SOME REMARKS ON THE INVARIANT SUBSPACE PROBLEM
FOR HYPONORMAL OPERATORS

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ABSTRACT. We make some remarks concerning the invariant subspace problem for hyponormal operators. In particular, we bring together various hypotheses that must hold for a hyponormal operator without nontrivial invariant subspaces, and we discuss the existence of such operators.

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Let $\mathcal{H}$ be a separable, infinite-dimensional, complex Hilbert space and denote by $\mathcal{L}(\mathcal{H})$ the algebra of all linear and bounded operators on $\mathcal{H}$. An operator $T \in \mathcal{L}(\mathcal{H})$ is called hyponormal (notation: $T \in H(\mathcal{H})$) if $[T^*, T] := T^* T - TT^* \succeq 0$, or equivalently, if $\|T^* x\| \leq \|Tx\|$ for every $x \in \mathcal{H}$.

The purpose of this paper is to use several results that may be applied to the invariant subspace problem (ISP) for hyponormal operators and thus to bring into focus what remains to be done to solve the problem completely. We begin by recalling some standard notation and terminology to be used. For a (nonempty) compact subset $K \subset \mathbb{C}$, we denote by $C(K)$ the Banach algebra of all continuous complex-valued functions on $K$ with the supremum norm, by $\text{Rat}(K)$ the subalgebra of $C(K)$ consisting of all rational functions with poles off the set $K$, and by $R(K)$ the closure in $C(K)$ of $\text{Rat}(K)$. For $T \in \mathcal{L}(\mathcal{H})$, the spectrum of $T$ is denoted by $\sigma(T)$ and the algebra $\{r(T) : r \in \text{Rat}(\sigma(T))\}$ by $\text{Rat}(T)$. The rational cyclic multiplicity of $T$ (notation: $m(T)$) is the smallest cardinal number $m$ with the property that there are $m$ vectors $\{x_i\}_{0 \leq i < m}$ in $\mathcal{H}$ such that $\vee \{Ax_i \mid 0 \leq i < m, A \in \text{Rat}(T)\} = \mathcal{H}$. For a bounded (nonempty) open subset $U \subset \mathbb{C}$, one denotes by $H^\infty(U)$ the Banach algebra of those analytic complex-valued functions on $U$ with the property that $\sup_{z \in U} |f(z)| < \infty$. The ideal of all compact operators on $\mathcal{H}$ will be denoted by $\mathcal{K} = \mathcal{K}(\mathcal{H})$. Since $\mathcal{K}$ is a two-sided, norm-closed ideal in $\mathcal{L}(\mathcal{H})$, the quotient algebra $\mathcal{L}(\mathcal{H})/\mathcal{K}$ is a $C^*$-algebra, which is called the Calkin algebra, and the quotient map from $\mathcal{L}(\mathcal{H})$ to $\mathcal{L}(\mathcal{H})/\mathcal{K}$ will be denoted by $\pi$. For $T \in \mathcal{L}(\mathcal{H})$, we write $\sigma_r(T)$ (resp., $\sigma_{re}(T), \sigma_{le}(T)$) for the essential (resp., right-, left-essential) spectrum of $T$ (i.e., the spectrum (resp., right, left spectrum) of $\pi(T)$).

An operator $A \in \mathcal{K}(\mathcal{H})$ is called a trace-class operator (notation: $A \in \mathcal{C}_1(\mathcal{H})$) if the series $\text{tr}(A) := \sum_{i \in \mathbb{N}} |A|e_i, e_i$ is convergent, where $|A| = (A^* A)^{1/2}$ and $\{e_i\}_{i \in \mathbb{N}}$ is an orthonormal basis of $\mathcal{H}$. An operator $A \in \mathcal{K}(\mathcal{H})$ is Hilbert-Schmidt operator (notation: $A \in \mathcal{C}_2(\mathcal{H})$) if $A^* A \in \mathcal{C}_1(\mathcal{H})$. For a selfadjoint operator $A \in \mathcal{L}(\mathcal{H})$, $A_-$ will denote its negative part $(|A| - A)/2$. Finally, $\mu$ will denote planar Lebesgue measure defined on the Borel subsets of $\mathbb{R}^2$. 
A beautiful generalization of the Berger-Shaw inequality for hyponormal operators [3] was given by Voiculescu.

