

## The Problem of Waldegrave

DAVID BELLHOUSE<sup>1</sup>

Resumé

Pierre Rémond de Montmort, dans son ouvrage de 1713 intitulé *Essay d'analyse sur les jeux de hazard*, mentionne un M. de Waldegrave qui correspondait avec lui à propos de divers problèmes de probabilité. C'est ce même Waldegrave qui a produit la première expression d'une solution de stratégie mixte en théorie des jeux et qui a prêté son nom au Problème de Waldegrave en probabilité. Or il n'a jamais été réellement déterminé qui était ce M. de Waldegrave. Trois auteurs se sont hasardés à l'identifier; tous trois se sont approchés de la vérité, mais aucun n'avait raison. Le Waldegrave en question se révèle être Charles Waldegrave, Jacobite actif et oncle de James, 1<sup>er</sup> comte Waldegrave, un diplomate britannique. Un très rapide biographie de Charles Waldegrave est fournie après un passage en revue de ses contributions à la probabilité. Le contexte politique et social dans lequel Charles Waldegrave a vécu est également décrit en quelques lignes.

Abstract

Pierre Rémond de Montmort, in his 1713 book *Essay d'analyse sur les jeux de hazard*, mentions a M. de Waldegrave, a man who was corresponding with him on probability problems. This Waldegrave gave the first expression to a mixed strategy solution in game theory and lends his name to Waldegrave's Problem in probability. Who this Waldegrave was has never been properly determined. Three authors have ventured to identify the man; all three were close, but incorrect. The correct Waldegrave turns out to be Charles Waldegrave, an active Jacobite, and uncle to James, 1<sup>st</sup> Earl Waldegrave, a British diplomat. A very sketchy biography of Charles Waldegrave is provided after reviewing his contributions to probability. The political and social context in which Charles Waldegrave lived is also briefly described.

### 1. WALDEGRAVE AND PROBABILITY

In the second edition of his book *Essay d'analyse sur les jeux de hazard* Pierre Rémond de Montmort mentions a gentleman named Waldegrave who is associated with two problems in probability [Montmort, 1713, p. 338]. The first is known today as Waldegrave's Problem and the second is the first apparent use in game theory of a mixed strategy solution. Both problems are discussed in the *Essay d'analyse* [Montmort, 1713, pp. 318 and 409]. Detailed technical analyses

---

<sup>1</sup> Department of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario, Canada  
N6A5B7, [bellhouse@stats.uwo.ca](mailto:bellhouse@stats.uwo.ca)

of these problems, as they are discussed in the *Essay d'analyse* and elsewhere in the eighteenth century, appear in Anders Hald's *History of Probability and Statistics* [Hald, 1990, pp. 314 – 322 and 378 – 392].

Montmort originally called Waldegrave's Problem the "Problème de la Poulle" or the Problem of the Pool [Montmort, 1713, p. 366]. It was Isaac Todhunter who called it Waldegrave's Problem [Todhunter, 1865, p. 124]. Here is a general description of the problem. Suppose there are  $n + 1$  players with each player putting one unit into the pot or pool. The first two players play each other and the winner plays the third player. The loser of each game puts one unit into the pot. Play continues in like fashion through all the players until one of the players has beaten all the others in succession. The original problem, stated in a letter dated 10 April, 1711, from Montmort to Nicholas Bernoulli [Montmort, 1713, p. 318] is for  $n = 2$  and is attributed to "M. de Waldegrave". The problem, according to Montmort, is to find the expectation of each player and the probability that the pool will be won within a specified number of games.

Waldegrave's Problem was motivated by a real gambling situation in the card game piquet and the board game trictrac [Montmort, 1713, p. 318]. These games are normally played with two players and the pool was introduced to allow for more players. Waldegrave was English, Montmort French. The pool appears as a variation of piquet as played in England and as a variation of the French game trictrac.

Piquet is a card game for two players using 36 cards from the standard deck (the sixes through aces). There is a scoring system based on the play of the cards; the first player to reach 100 points wins the game. In probably the earliest English rulebook for games, Charles Cotton in *The Compleat Gamester* describes the rules for piquet, but makes no mention of the possibility of having more than two players [Cotton, 1674, Ch. 6]. It is Richard Seymour writing about 45 years later in *The Court Gamester* who describes playing piquet in a pool. He writes:

"The Pool is another way of playing *Picquet*, only invented for Society; it is in every Way play'd the same with the other Game; but is a Contrivance to bring in a third. As for example,

Three persons are to cut, he who cuts the highest Card, stands out the first Game, for it is held an Advantage to be out first. Then the others are cut for Deal, as is before directed; If they play for Guineas, they are to lay down a Guinea apiece, which makes three Guineas: then he who loses the first Game lays down a Guinea more, and goes out, and he who stood out before, sits down; if the first Gamester beats him also, he sweeps the Board, which is called winning the *Pool*; and the loser must lay another guinea to it.

