became the chairman of the department of mathematics, and although he liked to have younger colleagues participating in the administration, he followed up on all details. He remained chairman until 1967, when he turned 60 and was replaced by Sparre Andersen. By the end of the 1950s he took part in the decision and planning to build new premises for chemistry, physics and mathematics in what became the H. C. Ørsted Institute, inaugurated in 1963.

Jessen was also influential in developing the new study reform and mathematics curriculum of 1960 which was coordinated with the University of Aarhus. At that time Denmark had only two universities, not counting specialized schools such as the Polytechnical School. Although the University of Aarhus was inaugurated in 1928, mainly for education in medicine, a Faculty of Natural Sciences was added only in 1954. Up to 1960 the main focus of the teaching of mathematics at the University of Copenhagen was the formation of teachers for the gymnasium, having the subjects astronomy, chemistry, mathematics and physics as a common basis and a specialization in one of the topics. For the new degree called cand. scient. (candidate of science) it became possible to study only mathematics, although many still combined it with physics. This required the creation of new basic courses built upon the new wave of abstraction.

The expansion in the 1960s in the number of students, and consequently also in positions, was met by a cautious and wise hiring policy of Jessen and his colleagues. In spite of the increased teaching load, not all positions were filled, and instead a number of them were kept for future talents. As chairman for the institute, Jessen felt a social obligation, and every year all employees from instructor to professor, were invited to a tea party at his home, in the 1960s necessarily distributed over several evenings, with a convenient mixture of junior

and senior staff. During such a party Jessen liked to tell mathematical anecdotes. As a repeated event he played the Hilbert record, containing the famous speech by Hilbert at the meeting of natural scientists in Königsberg in 1930. The speech ended with the words: “Wir müssen wissen, wir werden wissen”.

Jessen was elected a member of the Danish Academy of Technical Sciences in 1937, and two years later he was elected a member of the Royal Danish Academy of Sciences and Letters. The membership of the latter led to his election in 1950 to the board of the Carlsberg Foundation, where he acted as chairman for the years 1955–63. This gave him an enormous amount of work to do, thus leaving him only little time for mathematics.

Through the years Jessen had many honorary posts on a national as well as international level. He was secretary of the Danish Mathematical Society for 1930–42, again a member of the board in 1952–58, and president from 1954–58. He was elected an honorary member of the Society in connection with its centennial celebration in 1973. As a tribute to his eminent skill of lecturing, the Society decided on his 70th birthday to establish the “Børge Jessen Diploma”. For more than a decade, it was awarded every year on his birthday to a mathematician having recently given a lecture “rich in content and nice in presentation”.

In the years 1935–49, Jessen was editor of *Matematisk Tidsskrift B*, and for the period 1948–1988 he was a member of the editorial board of *Acta Mathematica*.

The scientific contributions of Jessen in the 1930s and 40s made him internationally known, and brought him into contact with leading mathematicians. Jessen spent the year 1949 in the USA with longer stays at Harvard, Chicago and Princeton. On part of the trip he was accompanied by Ellen, but their son Lars had to stay home and go to school. Paul Halmos writes in “I want to be a mathematician” about his impression of Jessen’s lectures: “A spectacularly clear and beautifully organized lecturer”. Jessen became a member of the Executive Committee of the International Mathematical Union in 1950. At the ICM in Amsterdam in 1954, Jessen was among the 20 plenary speakers.

In the fall of 1977, the Department of Mathematics hosted a celebration in connection with Jessen’s retirement. He gave a lecture: “Fra mine læreår” [English: From my years of learning], and the manuscript was published in Danish at the Institute, cf. [87].

Børge Jessen lost Ellen in 1979, and a worsening Parkinson’s disease made his last years difficult. His much reduced eyesight and hearing were two severe handicaps, and prevented activities which he appreciated: reading biographies and listening to music. His son Lars, who had been an essential support for him after the death of Ellen, died prematurely in 1990, and shortly after Børge gave up his big apartment in the centre of Copenhagen and moved to a rest home. He asked me to distribute his large mathematical library in the best way possible. A major part of his books were given to the youngest mathematics department in Denmark at Aalborg University.

