ON CHARACTERIZATIONS OF A CENTER GALOIS EXTENSION

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(Received 16 June 1999)

ABSTRACT. Let $B$ be a ring with 1, $C$ the center of $B$, $G$ a finite automorphism group of $B$, and $B^G$ the set of elements in $B$ fixed under each element in $G$. Then, it is shown that $B$ is a center Galois extension of $B^G$ (that is, $C$ is a Galois algebra over $C^G$ with Galois group $G|C \cong G$) if and only if the ideal of $B$ generated by $\{c - g(c) \mid c \in C\}$ is $B$ for each $g \neq 1$ in $G$. This generalizes the well known characterization of a commutative Galois extension $C$ that $C$ is a Galois extension of $C^G$ with Galois group $G$ if and only if the ideal generated by $\{c - g(c) \mid c \in C\}$ is $C$ for each $g \neq 1$ in $G$. Some more characterizations of a center Galois extension $B$ are also given.

Keywords and phrases. Galois extensions, center Galois extensions, central extensions, Galois central extensions, Azumaya algebras, separable extensions, $H$-separable extensions.

2000 Mathematics Subject Classification. Primary 16S30, 16W20.

1. Introduction. Let $C$ be a commutative ring with 1, $G$ a finite automorphism group of $C$ and $C^G$ the set of elements in $C$ fixed under each element in $G$. It is well known that a commutative Galois extension $C$ is characterized in terms of the ideals generated by $\{c - g(c) \mid c \in C\}$ for $g \neq 1$ in $G$, that is $C$ is a Galois extension with Galois group $G$ if and only if the ideal generated by $\{c - g(c) \mid c \in C\}$ is $C$ for each $g \neq 1$ in $G$ (see [3, Proposition 1.2, page 80]). A natural generalization of a commutative Galois extension is the notion of a center Galois extension, that is, a noncommutative ring $B$ with a finite automorphism group $G$ and center $C$ is called a center Galois extension of $B^G$ with Galois group $G$ if $C$ is a Galois extension of $C^G$ with Galois group $G|C \cong G$. Ikehata (see [4, 5]) characterized a center Galois extension with a cyclic Galois group $G$ of prime order in terms of a skew polynomial ring. Then, the present authors generalized the Ikehata characterization to center Galois extensions with Galois group $G$ of any cyclic order [7] and to center Galois extensions with any finite Galois group $G$ [8]. The purpose of the present paper is to generalize the above characterization of a commutative Galois extension to a center Galois extension. We shall show that $B$ is a center Galois extension of $B^G$ if and only if the ideal of $B$ generated by $\{c - g(c) \mid c \in C\}$ is $B$ for each $g \neq 1$ in $G$. A center Galois extension $B$ is also equivalent to each of the following statements:

(i) $B$ is a Galois central extension of $B^G$, that is, $B = B^G C$ which is $G$-Galois extension of $B^G$.

(ii) $B$ is a Galois extension of $B^G$ with a Galois system $\{b_i \in B, c_i \in C, i = 1, 2, \ldots, m\}$ for some integer $m$.

(iii) the ideal of the subring $B^G C$ generated by $\{c - g(c) \mid c \in C\}$ is $B^G C$ for each $g \neq 1$ in $G$. 
2. Definitions and notations. Throughout this paper, $B$ will represent a ring with $1$, $G = \{g_1 = 1, g_2, \ldots, g_n\}$ an automorphism group of $B$ of order $n$ for some integer $n$, $C$ the center of $B$, $B^G$ the set of elements in $B$ fixed under each element in $G$, and $B \ast G$ a skew group ring in which the multiplication is given by $gb = g(b)g$ for $b \in B$ and $g \in G$.