But if he who won the first Game, loses the second, he pays his Guinea, and makes room for the other; thus it goes round sometimes, till the *Pool* amounts to a great Sum.

You must observe, the *Pool* is never won, till one Person gets two Games successively.

Every Person that loses a Game, lays down a Guinea to the *Pool*.

When any person is lurch'd at this Play, he lays down one Guinea to the *Pool*, and pays another to him who lurch'd him." [Seymour, 1719, pp. 92 – 93]

The French literature, expressed in publications entitled *Académie universelle des jeux*, first published in 1718 [Académie, 1718], have descriptions of piquet for more than two players, typically three or five or higher odd numbers. The method of play, called "piquet à écrire", is completely different from the pool described by Seymour. For this variation in the game, a

method is given for totalling scores of the players in the game. Consequently, the pool in piquet appears to be peculiarly English.

Trictric is a French game similar to, but not the same as, backgammon. Each player has fifteen checkers that can be moved around a board by the roll of two dice. Unlike backgammon, which is a racing game around the board, trictrac has a system of scoring points for various plays and positions on the board. The winner in trictrac is the first to take twelve games (or trous) of twelve points each. The game is usually between two players. Similar to piquet à écrire there is a variation of trictrac called “trictrac à écrire” that is described in another edition of *Académie universelle des jeux*, [Nouvelle académie, 1721]. Interestingly there is no description of a pool in trictrac in the various eighteenth century editions of *Académie universelle des jeux* that I have seen. It is in the nineteenth century that a variation for more than two players using a pool is finally described in these publications [Cousin d’Avallon, 1842, p. 307]:

“On ne peut jouer que deux personnes à la fois: mais pour en amuser un plus grand number, on joue ce qu’on appelle la poule; c’est à dire que l’on peut être trois, quatre, cinq ou six à jouer; que chacun en entrant met au jeu; que, pour avoir la poule, il faut gagner autant de parties qu’on a de joueurs contre soi; et que chaque joueur qui renter est obligé de metre de nouveau au jeu. Ainsi, il peut arriver qu’une soirée entière se passe sans qu’aucun y réussisse: alors on partage la poule. Elle peut être très forte, arrive ainsi à ce point, sans avoir été gagnée.”

This quotation is immediately followed by a description of trictrac à écrire. In an earlier section on piquet, there is a description of piquet à écrire but no mention is made of the variation using a pool [Cousin d’Avallon, 1842, p. 207].

It was actually Abraham De Moivre who gave the first printed solution to the Waldegrave’s Problem for  $n = 2$  in Problem 15 of his *De Mensura Sortis* [De Moivre, 1711]. He addresses only the expectations of each of the players and also suggests in a corollary that a solution could be found for more than three players. The general solution turns out to be quite difficult but was soon obtained by both De Moivre and Bernoulli [De Moivre, 1714] and [Bernoulli, 1714]. Note from the quotation on the pool for trictrac, the game could go on all evening without the pool being won. There is no mention of this happening in piquet in *The Court Gamester*. This perhaps explains Montmort’s interest in the probability that the pool will be won in a certain number of games while De Moivre shows no such interest.

The quotation from Seymour’s *The Court Gamester* leads to some insights into the mathematics of probability problems and the actual play of the games. These insights are: (1) simplification and generalization are standard approaches to a mathematical solution, and (2) gamblers do not necessarily read or benefit from the mathematical literature. As described in *The Court Gamester*, the actual problem for three players in a pool is more difficult than the problem solved by De Moivre, Montmort, Waldegrave or Bernoulli. In piquet, a player is lunched when that player fails to obtain 50 points before the opponent obtains the 100 points required to win [Seymour, 1719, p. 91]. The winnings that one player obtains by lunching another makes the problem much more complicated and probably requires simulation to obtain a solution. Regarding generalization, the pool in *The Court Gamester* was described for only three players; there is no hint of increasing the pool size. However, the problem of the pool generalized to four or more players is a challenging, yet tractable, problem. Nothing is known about the writer Richard Seymour. What can be said about him is that he had not read De Moivre’s *De Mensura*

*Sortis*. According to the quotation, the player who sits out the first game has the advantage over the other two. De Moivre shows mathematically that it is actually the other way round. For an initial contribution to the pool of one unit by each player with an additional amount  $p$  given to the pool by a player for a loss on a round of play, the player initially sitting out has a disadvantage if  $p < 7/6$ . In the situation described by Seymour,  $p = 1$ .