**Theorem 1** (see [17]). If \( [T^*, T]_\lambda \in \mathcal{E}_1(\mathcal{H}) \) and \( X \in \mathcal{E}_2(\mathcal{H}) \) is such that \( m(T + X) < +\infty \), then \( [T^*, T] \in \mathcal{E}_1(\mathcal{H}) \) and \( \pi \text{tr}([T^*, T]) < m(T + X)\mu(\sigma(T + X)) \).

For purposes of finding a nontrivial invariant subspace (n.i.s.) for an arbitrary operator \( T \in \mathcal{L}(\mathcal{H}) \), one may assume that every nonzero vector in \( \mathcal{H} \) is cyclic for \( T \), and hence that \( m(T) = 1 \). Thus the following corollary (of Theorem 1 or the earlier Berger-Shaw inequality) is useful.

**Corollary 2.** Every hyponormal operator \( T \in \mathcal{L}(\mathcal{H}) \) with a rational cyclic vector (i.e., \( m(T) = 1 \)) is essentially normal (i.e., \( \pi(T) \) is normal in the Calkin algebra).

When looking for a n.i.s. for an arbitrary \( T \in \mathcal{L}(\mathcal{H}) \), one knows (cf. [10]) from a deep theorem of Apostol, Foiaş, and Voiculescu [2] that \( T \) may be assumed to belong to the class \( \mathcal{B}\mathcal{T}(\mathcal{H}) \) of biquasitriangular operators, (see [10] for the definition and a characterization). If we denote by \( \mathcal{E}N(\mathcal{H}) \) the collection of essentially normal operators on \( \mathcal{H} \) and by \( (N + K)(\mathcal{H}) \) the collection of those operators in \( \mathcal{L}(\mathcal{H}) \) which can be written as a sum of a normal and a compact operator, then a deep result of Brown-Douglas-Fillmore can also be applied.

**Theorem 3** (see [5]). The equality \( \mathcal{B}\mathcal{T}(\mathcal{H}) \cap \mathcal{E}N(\mathcal{H}) = (N + K)(\mathcal{H}) \) holds.

It is thus immediate from Theorems 1 and 3 and Corollary 2 that if there exists \( T \) in \( H(\mathcal{H}) \) without a n.i.s., then \( [T^*, T] \in \mathcal{E}_1(\mathcal{H}) \) and \( T \in (N + K)(\mathcal{H}) \).

A hyponormal operator is called *pure* if it does not have a nonzero reducing subspace to which its restriction is a normal operator. Obviously an operator in \( H(\mathcal{H}) \) without a n.i.s. is pure. The following result of Putnam [14] leads to another reduction of the ISP for hyponormal operators.

**Theorem 4.** Let \( T \in \mathcal{L}(\mathcal{H}) \) be a pure hyponormal operator. If \( \Delta \subset \mathbb{C} \) is an open set, then \( \mu(\Delta \cap \sigma(T)) > 0 \) whenever \( \Delta \cap \sigma(T) \neq \emptyset \).

This says that each point of the spectrum of such a \( T \) has *positive planar density*, and thus we may assume of a hyponormal operator \( T \) without a n.i.s. that \( \sigma(T) \) has not only positive \( \mu \)-measure but positive planar density at each point.

Let \( A \in \mathcal{L}(\mathcal{H}) \) be a selfadjoint operator and denote by \( E \) the spectral measure of the operator \( A \). To every vector \( x \in \mathcal{H} \) one may associate the Borel measure \( \nu_x \) on \( \mathbb{R} \) defined by \( \nu_x(\Omega) = \langle E(\Omega)x, x \rangle \) for every Borel set \( \Omega \subset \mathbb{R} \). The vector \( x \) is called *absolutely continuous* with respect to \( A \) if the measure \( \nu_x \) is absolutely continuous with respect to Lebesgue measure on \( \mathbb{R} \). The selfadjoint operator \( A \) is called *absolutely continuous* if every vector of \( \mathcal{H} \) is absolutely continuous with respect to \( A \). The following result can be found in [13], (see also [9, page 135]).