Børge Jessen enjoyed visits to the very end of his life. His mind was clear and his memory intact, but his reduced hearing made communication quite complicated. He carried these difficult years with much dignity.

List of publications of Jessen²

- [1] Løsning til Opgave 14. Aarg. 1926. *Mat. Tidsskr. A* 1926, 46–47.
- [2] To Trekantsætninger. *Mat. Tidsskr. A* 1926, 56–59.
- [3] Bemærkning om den Eulerske Polyedersætning. *Mat. Tidsskr. A* 1928, 9.
- [4] Om Delingslighed for maalelige Punktmængder. *Mat. Tidsskr. B* 1929, 85–91.
- [5] Om konvekse Kurvers Krumning. *Mat. Tidsskr. B* 1929, 50–62.
- [6] Über konvexe Punktmengen konstanter Breite. *Math. Z.* 29, 1929, 378–380.
- [7] Über monotone Funktionen. *Danske Vid. Selsk., Math.-fys. Medd.* 10, 6, 1929, 1–13.
- [8] With H. Bohr. Om Sandsynlighedsfordelinger ved Addition af konvekse Kurver. *Danske Vid. Selsk. Skrifter* (8) 12, 3, 1929, 1–82.
- [9] *Bidrag til Integralteorien for Funktioner af uendelig mange Variable*. G. E. C. Gads Forlag, København 1930.
- [10] Über eine Lebesguesche Integrationstheorie für Funktionen unendlich vieler Veränderlichen. pp. 127–138 in *Den syvende skandinaviske matematikerkongres i Oslo 19-22 august 1929*. A. W. Brøggers Bogtrykkeri, Oslo 1930.
- [11] With H. Bohr. Über die Werteverteilung der Riemannsches Zetafunktion. I. *Acta Math.* 54, 1930, 1–35.
- [12] Bemærkninger om konvekse Funktioner og Uligheder imellem Midelværdier. I. *Mat. Tidsskr. B* 1931, 17–28.
- [13] Bemærkninger om konvekse Funktioner og Uligheder imellem Midelværdier. II. *Mat. Tidsskr. B* 1931, 84–95.
- [14] Über die Verallgemeinerungen des arithmetischen Mittels. *Acta Litt. Sci. Szeged* 5, 1931, 108–116.

²This list comprises 93 publications followed by three textbooks in Danish numbered I-III