$B$ is called a $G$-Galois extension of $B^G$ if there exist elements $\{a_i, b_i \in B, i = 1, 2, \ldots, m\}$ for some integer $m$ such that $\sum_{i=1}^{m} a_i g(b_i) = \delta_1$, such a set $\{a_i, b_i\}$ is called a $G$-Galois system for $B$. $B$ is called a center Galois extension of $B^G$ if $C$ is a Galois algebra over $C^G$ with Galois group $G|_C \cong G$. $B$ is called a central extension of $B^G$ if $B = B^G C$, and $B$ is called a Galois central extension of $B^G$ if $B = B^G C$ is a Galois extension of $B^G$ with Galois group $G$.

Let $A$ be a subring of a ring $B$ with the same identity $1$. We denote $V_B(A)$ the commutator subring of $A$ in $B$. We call $B$ a separable extension of $A$ if there exist $\{a_i, b_i \in B, i = 1, 2, \ldots, m\}$ for some integer $m$ such that $\sum a_i b_i = 1$, and $\sum b_i a_i \otimes b_i b$ for all $b \in B$ where $\otimes$ is over $A$. $B$ is called $H$-separable extension of $A$ if $B \otimes_A B$ is isomorphic to a direct summand of a finite direct sum of $B$ as a $B$-bimodule. $B$ is called centrally projective over $A$ if $B$ is a direct summand of a finite direct sum of $A$ as a $A$-bimodule.

3. The characterizations. In this section, we denote $J_j^C = \{ c - g_j(c) \mid c \in C \}$. We shall show that $B$ is a center Galois extension of $B^G$ if and only if $B = B J_j^C$, the ideal of $B$ generated by $J_j^C$, for each $g_j \neq 1$ in $G$. Some more characterizations of a center Galois extension $B$ are also given. We begin with a lemma.

**Lemma 3.1.** If $B = B J_j^C$ for each $g_j \neq 1$ in $G$ (that is, $j \neq 1$), then

1. $B$ is a Galois extension of $B^G$ with Galois group $G$ and a Galois system $\{b_i \in B; c_i \in C, i = 1, 2, \ldots, m\}$ for some integer $m$.
2. $B$ is a centrally projective over $B^G$.
3. $B \ast G$ is $H$-separable over $B$.
4. $V_B(C) = C$.

**Proof.** (1) Since $B = B J_j^C$ for each $j \neq 1$, there exist $\{b_i \in B, c_i \in C, i = 1, 2, \ldots, m\}$ for some integer $m$, $j = 2, 3, \ldots, n$ such that $\sum b_i (c_i - g_j(c_i)) = 1$. Therefore, $\sum_{i=1}^{m} b_i^{(j)} c_i^{(j)} = 1 + \sum_{i=1}^{m} b_i^{(j)} g_j(c_i^{(j)})$. Let $b_i^{(j+1)} = -\sum_{i=1}^{m} b_i^{(j)} g_j(c_i^{(j)})$ and $c_i^{(j+1)} = 1$. Then $\sum_{i=1}^{m} b_i^{(j)} c_i^{(j)} = 1$ and $\sum b_i^{(j)} g_j(c_i^{(j)}) = 0$. Let $b_{i_2, i_3, \ldots, i_n} = b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)}$ and $c_{i_2, i_3, \ldots, i_n} = c_{i_2}^{(2)} c_{i_3}^{(3)} \cdots c_{i_n}^{(n)}$ for $i_2, i_3, \ldots, i_n \neq 1, 2, \ldots, m_j + 1$ and $j = 2, 3, \ldots, n$. Then

$$
\sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \cdots \sum_{i_n=1}^{m_n} b_{i_2, i_3, \ldots, i_n} c_{i_2, i_3, \ldots, i_n} = \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \cdots \sum_{i_n=1}^{m_n} b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)} c_{i_2}^{(2)} c_{i_3}^{(3)} \cdots c_{i_n}^{(n)}
= \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} c_{i_2}^{(2)} b_{i_3}^{(3)} c_{i_3}^{(3)} \cdots b_{i_n}^{(n)} c_{i_n}^{(n)}
= \sum_{i_2=1}^{m_2} b_{i_2}^{(2)} \sum_{i_3=1}^{m_3} b_{i_3}^{(3)} c_{i_3}^{(3)} \cdots \sum_{i_n=1}^{m_n} b_{i_n}^{(n)} c_{i_n}^{(n)} = 1
$$

