A characterization of intuitionistic fuzzy $\alpha$-open set is given, and conditions for an IFS to be an intuitionistic fuzzy $\alpha$-open set are provided. Characterizations of intuitionistic fuzzy precontinuous (resp., $\alpha$-continuous) mappings are given.

1. Introduction

After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them. Using the notion of intuitionistic fuzzy sets, Çoker [5] introduced the notion of intuitionistic fuzzy topological spaces. In this paper, we define the notion of intuitionistic fuzzy semiopen (resp., preopen and $\alpha$-open) mappings and investigate relation among them. We give a characterization of intuitionistic fuzzy $\alpha$-open set, and provide conditions for an IFS to be an intuitionistic fuzzy $\alpha$-open set. We discuss characterizations of intuitionistic fuzzy precontinuous (resp., $\alpha$-continuous) mappings. We give a condition for a mapping of IFTSs to be an intuitionistic fuzzy $\alpha$-continuous mapping.

2. Preliminaries

Definition 2.1 (Atanassov [1]). An intuitionistic fuzzy set (IFS) $A$ in $X$ is an object having the form

$$A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \},$$

(2.1)

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 (Atanassov [1]). Let $A$ and $B$ be IFSs of the forms $A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \}$ and $B = \{ (x, \mu_B(x), \gamma_B(x)) \mid x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) \( \hat{A} = \{ \langle x, y_A(x), \mu_A(x) \rangle \mid x \in X \} \),
(d) \( A \cap B = \{ \langle x, \mu_A(x) \land \mu_B(x), y_A(x) \lor y_B(x) \rangle \mid x \in X \} \),
(e) \( A \cup B = \{ \langle x, \mu_A(x) \lor \mu_B(x), y_A(x) \land y_B(x) \rangle \mid x \in X \} \).

For the sake of simplicity, we will use the notation \( A = \langle x, \mu_A, \gamma_A \rangle \) instead of \( A = \{ \langle x, \mu_A(x), y_A(x) \rangle \mid x \in X \} \). A constant fuzzy set taking value \( \alpha \in [0,1] \) will be denoted by \( \alpha \). The IFSs \( 0 \sim \) and \( 1 \sim \) are defined to be \( 0 \sim = \langle x, 0, 1 \rangle \) and \( 1 \sim = \langle x, 1, 0 \rangle \), respectively. Let \( \alpha, \beta \in [0,1] \) with \( \alpha + \beta \leq 1 \). An intuitionistic fuzzy point (IFP), written as \( p(\alpha, \beta) \), is defined to be an IFS of \( X \) given by

\[
p(\alpha, \beta)(x) := \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0,1) & \text{otherwise}. \end{cases}
\] (2.2)

Let \( f \) be a mapping from a set \( X \) to a set \( Y \). If

\[
B = \left\{ \langle y, \mu_B(y), y_B(y) \rangle : y \in Y \right\}
\] (2.3)

is an IFS in \( Y \), then the preimage of \( B \) under \( f \), denoted by \( f^{-1}(B) \), is the IFS in \( X \) defined by

\[
f^{-1}(B) = \left\{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(y_B)(x) \rangle : x \in X \right\}
\] (2.4)

and the image of \( A \) under \( f \), denoted by \( f(A) \), is an IFS of \( Y \) defined by

\[
f(A) = \langle y, f(\mu_A), f(\gamma_A) \rangle,
\] (2.5)

where

\[
f(\mu_A)(y) := \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise}, \end{cases}
\] (2.6)

\[
f(\gamma_A)(y) := \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise}, \end{cases}
\] (2.7)

for each \( y \in Y \). Çoker [5] generalized the concept of fuzzy topological space, first initiated by Chang [4], to the case of intuitionistic fuzzy sets as follows.
Definition 2.3 (Çoker [5, Definition 3.1]). An intuitionistic fuzzy topology (IFT) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:

(T1) $0_-, 1_- \in \tau$,
(T2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(T3) $\bigcup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case, the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS) in $X$. The complement $\tilde{A}$ of an IFOS $A$ in IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS) in $X$.

Definition 2.4 (Çoker [5, Definition 3.13]). Let $(X, \tau)$ be an IFTS and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in $X$. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of $A$ are defined by

$$\text{int}(A) = \bigcup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$

$$\text{cl}(A) = \bigcap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}. \quad (2.8)$$

Note that for any IFS $A$ in $(X, \tau)$, we have

$$\text{cl}(\tilde{A}) = \overline{\text{int}(A)}, \quad \text{int}(\tilde{A}) = \underline{\text{cl}(A)}. \quad (2.9)$$