- [15] Eine Integrationstheorie für Funktionen unendlich vieler Veränderlichen, mit Anwendung auf das Werteverteilungsproblem für fastperiodische Funktionen, insbesondere für die Riemannsche Zetafunktion. pp. 135–136 in *Verhandlungen des internationalen Mathematiker-Kongresses Zürich 1932*, 2. Orell Füssli Verlag, Zürich und Leipzig. Year of publication omitted.
- [16] Eine Integrationstheorie für Funktionen unendlich vieler Veränderlichen, mit Anwendung auf das Werteverteilungsproblem für fastperiodische Funktionen, insbesondere für die Riemannsche Zetafunktion. *Mat. Tidsskr. B* 1932, 59–65.
- [17] Über die Nullstellen einer analytischen fastperiodischen Funktion. Eine Verallgemeinerung der Jensenschen Formel. *Math. Ann.* 108, 1932, 485–516.
- [18] With H. Bohr. One More Proof of Kronecker's Theorem. *J. London Math. Soc.* 7, 1932, 274–275.
- [19] With H. Bohr. To nye simple Beviser for Kronecker's Sætning. *Mat. Tidsskr. B* 1932, 53–58.
- [20] With H. Bohr. Über die Werteverteilung der Riemannschen Zetafunktion. II. *Acta Math.* 58, 1932, 1–55.
- [21] With H. Bohr. Über fastperiodische Bewegungen auf einem Kreis. *Ann. Scuola Norm. Sup. Pisa (2)* 1, 1932, 385–398.
- [22] A Note on Distribution Functions. *J. London Math. Soc.* 8, 1933, 247–250.
- [23] Elementare Beweise einiger Sätze über stetige Abbildungen einer Kugel auf sich. *Mat. Tidsskr. B* 1933, 55–58.
- [24] Om Uligheder imellem Potensmiddelverdier. *Mat. Tidsskr. B* 1933, 1–19.
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- [26] With H. Bohr. Zum Kroneckerschen Satz. *Rend. Circ. Mat. Palermo* 57, 1933, 123–129.
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- [28] On the Approximation of Lebesgue Integrals by Riemann Sums. *Ann. of Math. (2)* 35, 1934, 248–251.
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- [30] With S. Bochner. Distribution Functions and Positive-Definite Functions. *Ann. of Math. (2)* 35, 1934, 252–257.
- [31] With H. Bohr. Mean-Value Theorems for the Riemann Zeta-Function. *Quart. J. Math., Oxford Series* 5, 1934, 43–47.
- [32] Abstrakt Maal- og Integralteori. 2. *Mat. Tidsskr. B* 1935, 60–74.
- [33] Remark on the Theorems of R. Petersen and S. Takahashi. *Mat. Tidsskr. B* 1935, 85–86.
- [34] Some Analytical Problems Relating to Probability. *J. Math. Physics, Mass. Inst. Technol.* 14, 1935, 24–27.
- [35] Über die Säkularkonstanten einer fastperiodischen Funktion. *Math. Ann.* 111, 1935, 355–363.
- [36] With W. Fenchel. Über fastperiodische Bewegungen in ebenen Bereichen und auf Flächen. *Danske Vid. Selsk., Math.-fys. Medd.* 13, 6, 1935, 1–28.
- [37] With J. Marcinkiewicz and A. Zygmund. Note on the Differentiability of Multiple Integrals. *Fundam. Math.* 25, 1935, 217–234.
- [38] With A. Wintner. Distribution Functions and the Riemann Zeta Function. *Trans. Amer. Math. Soc.* 38, 1935, 48–88.
- [39] Om Beviset for den elementære Talteoris Hovedsætning. *Mat. Tidsskr. A* 1936, 85–86.
- [40] With H. Bohr. On the Distribution of the Values of the Riemann Zeta Function. *Amer. J. Math.* 58, 1936, 35–44.
- [41] With H. Bohr. Über die Umkehrung von analytischen fastperiodischen Funktionen. *Math. Ann.* 113, 1936, 461–488.
- [42] Om Sekularkonstanten for en næstenperiodisk Funktion. *Mat. Tidsskr. B* 1937, 45–48.
- [43] Abstrakt Maal- og Integralteori. 3. *Mat. Tidsskr. B* 1938, 13–26.
- [44] Abstrakte Mass- und Integraltheorie. pp. 121–134 in *Nionde skandinaviske matematikerkongressen i Helsingfors 23-26 augusti 1938*. Mercators Tryckeri, Helsingfors 1939.
- [45] Anhang to E. Schneider. Absatz, Produktion und Lagerhaltung bei einfacher Produktion. *Archiv für mathematische Wirtschafts- und Sozialforschung* 4, 1938, 113–120.

- [46] Hvorledes behandler man simplest Potenslæren? *Mat. Tidsskr. A* 1938, 25–34.
- [47] Sur un problème de Lagrange concernant l'argument d'un polynome trigonométrique. *Comptes Rendus Acad. Sci., Paris* 207, 1938, 1081–1084.
- [48] With W. Fenchel. Mengenfunktionen und konvexe Körper. *Danske Vid. Selsk., Math.-fys. Medd.* 16, 3, 1938, 1–31.
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