**Proposition 5.** If \( T = X + iY \in \mathcal{L}(\mathcal{H}) \) is the Cartesian decomposition of a pure hyponormal operator, then \( X \) and \( Y \) are both absolutely continuous operators.

Next, recall that a subset \( \Delta \) of a nonempty open set \( U \) in \( \mathbb{C} \) is called *dominating* for \( U \) if \( \|f\|_{\infty, U} = \sup_{\lambda \in \Delta} |f(\lambda)|, \ f \in \mathcal{H}^{\infty}(U) \).
The deep invariant subspace theorem for hyponormal operators obtained by Brown in [6] on the basis of the beautiful structure theorem for such operators by Putinar [12] is the following.

**THEOREM 6.** Let $T \in \mathcal{L}(\mathcal{H})$ be a hyponormal operator. If there is a nonempty open set $U \subset \mathbb{C}$ such that $\sigma(T) \cap U$ is dominating for $U$, then $T$ has a n.i.s.

Since one knows (cf. [1] or [6]) that if $K$ is a (nonempty) compact set in $\mathbb{C}$ such that $R(K) \neq C(K)$, then $K$ is dominating on some nonempty open set, one gets immediately the following corollary.

**COROLLARY 7** (see [6]). Any hyponormal operator $T \in \mathcal{L}(\mathcal{H})$ with $R(\sigma(T)) \neq C(\sigma(T))$ has a n.i.s.

Thus if there exist hyponormal operators $T$ without a n.i.s., then as noted above, $T \in (N+K)(\mathcal{H})$ and $\sigma(T)$ must satisfy $R(\sigma(T)) = C(\sigma(T))$. Moreover, it is a consequence of elementary Fredholm theory (cf. [10]) that if $T \in \mathcal{L}(\mathcal{H})$ and $\sigma_{e}(T) \neq \sigma(T)$, then $T^{*}$ has point spectrum and thus $T$ has a n.i.s. Hence when looking for invariant subspaces for an arbitrary operator $T$ we may always suppose that $\sigma_{e}(T) = \sigma_{re}(T) = \sigma(T)$. This allows one to apply a result of Stampfli [16] to the problem.

**THEOREM 8.** Suppose $T \in \mathcal{L}(\mathcal{H})$ is such that $\sigma(T) = \sigma_{e}(T)$ and the Calkin map $\pi : \text{Rat}(T) \to \mathcal{L}(\mathcal{H})/\mathbb{K}$ is bounded below. Then $T$ has a n.i.s.

**Proof.** By hypothesis, there exists a constant $M > 0$ such that $\|\pi(r(T))\|_{e} \geq M\|r(T)\|$ for every $r \in \text{Rat}(\sigma(T))$, where $\|\cdot\|_{e}$ is the norm in the Calkin algebra. On the other hand,

$$\|\pi(r(T))\|_{e} = \|\pi(T)\|_{e} = \sup_{z \in \sigma_{e}(T)} |r(z)| = \sup_{z \in \sigma(T)} |r(z)|. \quad (1)$$

Thus $\|r(T)\| \leq (1/M)\|r\|_{\sigma(T)}$, $r \in \text{Rat}(\sigma(T))$, so $\sigma(T)$ is a $(1/M)$-spectral set for $T$. The result now follows from [16].

**COROLLARY 9.** If $T \in H(\mathcal{H})$ and $T$ has no n.i.s., then there exist sequences $\{r_{n}(T)\}_{n \in \mathbb{N}}$ in the algebra $\text{Rat}(T)$ and $\{K_{n}\}_{n \in \mathbb{N}}$ in $\mathbb{K}$ such that $\|r_{n}(T)\| = 1$, $n \in \mathbb{N}$, and $\|r_{n}(T) - K_{n}\| \to 0$. Moreover, any such sequence $\{r_{n}(T)\}_{n \in \mathbb{N}}$ has no subnet converging in the weak operator topology (WOT) to a nonzero operator.