(3.1)
and for each \( j \neq 1 \)

\[
\sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2,i_3,\ldots,i_n} g_j(c_{i_2,i_3,\ldots,i_n})
\]

\[
= \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)} g_j(c_{i_2}^{(2)} c_{i_3}^{(3)} \cdots c_{i_n}^{(n)})
\]

\[
= \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)} g_j(c_{i_2}^{(2)} g_j(c_{i_3}^{(3)}) \cdots g_j(c_{i_n}^{(n)}))
\]

\[
= \sum_{i_2=1}^{m_2+1} b_{i_2}^{(2)} g_j(c_{i_2}^{(2)}) \sum_{i_3=1}^{m_3+1} b_{i_3}^{(3)} g_j(c_{i_3}^{(3)}) \cdots \sum_{i_n=1}^{m_n+1} b_{i_n}^{(n)} g_j(c_{i_n}^{(n)}) = 0.
\]

Thus, \( \{b_{i_2,i_3,\ldots,i_n} \in B; c_{i_2,i_3,\ldots,i_n} \in C, i_j = 1,2,\ldots,m_j + 1 \text{ and } j = 2,3,\ldots,n \} \) is a Galois system for \( B \). This complete the proof of (1).

(2) By (1), \( B \) is a Galois extension of \( B^G \) with a Galois system \( \{b_i \in B, c_i \in C, i = 1,2,\ldots,m \} \) for some integer \( m \). Let \( f_i : B \to B^G \) given by \( f_i(b) = \sum_{j=1}^n g_j(c_i b) \) for all \( b \in B \), \( i = 1,2,\ldots,m \). Then it is easy to check that \( f_i \) is a homomorphism as \( B^G \)-bimodule and \( b = \sum_{i=1}^m b_i c_i b = \sum_{i=1}^m b_i g_j(c_i) g_j(b) = \sum_{i=1}^m b_i \sum_{j=1}^n g_j(c_i b) = \sum_{i=1}^m b_i f_i(b) \) for all \( b \in B \). Hence \( \{b_i; f_i, i = 1,2,\ldots,m \} \) is a dual bases for \( B \) as \( B^G \)-bimodule, and so \( B \) is finitely generated and projective as \( B^G \)-bimodule. Therefore, \( B \) is a direct summand of a finite direct sum of \( B^G \) as a \( B^G \)-bimodule. Thus \( B \) is centrally projective over \( B^G \).

(3) By (1), \( B \) is a Galois extension of \( B^G \) with Galois group \( G \). Hence \( B \ast G \cong \text{Hom}_{B^G}(B, B) \) [2, Theorem 1]. By (2), \( B \) is centrally projective over \( B^G \). Thus, \( B \ast G \cong \text{Hom}_{B^G}(B, B) \) is \( H \)-separable over \( B \) [6, Proposition 11].

(4) We first claim that \( V_{B \ast G}(C) = B \). Clearly, \( B \subset V_{B \ast G}(C) \). Let \( \sum_{i=1}^m b_i g_j \in V_{B \ast G}(C) \) for some \( b_j \in B \). Then \( c(\sum_{j=1}^n b_j g_j) = (\sum_{j=1}^n b_j g_j) c \) for each \( c \in C \), so \( c b_j = b_j g_j(c) \), that is, \( b_j (c - g_j(c)) = 0 \) for each \( g_j \in G \) and \( c \in C \). Since \( B = B_j^{(C)} \) for each \( g_j \neq 1 \), there exist \( b_i^{(j)} \in B \) and \( c_i^{(j)} \in C \), \( i = 1,2,\ldots,m \) such that \( \sum_{i=1}^m b_i^{(j)} (c_i^{(j)} - g_j(c_i^{(j)})) = 1 \). Hence \( b_j = \sum_{i=1}^m b_i^{(j)} (c_i^{(j)} - g_j(c_i^{(j)})) b_j = \sum_{i=1}^m b_i^{(j)} b_j (c_i^{(j)} - g_j(c_i^{(j)})) = 0 \) for each \( g_j \neq 1 \). This implies that \( \sum_{i=1}^m b_j g_j = b_1 \in B \). Hence \( V_{B \ast G}(C) \subseteq B \), and so \( V_{B \ast G}(C) = B \). Therefore, \( V_{B \ast G}(B) \subset V_{B \ast G}(C) = B \). Thus \( V_{B \ast G}(B) = V_B(B) = C \).