The second problem associated with Waldegrave is related to the card game Le Her which is played with a standard deck where kings are counted as the highest card and aces the lowest in the deck. For two players,  $A$  and  $B$ ,  $A$  deals a card face down to  $B$  and then one, again face down, to himself. If  $B$  chooses, he may switch his card with  $A$  unless  $A$  holds a king. Then if  $A$  so chooses, he may switch his card with one randomly chosen from the remaining cards in the deck unless the card he draws is a king. The winner is the player holding the highest card. Montmort describes Le Her for three players [Montmort, 1713, pp. 278 – 279]. In this situation,  $B$  can switch with  $A$ ,  $C$  with  $B$  and  $A$  can draw another card from the deck. The problem is to find a strategy that maximizes the probability of winning.

Only the problem of two players  $A$  and  $B$  was tackled by Montmort and his friends. It is first mentioned in *Essay d'analyse* in a letter of November 10, 1711 from Nicolaus Bernoulli to Montmort [Montmort, 1713, p. 334]. In the letter Bernoulli says that he has solved the problems related to Le Her. A few months later, in March of 1712, Montmort wrote back to Bernoulli [Montmort, 1713, p. 338] saying that he has shown the solution to his friend and neighbour, the Abbé de Monsoury (also called Abbé d'Orbais in other letters) and that the abbot has written Montmort a letter pointing out that Bernoulli's solution is incorrect. Later in September [Montmort, 1713, p. 361] Waldegrave wrote to the abbot saying that he agrees with him. Bernoulli tried to fix up his solution [Montmort 1713 p. 376], but in August of 1713 Montmort wrote to Bernoulli saying that his new arguments on Le Her have not convinced Waldegrave and the abbot [Montmort 1713, p. 400]. Finally in a letter of November 13, 1713 to Montmort, Waldegrave comes up with the strategy to solve the problem. Two days later Montmort communicated the solution to Bernoulli [Montmort, 1713, pp. 409 – 412]. The solution is to use a randomized strategy that maximizes each player's probability of winning. In particular, with probability  $5/8$   $B$  uses the strategy that he will switch cards of value 7 and under, and with probability  $3/8$  he uses the strategy that he will retain cards of 7 and over. With probability  $3/8$   $A$  uses the strategy that he will draw from the deck when he holds 8 and lower, and with probability  $5/8$  he uses the strategy that he will keep what he had when he holds 8 and higher.