**Proof.** Since $T \in H(\mathcal{H})$ has no n.i.s., $m(T) = 1$, and according to Corollary 2, $T \in \mathcal{L}(\mathcal{H})/\mathbb{K}$. According to Theorem 8, $\pi : \text{Rat}(T) \to \mathcal{L}(\mathcal{H})/\mathbb{K}$ must not be bounded below. Thus, there are sequences $\{r_{n}(T)\}_{n \in \mathbb{N}}$ in the algebra $\text{Rat}(T)$ and $\{K_{n}\}_{n \in \mathbb{N}}$ in $\mathbb{K}$ such that $\|r_{n}(T)\| = 1$, $n \in \mathbb{N}$, and $\|r_{n}(T) - K_{n}\| \to 0$. Moreover, if there is a subnet $\{r_{k}(T)\}_{k \in \mathbb{N}}$ converging in the WOT to a nonzero operator, then $T$ has a nontrivial hyperinvariant subspace according to [8].

The following proposition simply summarizes the results mentioned above.

**PROPOSITION 10.** If there exists $T \in H(\mathcal{H})$ such that $T$ has no n.i.s., then $T$ has the following properties:

(a) $\sigma_{e}(T) = \sigma_{re}(T) = \sigma(T),$
(b) \( \sigma(T) \) is a connected and perfect subset of \( \mathbb{C} \) such that every point of \( \sigma(T) \) has positive planar density,

c) \( R(\sigma(T)) = C(\sigma(T)) \), and, more generally, for every nonempty bounded open set \( U \subset \mathbb{C} \), \( U \cap \sigma(T) \) is not dominating for \( U \),

d) \( T = N + K \), where \( N \) is a normal operator and \( K \) is compact,

e) \( T = X + iY \), where \( X \) and \( Y \) are absolutely continuous selfadjoint operators,

(f) \( [T^*, T] \) is a positive semi-definite operator in \( \mathcal{E}_1(\mathcal{H}) \) with \( \text{tr}([T^*, T]) > 0 \),

(g) \( m(T) = 1 \),

(h) there exist sequences \( \{r_n(T)\}_{n \in \mathbb{N}} \subset \text{Rat}(T) \) and \( \{K_n\}_{n \in \mathbb{N}} \in \mathbb{K} \) such that \( \|r_n(T)\| = 1, n \in \mathbb{N}, \) and \( \|r_n(T) - K_n\| \to 0 \). Moreover, any such sequence has no subnet converging in the WOT to a nonzero operator.

This proposition raises the interesting question:

**Problem 11.** Are there any operators in \( H(\mathcal{H}) \) satisfying (a)-(h)?

This is perhaps a difficult question, which we are unable to answer at present. Moreover, the list of necessary conditions for a hyponormal operator that has no n.i.s. is larger and, of course, not all results are included in this paper. The remainder of this paper is devoted to making some progress on this question. We first recall a result from [11].

**Proposition 12.** Given a nonempty compact set \( K \subset \mathbb{C} \) with positive density at each point, there is an irreducible, hyponormal operator with rank one self-commutator whose spectrum is \( K \).

This proposition shows that to make a start toward answering Problem 11 in the affirmative, we need to construct a compact set \( K \) which has properties (b) and (c) of Proposition 10 (with \( K = \sigma(T) \)). A collection of such sets \( K \) may be constructed by slight variation of the following.

**Example 13.** We first specify a Cantor set \( C_{\{\theta_n\}} \subset [0, 1] \) which has positive linear Lebesgue measure and, in fact, has positive linear density at each point. For this purpose we follow the notation and terminology of [4, Example 6P] and set \( \theta_n = 1/3^n \) for each \( n \in \mathbb{N} \). Note that for each \( n \) the closed intervals in the collection \( \mathcal{F}_n \) have the same length—say \( l_n \). Let \( p \in C_{\{\theta_n\}} \) and let \( (a, b) \) be an open interval containing the point \( p \). Since \( C_{\{\theta_n\}} = \cap_n (\bigcup I : I \in \mathcal{F}_n) \) and \( l_n \to 0 \), there exists \( n_0 \) sufficiently large that some interval \( I_0 \in \mathcal{F}_{n_0} \) satisfies \( p \in I_0 \subset (a, b) \). Since \( C_{\{\theta_n\}} \cap I_0 \) is another Cantor set, its measure can be easily calculated to be greater than \( |l_{n_0} - l_{n_0}(\sum_{k \in \mathbb{N}} 1/3^{n_0 + k})| = l_{n_0}(1 - 1/(2 \cdot 3^{n_0})) > 0 \), and thus \( C_{\{\theta_n\}} \) has positive density at each point \( p \). Let now \( z_0 \) be the point \((1/2, 1) \) in \( \mathbb{C} \), and consider the planar set

\[
K_1 := \{tp + (1 - t)z_0 | 0 \leq t \leq 1, p \in C_{\{\theta_n\}}\}. \tag{2}
\]