We now show some characterizations of a center Galois extension \( B \).

**Theorem 3.2.** The following statements are equivalent.

1. \( B \) is a center Galois extension of \( B^G \).
2. \( B = B_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \).
3. \( B \) is a Galois extension of \( B^G \) with a Galois system \( \{b_i \in B, c_i \in C, i = 1,2,\ldots,m \} \) for some integer \( m \).
4. \( B \) is a Galois central extension of \( B^G \).
5. \( B^G = B^G \ast C \) \( f_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \).
PROOF. (1)⇒(2). By hypothesis, \( C \) is a Galois extension of \( C^G \) with Galois group \( G|_C \cong G \). Hence \( C = C_{g_j}^{(C)} \) for each \( g_j \neq 1 \) in \( G \) [3, Proposition 1.2, page 80]. Thus, \( B = BJ_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \).

(2)⇒(1). Since \( B = BJ_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \), \( B \ast G \) is \( H \)-separable over \( B \) by Lemma 3.1(3) and \( V_{B \ast G}(B) = C \) by Lemma 3.1(4). Thus \( C \) is a Galois extension of \( C^G \) with Galois group \( G|_C \cong G \) by [1, Proposition 4].

(1)⇒(3). This is Lemma 3.1(1).

(3)⇒(1). Since \( B \) is Galois extension of \( B^G \) with a Galois system \( \{b_i \in B, c_i \in C, i = 1, 2, \ldots, m \} \) for some integer \( m \), we have \( \sum_{i=1}^{m} b_i g_j(c_i) = \delta_{1,j} \). Hence \( \sum_{i=1}^{m} b_i (c_i - g_j(c_i)) = 1 \) for each \( g_j \neq 1 \) in \( G \). So for every \( b \in B, b = \sum_{i=1}^{m} b_i (c_i - g_j(c_i)) \in BJ_j^{(C)} \). Therefore, \( B = BJ_j^{(C)} \) for each \( g_i \neq 1 \) in \( G \). Thus, \( B \) is a center Galois extension of \( B^G \) by (2)⇒(1).

(1)⇒(4). Since \( C \) is a Galois algebra with Galois group \( G|_C \cong G \), \( B \) and \( B^G \) are Galois extensions of \( B^C \) with Galois group \( G|_B^C \cong G \). Noting that \( B^G \subset B \), we have \( B = B^G \), that is, \( B \) is a central extension of \( B^G \). But \( B \) is a Galois extension of \( B^G \), so \( B \) is a Galois central extension of \( B^G \).

(4)⇒(1). By hypothesis, \( B = B^G \) is a Galois extension of \( B^G \). Hence there exists a Galois system \( \{a_i; b_l \in B, i = 1, 2, \ldots, m \} \) for some integer \( m \) such that \( \sum_{i=1}^{m} a_i g_j(b_l) = \delta_{1,j} \). But \( B = B^G \), \( C \), so \( a_i = \sum_{k=1}^{n_{a_i}} b_k^{(a_i)} c_k^{(a_i)} \) and \( b_l = \sum_{i=1}^{n_{b_i}} b_l^{(b_i)} c_i^{(b_i)} \) for some \( a_i^{(a_i)}, b_l^{(b_i)} \) in \( B^G \) and \( c_k^{(a_i)}, c_i^{(b_i)} \) in \( C \), \( k = 1, 2, \ldots, n_{a_i}, l = 1, 2, \ldots, n_{b_i}, i = 1, 2, \ldots, m \). Therefore,