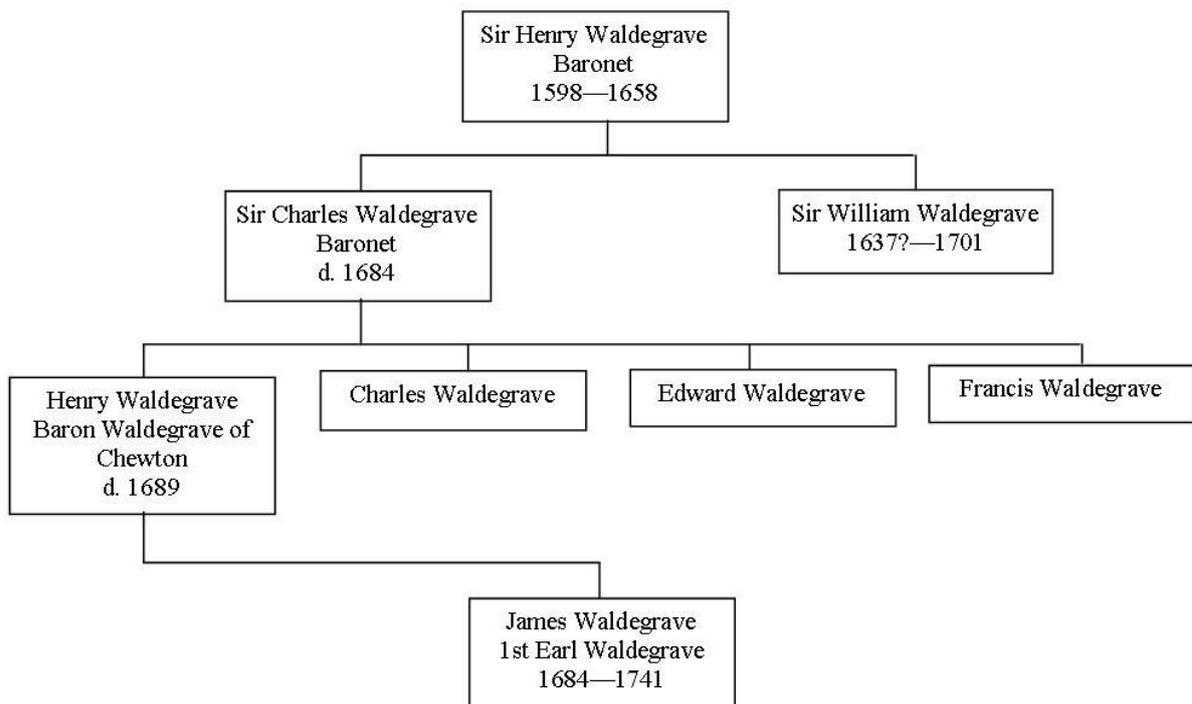
## 2. SOME ERRORS IN IDENTIFICATION AND A PROBABLE SOLUTION

Who was Montmort's M. de Waldegrave and where did he come from? Three authors have ventured to identify the man. All three were close, but incorrect. Harold Kuhn, in the preface to *Precursors in Mathematical Economics: An Anthology* [Baumol and Goldfeld, 1968, pp. 3 – 9], made the first attempt. He states that the Waldegrave in question is the English aristocrat James Waldegrave (1684 – 1741), 1<sup>st</sup> Earl Waldegrave. This Waldegrave was initially educated in France and later served as the British ambassador to Paris and Vienna [Woodfine 2004]. There is widespread acceptance of Kuhn's identification, at least on the Internet. Another author, Julian Henny appears to make two conflicting identifications. In the index to the book in which his article is published, Waldegrave is identified as the first earl or his brother. In a footnote to the actual article that Henny wrote [Henny, 1975, p. 495], there is an indication that

the Waldegrave of interest is James Waldegrave's uncle. George Barnard puts forward the claim that the uncle was William Waldegrave who came from a Waldegrave family of 23 children [Barnard, 1991, p. 288]. William Waldegrave was physician to Queen Mary of Modena, wife of James II of England [Bevan, 2004]. This Waldegrave, who was knighted by James II, fled with the queen and her newborn son to France during the revolution of 1688 that put William and Mary on the throne of England. At least all the Waldegraves named so far were in France at one time or another and could have talked to Montmort.

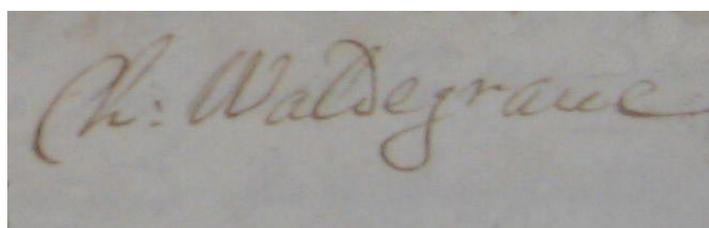
It is Montmort who gives the clue to Waldegrave's identity [Montmort, 1713, p. 388]. He says that Waldegrave was "un Gentilhomme Anglois, frere de seu Milord Waldegrave, qui avoit épousé une fille naturelle du Roy Jacques." Thus Waldegrave is the brother of an aristocrat names Waldegrave who married an illegitimate daughter of King James II of Britain. The "Milord Waldegrave" is Henry Waldegrave (1661 – 1690) who was made Baron Waldegrave of Chewton in 1686. He married Henrietta Fitzjames in 1683; she is the illegitimate daughter of Arabella Churchill and James, Duke of York, later James II of England. The eldest son of Henry and Henrietta Waldegrave is James Waldegrave, who was created 1<sup>st</sup> Earl Waldegrave in 1729 [Woodfine, 2004]. Consequently, Kuhn must be wrong; he is out by a generation. Also, Barnard is out by a generation in the other direction. Henry, 1<sup>st</sup> Baron Waldegrave, in his will [Public Record Office], states that he gives "guardianship and tuition of my eldest sonn unto my well beloved wife and my uncle Sir William Waldegrave". Besides being Henry's uncle and not his brother, Sir William died in 1701 [Lart, 1910, p. 133] and so could not have been writing letters to Montmort from 1711 to 1713.

Figure 1: The Waldegrave Family



The standard reference for British aristocracy [Burke, 2005] states that Henry, 1<sup>st</sup> Baron Waldegrave is the son of Sir Charles Waldegrave, Baronet (d. 1683), but does not list any of Henry's siblings. Further Sir Charles is the son of Sir Henry Waldegrave, Baronet (1598 – 1658). Again no siblings of Sir Charles are given, only that Sir Henry had the distinction of siring 23 children. Sir William Waldegrave is one of these children. Another source for genealogy of the British aristocracy [Collins, 1970, p. 242] gives Charles and Eleanor Waldegrave as the younger son and daughter of Sir Charles Waldegrave and quotes Sir Henry Waldegrave's monumental inscription which shows that of his 23 children, 13 were sons and 10 were daughters. Charles and Eleanor Waldegrave also show up as the younger siblings of Henry, 1<sup>st</sup> Baron Waldegrave in another genealogical work [Jacob 1767, p. 115]. These printed sources are confirmed by Henry's will [Public Record Office] which identifies Charles Waldegrave as his brother. The Waldegrave pedigree, hand written and hanging on the wall in the Waldegrave family home in Chewton, shows only Charles Waldegrave as Henry's sibling. A genealogical chart showing all the family connections is shown in Figure 1.