We will show that \( K_1 \) has the properties (b) and (c) above. By construction, the set \( K_1 \) is arcwise connected and perfect (since \( C_{\{\theta_n\}} \) is perfect). Next, we show that every point \( q \) of \( K_1 \) has positive planar density. Let \( \Delta \) be an open disc in \( \mathbb{C} \) such that \( q \in \Delta \). Since \( K_1 \) is perfect, there exists a point \( t_0p_0 + (1 - t_0)x_0 \in \Delta \cap K_1 \) such that
0 < t_0 < 1. Clearly there exist positive real numbers t_1, t_2 such that 0 < t_1 < t_0 < t_2 < 1 and some nondegenerate interval [a, b] with p_0 \in [a, b] such that the trapezoid-like figure \( \gamma := \{(p + (1-t)z_0) : t_1 \leq t \leq t_2, p \in C([\theta_n] \cap [a, b]) \} \) is contained in \( \Delta \). Let \( \alpha > 0 \) be the linear measure of the set \( C([\theta_n] \cap [a, b]) \). Then the intersection of the set \( K_0 \cap \Gamma \) with each horizontal line \( y = t, t_1 \leq t \leq t_2 \) has (linear) measure \( (1-t)\alpha \). Thus, by Fubini’s theorem, the \( \mu \)-measure of the set \( K_1 \cap \Gamma \) is \( \int_{t_1}^{t_2} \alpha \, dt = (t_2 - t_1) \alpha / 2 > 0 \), and hence \( K_1 \) has property (b). To check that \( K_1 \) has property (c), the following lemma is useful.

**Lemma 14.** If \( U \) is any bounded open set such that \( U \cap K_1 \neq \emptyset \), then there exist a point \( p_0 \) belonging to the outer boundary of \( U \), an \( \varepsilon > 0 \), and a disc \( D = \{ z \in \mathbb{C} : |z - p_0| < \varepsilon \} \) such that \( D \cap K_1 = \emptyset \).

**Proof.** By construction, \( [0, 1] \setminus C[\theta_n] = \bigcup_{n=1}^{\infty} (a_n, b_n) \) where \( \{ (a_n, b_n) \}_{n=1}^{\infty} \) is the disjoint sequence of “excluded” open intervals. Thus each open triangular domain \( T_n = \{ (z_0 + (1-t)p) : -\infty < t < 1, p \in (a_n, b_n) \} \) in \( \mathbb{C} \) is disjoint from \( K_1 \). Since \( U \cap K_1 \neq \emptyset \) and every point of \( C([\theta_n]) \) is a limit point of end points of arbitrarily short excluded intervals, there exists some triangular domain \( T_n_0 \) such that \( U \cap T_n_0 \neq \emptyset \), and clearly any half-line joining a point of \( U \cap T_n_0 \) to the ideal point \( |z| = +\infty \) and lying entirely in \( T_n_0 \) must intersect \( \partial U \) in some last point (since \( \partial U \) is compact), which clearly satisfies the desired conclusions.