\[
\delta_{1,j} = \sum_{i=1}^{m} a_i g_j(b_l) = \sum_{i=1}^{m} \sum_{k=1}^{n_{a_i}} b_k^{(a_i)} c_k^{(a_i)} g_j \left( \sum_{l=1}^{n_{b_i}} b_l^{(b_i)} c_l^{(b_i)} \right) = \sum_{i=1}^{m} \sum_{k=1}^{n_{a_i}} \sum_{l=1}^{n_{b_i}} b_k^{(a_i)} c_k^{(a_i)} b_l^{(b_i)} g_j(c_l^{(b_i)}) = \sum_{i=1}^{m} \sum_{k=1}^{n_{a_i}} \sum_{l=1}^{n_{b_i}} (b_k^{(a_i)} c_k^{(a_i)} b_l^{(b_i)}) g_j(c_l^{(b_i)}).
\]

This shows that \( \{b_k^{(a_i)} b_l^{(b_i)} = b_k^{(a_i)} c_k^{(a_i)} b_l^{(b_i)} \in B; c_k^{(a_i)} b_l^{(b_i)} = c_l^{(b_i)} \in C, k = 1, 2, \ldots, n_{a_i}, l = 1, 2, \ldots, n_{b_i}, i = 1, 2, \ldots, m \} \) is a Galois system for \( B \). Thus, \( B \) is a center Galois extension of \( B^G \) by (3)⇒(1).

(1)⇒(5). Since \( B \) is a center Galois extension of \( B^G, B = BJ_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \) by (1)⇒(2) and \( B = B^G \) by (1)⇒(4). Thus, \( B^G = BJ_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \).

(5)⇒(1). Since \( B^G = BJ_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \), \( B = BJ_j^{(C)} \) for each \( g_j \neq 1 \) in \( G \). Thus, \( B \) is a center Galois extension of \( B^G \) by (2)⇒(1).

The characterization of a commutative Galois extension \( C \) in terms of the ideals generated by \( \{c - g(c) \mid c \in C\} \) for \( g \neq 1 \) in \( G \) is an immediate consequence of Theorem 3.2.

**Corollary 3.3.** A commutative ring \( C \) is a Galois extension of \( C^G \) if and only if \( C = CJ_j^{(C)} \), the ideal generated by \( \{c - g(c) \mid c \in C\} \) is \( C \) for each \( g_j \neq 1 \) in \( G \).

**Proof.** Let \( B = C \) in Theorem 3.2. Then, the corollary is an immediate consequence of Theorem 3.2(2).

By Theorem 3.2, we derive several characterizations of a Galois centreal extension \( B \).
Corollary 3.4. If $B$ is a central extension of $B^G$ (that is, $B = B^G C$), then the following statements are equivalent.

1. $B$ is a Galois extension of $B^G$.
2. $B$ is a center Galois extension of $B^G$.
3. $B \star G$ is $H$-separable over $B$.
4. $B = CJ_j(B)$ for each $g_j \neq 1$ in $G$.
5. $B = BJ_j(B)$ for each $g_j \neq 1$ in $G$.

Proof. (1)$\iff$(2). This is given by (1)$\iff$(4) in Theorem 3.2.

(2)$\iff$(3). This is Lemma 3.1(3).

(3)$\iff$(1). Since $B \star G$ is $H$-separable over $B$, $B$ is a Galois extension of $B^G$ [1, Proposition 2].

Since $B = B^G C$ by hypothesis, it is easy to see that $J_j^B = B^G J_j(C)$ for each $g_j$ in $G$. Thus, $B = CJ_j(B)$, $B = BJ_j(B)$, and $B = BJ_j(C)$ are equivalent. This implies that (2)$\iff$(4)$\iff$(5) by Theorem 3.2(2). \qed

We call a ring $B$ the DeMeyer-Kanzaki Galois extension of $B^G$ if $B$ is an Azumaya $C$-algebra and $B$ is a center Galois extension of $B^G$ (for more about the DeMeyer-Kanzaki Galois extensions, see [2]). Clearly, the class of center Galois extensions is broader than the class of the DeMeyer-Kanzaki Galois extensions. We conclude the present paper with two examples. (1) The DeMeyer-Kanzaki Galois extension of $B^G$ and (2) a center Galois extension of $B^G$, but not the DeMeyer-Kanzaki Galois extension of $B^G$.