Figure 2: 1691 Signature of Charles Waldegrave



We are perhaps left with Charles Waldegrave as Montmort's M. de Waldegrave. There is, however, a complication. Sir Charles Waldegrave's will [Public Record Office] mentions his son and heir Henry (later Henry, 1<sup>st</sup> Baron Waldegrave) as well as three younger sons: Charles, Edward and Francis. Each of the younger sons was to receive a yearly annuity of £100 as well as a share in their mother's legacy when she died. Edward and Francis appear to be minor characters. They show up in only one other document [House of Lords Record Office] dated 1691 and may not have survived to 1711 to correspond with Montmort. We are left once again with Charles Waldegrave as the leading candidate. There are no known portraits of our leading candidate. All that can be offered is a copy of his signature from the 1691 document. It is shown in Figure 2.

### 3. A SKETCHY BIOGRAPHY

What do we know of Charles Waldegrave? The answer is not very much. The only mention of his date of birth may be incorrect and there is no record of his date of death. If he left a will, either it has not been found or it has not survived. Other than his time in France corresponding with or visiting Montmort, much of the construction of his biography is based on fragments of evidence and conjecture. He can, however, be placed firmly in a historical context.

The Waldegraves were a Catholic family in an officially anti-Catholic country; Catholics were subjected to large fines for non-attendance at Church of England services. James Stuart was a Catholic when he ascended the throne as James II in 1685. He tried to ease restrictions on Catholics and to promote them in the universities and at court [Willcox and Arnstein, 2001, pp. 4

– 6]. For example, Henry, 1<sup>st</sup> Baron Waldegrave was appointed to the court position of Comptroller of the Household in 1687 [Collins, 1970, pp. 242 – 243]. The general situation was tolerated by the politically powerful of the day since James was in his fifties and had no Catholic heir. A crisis occurred when James's son was born in 1688 and it was announced that the boy would be brought up Catholic and that the pope would be the boy's godfather. A small group of political leaders invited James's Protestant son-in-law, William of Orange to intervene and James's reign quickly fell to pieces. James and his family fled to France and, with the assistance and financial support of Louis XIV, set up a court in exile at Saint Germain-en-Laye, one of Louis's palaces. Several Waldegraves, including Henry, Lord Waldegrave, his uncle Sir William Waldegrave and their families, followed James to France and resided at Saint Germain-en-Laye [Lart, 1910] and [Lart, 1912].

Charles Waldegrave remained in England. He was there in 1691 after his older brother Henry died. In his will Henry, Lord Waldegrave, had made his uncle Sir William Waldegrave, his brother Charles and two others trustees of his estates in England. James Waldegrave, the son and heir living in France, later the 1<sup>st</sup> Earl Waldegrave, would not come of age until 1705. The trustees also received some property in the will. As a trustee for the estate Charles was the one who made application for an act of parliament that would enable the trustees to make leases on the lands in the estate. Several annuities based on rents from Henry, Lord Waldegrave's land were in arrears. Some of these annuities were established in the will of Henry's father from 1683. Charles was owed £257 and his brothers Edward and Francis were each owed £750 [House of Lords Record Office]. James Waldegrave did not return to England until 1706 or later [Woodfine 2004]. His uncle Charles was active as his trustee up to at least 1705 at which time he was involved in a conveyance of lands in Shropshire [Shropshire Archives]. As a trustee it is possible that Charles Waldegrave travelled to France to meet with his nephew about his estates.

Charles Waldegrave was a Catholic and so he was not baptized in the Church of England where the official records of baptism, marriage and burial were kept. Consequently, there are no official birth records for Charles Waldegrave and his date of birth can only be guessed at. Alexander Jacob states that the elder brother Henry was born August 14, 1659 and Charles was born August 21, 1660 [Jacob, 1767, p. 115]. This is at variance with Henry, Lord Waldegrave's burial record [Lart 1910, p. 133] which states that he was buried June 2, 1690 having attained the age of 28. This yields a birth year of 1661 or 1662 for Henry, which puts Charles's birth year at 1662 or later. Adding a bit to the confusion are some deeds related to the marriage settlement of Elizabeth Waldegrave in 1683 [Shakespeare Birthplace Trust Records Office]. She was apparently another sibling of Henry and Charles. The deeds have both Henry and Charles parties to the agreement. If Charles were of full age (21) when the documents were signed, then his birth year would be 1662 or before.