3. Intuitionistic fuzzy openness

Definition 3.1 [7]. An IFS $A$ in an IFTS $(X, \tau)$ is called

(i) an intuitionistic fuzzy semiopen set (IFSOS) if

$$A \subseteq \text{cl} (\text{int}(A)), \quad (3.1)$$

(ii) an intuitionistic fuzzy $\alpha$-open set (IF$\alpha$OS) [3] if

$$A \subseteq \text{int} (\text{cl} (\text{int}(A))), \quad (3.2)$$

(iii) an intuitionistic fuzzy preopen set (IFPOS) if

$$A \subseteq \text{int} (\text{cl}(A)), \quad (3.3)$$

(iv) an intuitionistic fuzzy regular open set (IFROS) if

$$\text{int} (\text{cl}(A)) = A. \quad (3.4)$$
An IFS $A$ is called an intuitionistic fuzzy semiclosed set, intuitionistic fuzzy $\alpha$-closed set, intuitionistic fuzzy preclosed set, and intuitionistic fuzzy regular closed set, respectively (IFS-SCS, IF$\alpha$CS, IFPCS, and IFRCS, resp.), if the complement of $A$ is an IFSOS, IF$\alpha$OS, IFPOS, and IFROS, respectively.

In the following diagram, we provide relations between various types of intuitionistic fuzzy openness (intuitionistic fuzzy closedness):

\[ \text{IFROS (IFRCS)} \]
\[ \downarrow \]
\[ \text{IFOS (IFCS)} \]
\[ \downarrow \]
\[ \text{IF} \alpha \text{OS (IF} \alpha \text{CS)} \]
\[ \downarrow \]
\[ \text{IFPOS (IFPCS)} \quad \text{IFSOS (IFSCS)} \]
\[ \downarrow \]
\[ \text{IF}\beta \text{OS (IF} \beta \text{CS)} \]

The reverse implications are not true in the above diagram (see [7]). The following is a characterization of an IF$\alpha$OS.

**Theorem 3.2.** An IFS $A$ in an IFTS $(X, \tau)$ is an IF$\alpha$OS if and only if it is both an IFSOS and an IFPOS.

**Proof.** Necessity follows from the diagram given above. Suppose that $A$ is both an IFSOS and an IFPOS. Then $A \subseteq \text{cl}(\text{int}(A))$, and so

$$\text{cl}(A) \subseteq \text{cl}(\text{cl}(\text{int}(A))) = \text{cl}(\text{int}(A)).$$

(3.6)

It follows that $A \subseteq \text{int}(\text{cl}(A)) \subseteq \text{int}(\text{cl}(\text{int}(A)))$, so that $A$ is an IF$\alpha$OS. □

We give condition(s) for an IFS to be an IF$\alpha$OS.

**Theorem 3.3.** Let $A$ be an IFS in an IFTS $(X, \tau)$. If $B$ is an IFSOS such that $B \subseteq A \subseteq \text{int}(\text{cl}(B))$, then $A$ is an IF$\alpha$OS.
Proof. Since $B$ is an IFSOS, we have $B \subseteq \text{cl}(\text{int}(B))$. Thus,

$$A \subset \text{int}(\text{cl}(B)) \subseteq \text{int}(\text{cl}(\text{int}(B))) = \text{int}(\text{cl}(\text{int}(B))) \subseteq \text{int}(\text{cl}(\text{int}(A))),$$  \hspace{1cm} (3.7)

and so $A$ is an IFαOS. \hfill \Box

**Lemma 3.4.** Any union of IFαOSs (resp., IFPOSs) is an IFαOS (resp., IFPOS).

The proof is straightforward.

**Theorem 3.5.** An IFS $A$ in an IFTS $X$ is intuitionistic fuzzy $\alpha$-open (resp., intuitionistic fuzzy preopen) if and only if for every IFP $p(\alpha, \beta) \in A$, there exists an IFαOS (resp., IFPOS) $B_{p(\alpha, \beta)}$ such that $p(\alpha, \beta) \in B_{p(\alpha, \beta)} \subseteq A$.

Proof. If $A$ is an IFαOS (resp., IFPOS), then we may take $B_{p(\alpha, \beta)} = A$ for every $p(\alpha, \beta) \in A$. Conversely assume that for every IFP $p(\alpha, \beta) \in A$, there exists an IFαOS (resp., IFPOS) $B_{p(\alpha, \beta)}$ such that $p(\alpha, \beta) \in B_{p(\alpha, \beta)} \subseteq A$. Then,

$$A = \bigcup \{p(\alpha, \beta) \mid p(\alpha, \beta) \in A\} = \bigcup \{B_{p(\alpha, \beta)} \mid p(\alpha, \beta) \in A\} \subseteq A,$$

and so $A = \bigcup \{B_{p(\alpha, \beta)} \mid p(\alpha, \beta) \in A\}$, which is an IFαOS (resp., IFPOS) by Lemma 3.4. \hfill \Box

**Definition 3.6.** Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then, $f$ is called

(i) an intuitionistic fuzzy open mapping if $f(A)$ is an IFOS in $Y$ for every IFOS $A$ in $X$,

(ii) an intuitionistic fuzzy $\alpha$-open mapping if $f(A)$ is an IFαOS in $Y$ for every IFOS $A$ in $X$,

(iii) an intuitionistic fuzzy preopen mapping if $f(A)$ is an IFPOS in $Y$ for every IFOS $A$ in $X$,

(iv) an intuitionistic fuzzy semiopen mapping if $f(A)$ is an IFSOS in $Y$ for every IFOS $A$ in $X$.