We next show that \( K_1 \) satisfies (c) of Proposition 10. Let \( U \) be a bounded open set in \( \mathbb{C} \) such that \( U \cap K_1 \neq \emptyset \) and set \( C = U^c \). Then the outer boundary of \( C \) coincides with the outer boundary of \( U \), and applying Lemma 14 to \( U \), we get a point \( p_0 \) of the outer boundary of \( U \) and an open disc \( D \) centered at \( p_0 \) with radius \( \varepsilon > 0 \) such that \( D \cap K_1 = \emptyset \). By [7, Corollary 13.3], \( p_0 \) is a peak point of \( R(C) \), that is, there exists an \( f_0 \in R(C) \) such that \( f_0(p_0) = 1 \) and \( |f_0(z)| < 1 \) for \( z \in C \setminus \{ p_0 \} \). Clearly \( f_0 \in H^\infty(U) \) and \( \sup_{\lambda \in \mathbb{D}} |f_0(\lambda)| = 1 \) (since \( p_0 \in \partial U \)) while \( \sup_{\lambda \in (K_1 \cap U)^{-}} |f_0(\lambda)| < 1 \) (since \( (K_1 \cap U)^{-} \) is at positive distance from \( p_0 \) and \( |f_0| < 1 \) on \( (K_1 \cap U)^{-} \subset C \setminus D \). Thus \( K_1 \cap U \) is not dominating for \( U \), and \( K_1 \) has property (c).

Let \( T_1 \) be an irreducible hyponormal operator with rank one self-commutator whose spectrum \( \sigma(T_1) \) is the compact set \( K_1 \) described in Example 13, and whose existence is guaranteed by Proposition 12. Thus property (f) is also satisfied. One observes that property (a) is satisfied too. Indeed, if one assumes that there exists \( \lambda_0 \in \sigma(T_1) \setminus \sigma_e(T_1) \), then \( T_1 - \lambda_0 \) is semi-Fredholm operator with nonpositive index (since \( T_1 \in H(\mathcal{H}) \)). Since \( T_1 \) is pure, \( \sigma_p(T_1) = \emptyset \), and thus the index is negative, which implies that \( \sigma(T_1) \) contains a nonempty open set. Obviously this is a contradiction since \( \sigma(T_1) = K_1 \) has no interior, and thus \( \sigma(T_1) = \sigma_{re}(T_1) \). In a similar way one shows that \( \sigma(T_1) = \sigma_{re}(T_1) \). Moreover, \( T_1 \in \mathcal{B}(\mathcal{H}) \) according to theorem of Apostol, Foiaș, and Voiculescu [2], since \( \sigma_e(T_1) \) contains no pseudoholes or holes associated with Fredholm index different from 0. Thus, by Theorem 3, the operator \( T_1 \) can be written \( T_1 = N + K \), where \( N \) is a normal operator and \( K \) is a compact operator. Moreover, since \( T_1 \) is pure, \( T_1 = X_1 + iY_1 \), where \( X_1 \) and \( Y_1 \) are absolutely continuous selfadjoint operators according to Proposition 5.

Thus we have shown that the operator \( T_1 \) has properties (a)-(f) of Proposition 10. Whether \( T_1 \) has properties (g) and (h) of this proposition the author is unable to
conclude. However, techniques and results of Stampfli [15] may be applied to obtain the following theorem.

**Theorem 15.** Any hyponormal operator $T$ in $\mathcal{B}(H)$ such that $\sigma(T)$ is the set $K_1$ of Example 13 has a nontrivial hyperinvariant subspace.

**Proof.** Consider the operator $T - (1/2 + i)$, which has spectrum $K_1 - (1/2 + i)$. Define $K_+ := K_1 \cap D_1$, where $D_1 := \{z \in \mathbb{C} : |z - 4| < 4\}$ and $K_- := K_1 \cap D_2$, where $D_2 := \{z \in \mathbb{C} : |z + 4| < 4\}$. Then $\sigma(T) = K_+ \cup K_-, K_+ \cap K_- = \{(0,0)\}$, and $\partial D_1 \cap K_1 = \partial D_2 \cap K_1 = \{(0,0)\}$. Choosing $f_1(z) = f_2(z) = z^2$, we may follow [15, Example 1] and observe that the operators $A_i := \int_{\partial D_i} f_i(z)(z - T)^{-1}dz, \; i = 1, 2$, commute with any operator $S$ with which $T$ commutes to conclude that $T$ has a nontrivial hyperinvariant subspace.

**Remark 16.** We note that it is quite easy to modify the construction of Example 13 to produce compact sets satisfying properties (b) and (c) of Proposition 10 such that the techniques of [15] of “integrating through the spectrum” are no longer available to produce nontrivial invariant subspaces for the corresponding operator $T$.

**References**


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Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

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