Prior to Charles Waldegrave showing up in France some time between 1711 and 1713, he probably lived on the Waldegrave estates in England. The will of Philip Waldegrave, a distant cousin, puts Charles living with Henry, 1<sup>st</sup> Baron Waldegrave in 1688 [Public Record Office]. Philip left all his lands to Henry, Lord Waldegrave and his heirs; Charles received a £20 annuity. Charles undoubtedly continued to reside on the Waldegrave estates while he acted as trustee for Henry's son James while he was in France. A 1714 codicil to Philip Waldegrave's will appointed James, Lord Waldegrave as the executor of the will (the previous executor had died). Although Henry, Lord Waldegrave had been dead for 24 years, there was no change to the main beneficiary of the will. James was the heir, the lands went to the heir and so there was no need for a change. Philip made no changes to Charles Waldegrave's description so that it might be

assumed not only that Charles was alive in 1714 but also that he continued to have his official residence on the Waldegrave estates. Philip Waldegrave died in 1720.

In April of 1711, Montmort was in contact with Charles (we assume) Waldegrave, but did not say where Waldegrave was at the time [Montmort, 1713, p. 318]. It was at this time that Waldegrave proposed the Problem of the Pool to Montmort and suggested a solution. While writing to Bernoulli, Montmort mentioned Waldegrave and his solution and then continued with his own solution to Bernoulli. At the time of Waldegrave's last correspondence with Montmort on November 13, 1713, he was staying at the Château du Breviande. This was probably owned by Montmort since his father was sieur de Breviande [Fontenelle, 1721, p. 83]. At the end of 1713, Waldegrave was probably in Paris since he had agreed to take Montmort's book to the printer in Paris [Montmort, 1713, p. 412].

At about this time, Britain was experiencing change and some upheaval. The last Stuart monarch, Queen Anne died in 1714. The crown went to a distant relative whose roots went back to the Tudors, the Elector George of Hanover who reigned as George I. Left out of the succession was the son of James II, James Stuart, known variously as the Pretender, the Old Pretender and the Chevalier de St. George, who was very much alive in Saint Germain-en-Laye [Gregg, 2004]. It was not an easy transition to the Hanoverian dynasty. There were conspiracies to get the Stuarts back on the throne with uprisings planned for both England and Scotland. Key figures in the planned uprising were James Butler, Duke of Ormond John Erskine, Earl of Mar. In August of 1715 the uprising began in Scotland under Mar. In October of 1715 James Stuart appointed Ormond, then in Paris, as captain-general of the Jacobite army to lead the rebellion in England. Later in October James Stuart intended to join the rebellion but waited on the Continent for the English uprising to begin. By December of 1715 the British government had the upper hand in the Scottish uprising and there was no English uprising. James Stuart finally arrived in Scotland very late in December of 1715. His military situation became hopeless and he returned to France in late February of 1716 [Gregg, 2004].

It is in this general context that a letter from Montmort to the English mathematician Brook Taylor can be read. The letter is dated January 2, 1715, probably by Taylor on his receipt of it. This is in old style dating prior to English calendar reform that put the English calendar in sync with most of Europe and had the year start on January 1 rather than the English style of March 25. Consequently, the date on the Gregorian calendar that was used in France, and in use today, is January 13, 1716. A translation of the relevant part of the letter is:

“Never mind, just give me news. It is thought here that the Duke of Ormond is still in France, having been unable of crossing the border, and we have not received any definite news as to whether the Chevalier de Saint George is in Scotland. M. de Waldegrave, had he not met with that fatal accident, would have had no choice but to follow. This is a consolation for us in his misfortune.”

Waldegrave was definitely a Jacobite. What Charles Waldegrave's accident was is not known, but it was not fatal in the common meaning of the word. On March 31, 1716, Montmort wrote again to Brook Taylor remarking that Waldegrave was with him having café au lait.

Charles Waldegrave probably continued to live in France for the next few years and then possibly returned to England. These were difficult times for Catholics in England. Because of the wording of the oath, many Catholics would not swear allegiance to George I. The British parliament passed legislation in 1716 that required the registration of all persons who would not

swear allegiance to King George and who derived income, through rent or annuities, from landed properties. James, 1<sup>st</sup> Earl Waldegrave (then 2<sup>nd</sup> Baron Waldegrave), his mother and his brother, as well as Philip Waldegrave all appear on these registration lists [Cosin 1746, pp. 28, 40, 99, 113].

In 1722 there was a dramatic change in the Waldegrave family. James Waldegrave declared that he was a Protestant and swore an oath of allegiance to King George. He was soon given a position in the king's court and his career began to move in an upward direction. He was sent on a diplomatic mission to Paris in 1725 to compliment Louis XV on his marriage and two years later he was appointed the ambassador to the emperor of Austria. Rewards for his work soon followed; he was elevated to an earldom in 1729 [Woodfine 2004].

These events may have brought Charles Waldegrave back to England where he could now live under his nephew's protection. The evidence is very slim. A "Mr. Waldegrave" shows up on a list in which Mr. Pierson, James, Lord Waldegrave's steward recorded various monetary payments that he made [Waldegrave Family]. This Mr. Waldegrave received payments of £15, £25, £25, £20 and £20 on June 18, August 7 and September 17 of 1724 and on July 6 and August 7 of 1725 respectively. It is impossible to determine who this Mr. Waldegrave is other than to eliminate the 1<sup>st</sup> Earl himself, his mother and his brother who all received payments from the steward.