Example 3.5. Let $C$ be the field of complex numbers, that is, $C = \mathbb{R} + \mathbb{R} \sqrt{-1}$, where $\mathbb{R}$ is the field of real numbers, $B = C[i,j,k]$ the quaternion algebra over $C$, and $G = \{1, g \mid g(c_1 + c_i i + c_j j + c_k k) = g(c_1) + g(c_i)i + g(c_j)j + g(c_k)k \}$ for each $b = c_1 + c_i i + c_j j + c_k k \in C[i,j,k]$ and $g(u + v \sqrt{-1}) = u - v \sqrt{-1}$ for each $c = u + v \sqrt{-1} \in C$. Then

1. The center of $B$ is $C$.
2. $B$ is an Azumaya $C$-algebra.
3. $C$ is a Galois extension of $C^G$ with Galois group $G_{|C} \cong G$ and a Galois system \[
\{a_1 = 1/\sqrt{2}, a_2 = (1/\sqrt{2}) \sqrt{-1}; b_1 = 1/\sqrt{2}, b_2 = -(1/\sqrt{2}) \sqrt{-1}\}.
\]
4. $B$ is the DeMeyer-Kanzaki Galois extension of $B^G$ by (2) and (3).
5. $B^G = \mathbb{R}[i,j,k]$.
6. $B = B^G C$, so $B$ is a centeral extension of $B^G$.
7. $J_j^G = \mathbb{R} \sqrt{-1}$.
8. $B = BJ_j^G$ since $1 = -\sqrt{-1} \sqrt{-1} \in BJ_j^G$.
9. $J_j^G(B) = \mathbb{R} \sqrt{-1} + \mathbb{R} \sqrt{-1} i + \mathbb{R} \sqrt{-1} j + \mathbb{R} \sqrt{-1} k$.
10. $B = CJ_j^G(B)$.

Example 3.6. By replacing in Example 3.5 the field of complex numbers $C$ with the ring $C = \mathbb{Z} \oplus \mathbb{Z}$ where $\mathbb{Z}$ is the ring of integers, $g(a,b) = (b,a)$ for all $(a,b) \in C$, and $G = \{1, g \mid g(c_1 + c_i i + c_j j + c_k k) = g(c_1) + g(c_i)i + g(c_j)j + g(c_k)k \}$ for each $b = c_1 + c_i i + c_j j + c_k k \in B = C[i,j,k]$. Then

1. The center of $B$ is $C$.
2. $C$ is a Galois extension of $C^G$ with Galois group $G_{|C} \cong G$ and a Galois system \[
\{a_1 = (1,0), a_2 = (0,1); b_1 = (1,0), b_2 = (0,1)\}.
\]
(3) $B$ is not an Azumaya $C$-algebra (for $1/2 \not\in C$), and so $B$ is not the DeMeyer-Kanzaki Galois extension of $B^G$.

(4) $C^G = \{(a,a) | a \in \mathbb{Z}\} \cong \mathbb{Z}$.

(5) $B^G = C^G[i,j,k]$.

(6) $B = B^G C$, so $B$ is a central extension of $B^G$.

(7) $J^{(C)}_\theta = \{(a,-a) | a \in \mathbb{Z}\} = \mathbb{Z}(1,-1)$.

(8) $B = BJ^{(C)}_\theta$ since $1 = (1,1) = (1,-1)(1,-1) \in BJ^{(C)}_\theta$.

(9) $J^{(B)}_\theta = Z(1,-1) + Z(1,-1)i + Z(1,-1)j + Z(1,-1)k$.

(10) $B = CJ^{(B)}_\theta$.

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<td>May 1, 2009</td>
</tr>
<tr>
<td>First Round of Reviews</td>
<td>August 1, 2009</td>
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