We have the following implications in which reverse implications are not valid, where “IF” means “intuitionistic fuzzy”:
Let $A = \langle x, \mu_A, \gamma_A \rangle$, $B = \langle x, \mu_B, \gamma_B \rangle$, and $C = \langle x, \mu_C, \gamma_C \rangle$ be IFSs in $I = [0,1]$ defined by

$$
\mu_A(x) = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{2}, \\
2x - 1, & \frac{1}{2} \leq x \leq 1,
\end{cases} \quad \gamma_A(x) = \begin{cases} 
1, & 0 \leq x \leq \frac{1}{2}, \\
2(1-x), & \frac{1}{2} \leq x \leq 1,
\end{cases} 
$$

$$
\mu_B(x) = \begin{cases} 
1, & 0 \leq x \leq \frac{1}{4}, \\
2 - 4x, & \frac{1}{4} \leq x \leq \frac{1}{2}, \\
0, & \frac{1}{2} \leq x \leq 1,
\end{cases} \quad \gamma_B(x) = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{4}, \\
4x - 1, & \frac{1}{4} \leq x \leq \frac{1}{2}, \\
1, & \frac{1}{2} \leq x \leq 1,
\end{cases} 
$$

$$
\mu_C(x) = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{4}, \\
\frac{1}{3}(4x - 1), & \frac{1}{4} \leq x \leq 1,
\end{cases} \quad \gamma_C(x) = \begin{cases} 
1, & 0 \leq x \leq \frac{1}{4}, \\
\frac{4}{3}(1-x), & \frac{1}{4} \leq x \leq 1.
\end{cases}
$$

Then $\tau_1 = \{0_-, 1_-, B, A \cup B\}$, $\tau_2 = \{0_-, 1_-, \overline{C}\}$, and $\tau_3 = \{0_-, 1_-, C\}$ are IFTSs on $I$. Define a mapping $f : I \to I$ by $f(x) = \min\{2x, 1\}$ for each $x \in I$. Then $f(0_-) = 0_-$, $f(1_-) = 1_-$, $f(A) = 0_-$, and $f(B) = \overline{A} = f(A \cup B)$. It is easy to verify that $\overline{A}$ is an IFαOS in $(I, \tau_2)$. Since $\overline{A} \notin \tau_2$, we know that the mapping $f : (I, \tau_1) \to (I, \tau_2)$ is intuitionistic fuzzy $\alpha$-open which is not intuitionistic fuzzy open. We also note that $\overline{A}$ is an IFOS but not an IFPOS in $(I, \tau_1)$. Hence, $f : (I, \tau_1) \to (I, \tau_1)$ is an intuitionistic fuzzy semiopen mapping which is not intuitionistic fuzzy preopen, and so, also not intuitionistic fuzzy $\alpha$-open. Further, $\overline{A}$ is an IFPOS which is not an IFOS in $(I, \tau_3)$. Therefore, $f : (I, \tau_1) \to (I, \tau_3)$ is an intuitionistic fuzzy preopen mapping which is not intuitionistic fuzzy semiopen, and thus, also not intuitionistic fuzzy $\alpha$-open.

**Theorem 3.7.** Let $f : (X, \tau) \to (Y, \kappa)$ and $g : (Y, \kappa) \to (Z, \delta)$ be mappings of IFTSs. If $f$ is intuitionistic fuzzy open and $g$ is intuitionistic fuzzy $\alpha$-open (resp., intuitionistic fuzzy preopen), then $g \circ f$ is intuitionistic fuzzy $\alpha$-open (resp., intuitionistic fuzzy preopen).

The proof is straightforward.

**Theorem 3.8.** A mapping $f : (X, \tau) \to (Y, \kappa)$ is intuitionistic fuzzy $\alpha$-open if and only if it is intuitionistic fuzzy preopen and intuitionistic fuzzy semiopen.