When James, Lord Waldegrave went to Vienna in 1727 to take up his diplomatic post, he stopped in Paris along the way. There he began cultivating informants in order to collect information on Jacobites still in France [Woodfine 2004]. In a list of payments to various French merchants and a few others with English names, a "Mr. Waldegrave" again shows up receiving 500 livres (about £20 to £21) on October 24, 1727 [Waldegrave Family]. With his Jacobite connections, the uncle would have made an ideal spy for the nephew. Again, it is impossible to know if this Mr. Waldegrave is indeed Charles Waldegrave.

If Charles Waldegrave indeed returned to England with his nephew, he may have lived on the Waldegrave estates into his late seventies or early eighties. A "Mr. Waldegrave" shows up in the Waldegrave family accounts in 1739 [Waldegrave Family]. There was a small annuity payment made to him as well as payments to two of his servants.

No other information on Charles Waldegrave can be found. He is a man whose early and later years were lived in obscurity. The only account, brief as it is, that we really have of this gentleman mathematician is through the few letters of Montmort that have survived. At least we now know who he is.

## Acknowledgements

This work was supported by a grant from the Social Sciences and Humanities Research Council of Canada. The author would like to thank Catherine Cox for her translation from French of the quotation from Montmort's letter to Brook Taylor and for the translation of the abstract. He would especially like to thank William Waldegrave, Baron Waldegrave of North Hill for his help with gaining access to the Waldegrave family papers and James Waldegrave, 13<sup>th</sup> Earl Waldegrave for his hospitality and permission to view the Waldegrave family papers.

## References

### Manuscripts

[House of Lords Record Office]

Acts of Parliament

HL/PO/JO/10/1/439/481(a): Lord Waldegrave's Estate Act, oath of Charles Waldegrave.

HL/PO/JO/10/1/439/481(b): Lord Waldegrave's Estate Act, oath of James Keith.

[Public Record Office]

Wills

PROB 11/376. Will of Sir Charles Waldegrave or Waldgrave of Chiswick, Middlesex, probate granted 12 June 1684.

PROB 11/407. Will of Henry Lord Waldegrave, probate granted 2 December 1691.

PROB 11/579. Will of Phillip Waldegrave of Kelvedon Hall, Essex, probate granted 3 March 1721.

[Royal Society Archives]

Correspondence of Dr Brook Taylor, Secretary of the Royal Society

MS/82/12: Letter from Remond Montmort 2 January 1715/16 (p. 88)

MS/82/13: Letter from Remond de Montmort 31 March 1716 (p. 84)

[Shakespeare Birthplace Trust Records Office]

Deeds relating to the Estates of the Leighs of Stoneleigh

DR 18/10/54/25: Deed relating to Flechamstead Manor

DR 18/1/2127a: Miscellaneous Deeds

[Shropshire Archives]

Collection from the Broseley Office of Messrs. Cooper & Co., Solicitors

1681/132/14: Conveyance

[Waldegrave Family]

Lord Waldegrave's Papers, Chewton House

Box labelled "Accounts Bills and Vouchers 1730–34", item "Mr. Piersons Accounts"

Box labelled "1727–8 Part 2", item "Lord Waldegrave's Journal of His Embassy to France"

Box labelled "Accounts, Bills and Vouchers", item "The Copy of James Underhills Account with the Right Hon<sup>ble</sup> the Earl Waldegrave from 1<sup>st</sup> October 1739 to the 29<sup>th</sup> December 1739"

### Printed Works

[Académie, 1718] *Académie universelle des jeux: contenant les regles des jeux de cartes permis, du trictrac, des echecs, de la Paulme, du mail, du billard, & autres : avec des instructions faciles pour apprendre à les bien jouer*, Paris, Le Gras, 1718.

[Barnard, 1991] G.A. Barnard: Review of "A History of Probability and Statistics: And Their Applications Before 1750", *SIAM Review*, 33, 286 – 290, 1991.

- [Baumol and Goldfeld, 1968] W.J. Baumol and S.M. Goldfeld, *Precursors in Mathematical Economics: An Anthology*, London, London School of Economics and Political Science, 1968.
- [Bernoulli, 1714] N. Bernoulli, *Solutio Generalis Problematis XV. Propositi a D. de Moivre, in tractatu de Mensura Sortis inserto Actis Philosophicis Anglicanis No 329. Pro Numero Quocunque Collusorum: per D. Nicolaum Bernoulli, Basiliensem, Reg. Soc. Sodalem*, *Philosophical Transactions*, 29, 133 – 144, 1714.
- [Bevan 2004] M. Bevan, *Sir William Waldegrave (1636? – 1701)*, in *Oxford Dictionary of National Biography* (Vol. 56), eds. H.C.G Matthew and B. Harrison, Oxford, Oxford University Press, 776 – 777, 2004
- [Burke, 2005] *Burke's Peerage & Gentry, Waldegrave*, <http://www.burkes-peerage.net/sites/peerageandgentry/sitepages/home.asp>, 2005.
- [Collins, 1970] A. Collins, *Collins's Peerage of England: genealogical, giographical, and historical. Greatly augmented, and continued to the present times by Sir Egerton Brydges (Volume IV)*, New York, AMS Press, 1970.
- [Cosin, 1746] A. Cosin, *The Names of the Roman Catholics, Nonjurors, and Others, Who Refus'd to Take the Oaths to His late Majesty King George*, London, Woodfall, 1746.
- [Cotton, 1674] C. Cotton, *The Compleat Gamester*, London: Cutler and Brome, 1674.
- [Cousin d'Avallon, 1842] C.-Y. Cousin d'Avallon, *Académie universelle des jeux*, 3e édition, Paris, Corbet aîné, 1842.
- [De Moivre, 1711] A. De Moivre, *De Mensura Sortis seu; de Probabilitate Eventuum in Ludis a Casu Fortuito Pendentibus*, *Philosophical Transactions*, 27, 213–264, 1711.
- [De Moivre, 1714] A. De Moivre, *Solutio Generalis Altera Praecedentis Problematis, ope Combinationum & Serierum Infinitarum*, per D. Abr. De Moivre. *Reg. Soc Sodalem*, *Philosophical Transactions*, 29, 145 – 158, 1714.
- [Fontenelle, 1721] B. de Fontenelle, *Éloge de M. de Montmort*, *Histoire de l'Académie royale des sciences ( Année 1719)*, 83–93, 1721.
- [Gregg, 2004] E. Gregg, *James Francis Edward*, in *Oxford Dictionary of National Biography* (Vol. 29), eds. H.C.G Matthew and B. Harrison, Oxford, Oxford University Press, 673 – 678, 2004.
- [Hald 1990] A. Hald, *A History of Probability and Statistics and Their Applications before 1750*, Wiley, New York, 1990.
- [Henny, 1975] J. Henny, *Niklaus und Johann Bernoullis Forschungen auf dem Gebiet der Wahrscheinlichkeitsrechnung in ihrem Briefwechsel mit Pierre Rémond de Montmort*, in *Die*

Werke von Jakob Bernoulli (Band 3), ed. B.L. van der Waerden, Basel, Birkhäuser, 457 – 507, 1975.

[Jacob, 1767], A. Jacob, A Complete English Peerage. Containing a Genealogical, Biographical, and Historical account of the Peers of this Realm (Volume II), London, Wilson, Fell and Robson, 1767.

[Lart, 1910] C.E. Lart, The Parochial Registers of Saint Germain-en-Laye: Jacobite Extracts of Birth, Marriages and Deaths (Volume I), London, St. Catherine Press, 1910.

[Lart, 1912] C.E. Lart, The Parochial Registers of Saint Germain-en-Laye: Jacobite Extracts of Birth, Marriages and Deaths (Volume II), London, St. Catherine Press, 1912.

[Montmort, 1713] P.R. de Montmort, Essay d'analyse sur les jeux de hazard. Seconde édition, Paris, Quillau, 1713.

[Nouvelle académie, 1721] La plus nouvelle académie universelle des jeux ou divertissemens innocens, contenant les regles des jeux de cartes permis; des echecs du trictrac, du revertier, du toute-table, du tourne-case des dames rabatues, du plain & du toc, comme aussi, celles du billiard, du mail, de la courte-paume, de la longue-paume, etc., avec des instructions faciles por apprendre à les bien jouer, Leiden: Pierre vander Aa, 1721.

[Seymour, 1719] R. Seymour, The Court Gamester, London, E. Curll, 1719.

[Taylor, 1793] B. Taylor, Contemplatio Philosophica: A Posthumous Work of the Late Brook Taylor LLD FRS, London, Bulmer, 1793.

[Todhunter, 1865] I. Todhunter, A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace, Cambridge, Cambridge University Press (reprinted 1965, New York: Chelsea Publishing), 1865.

[Willcox and Arnstein, 2001] W.B. Willcox and W.L. Arnstein, The Age of Aristocracy 1688 – 1830, eighth edition, Boston, Houghton Mifflin, 2001.

[Woodfine, 2004] P. Woodfine, James, first Earl Waldegrave (1684 – 1741), in Oxford Dictionary of National Biography (Vol. 56), eds. H.C.G Matthew and B. Harrison, Oxford, Oxford University Press, 768 – 771, 2004.