**Proof.** Necessity follows from the above second diagram (3.9). Assume that $f$ is intuitionistic fuzzy preopen and intuitionistic fuzzy semiopen and let $A$ be an IFOS in $X$. Then, $f(A)$ is an IFPOS as well as an IFOS in $Y$. It follows from Theorem 3.2 that $f(A)$ is an IFαOS so that $f$ is an intuitionistic fuzzy $\alpha$-open mapping. \(\square\)
4. Intuitionistic fuzzy continuity

**Definition 4.1** [7]. Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then $f$ is called an intuitionistic fuzzy precontinuous mapping if $f^{-1}(B)$ is an IFPOS in $X$ for every IFOS $B$ in $Y$.

**Theorem 4.2.** For a mapping $f$ from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$, the following are equivalent.

(i) $f$ is intuitionistic fuzzy precontinuous.

(ii) $f^{-1}(B)$ is an IFPCS in $X$ for every IFCS $B$ in $Y$.

(iii) $\text{cl}(\text{int}(f^{-1}(A))) \subseteq f^{-1}(\text{cl}(A))$ for every IFS $A$ in $Y$.

**Proof.** (i) $\Rightarrow$ (ii). The proof is straightforward.

(ii) $\Rightarrow$ (iii). Let $A$ be an IFS in $Y$. Then $\text{cl}(A)$ is intuitionistic fuzzy closed. It follows from (ii) that $f^{-1}(\text{cl}(A))$ is an IFPCS in $X$ so that

$$\text{cl}(\text{int}(f^{-1}(A))) \subseteq \text{cl}(\text{int}(f^{-1}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A)).$$

(4.1)

(iii) $\Rightarrow$ (i). Let $A$ be an IFOS in $Y$. Then $\overline{A}$ is an IFCS in $Y$, and so

$$\text{cl}(\text{int}(f^{-1}(\overline{A}))) \subseteq f^{-1}(\text{cl}(\overline{A})) = f^{-1}(\overline{A}).$$

(4.2)

This implies that

$$\overline{\text{int}(\text{cl}(f^{-1}(A)))} = \text{cl}(\overline{\text{cl}(f^{-1}(A))}) = \text{cl}(\text{int}(f^{-1}(A)))$$

$$= \text{cl}(\text{int}(f^{-1}(\overline{A}))) \subseteq f^{-1}(\overline{A}) = \overline{f^{-1}(A)},$$

(4.3)

and thus $f^{-1}(A) \subseteq \text{int}(\text{cl}(f^{-1}(A)))$. Hence $f^{-1}(A)$ is an IFPOS in $X$, and $f$ is intuitionistic fuzzy precontinuous. 

**Definition 4.3** [9]. Let $p_{(a,\beta)}$ be an IFP of an IFTS $(X, \tau)$. An IFS $A$ of $X$ is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(a,\beta)}$ if there exists an IFOS $B$ in $X$ such that $p_{(a,\beta)} \subseteq B \subseteq A$.

**Theorem 4.4.** Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then the following assertions are equivalent.

(i) $f$ is intuitionistic fuzzy precontinuous.

(ii) For each IFP $p_{(a,\beta)} \in X$ and every IFN $A$ of $f(p_{(a,\beta)})$, there exists an IFPOS $B$ in $X$ such that $p_{(a,\beta)} \subseteq B \subseteq f^{-1}(A)$.

(iii) For each IFP $p_{(a,\beta)} \in X$ and every IFN $A$ of $f(p_{(a,\beta)})$, there exists an IFPOS $B$ in $X$ such that $p_{(a,\beta)} \subseteq B$ and $f(B) \subseteq A$.

**Proof.** (i) $\Rightarrow$ (ii). Let $p_{(a,\beta)}$ be an IFP in $X$ and let $A$ be an IFN of $f(p_{(a,\beta)})$. Then there exists an IFOS $B$ in $Y$ such that $f(p_{(a,\beta)}) \subseteq B \subseteq A$. Since $f$ is intuitionistic fuzzy precontinuous,
we know that \( f^{-1}(B) \) is an IFPOS in \( X \) and

\[
P_{(\alpha, \beta)} \subseteq f^{-1}(f(p_{(\alpha, \beta)})) \subseteq f^{-1}(B) \subseteq f^{-1}(A). \quad (4.4)
\]

Thus (ii) is valid.

(ii) \( \Rightarrow \) (iii). Let \( p_{(\alpha, \beta)} \) be an IFP in \( X \) and let \( A \) be an IFN of \( f(p_{(\alpha, \beta)}) \). The condition (ii) implies that there exists an IFPOS \( B \) in \( X \) such that \( p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A) \) so that \( p_{(\alpha, \beta)} \in B \) and \( f(B) \subseteq f(f^{-1}(A)) \subseteq A \). Hence (iii) is true.

(iii) \( \Rightarrow \) (i). Let \( B \) be an IFOS in \( Y \) and let \( p_{(\alpha, \beta)} \in f^{-1}(B) \). Then \( f(p_{(\alpha, \beta)}) \in B \), and so \( B \) is an IFN of \( f(p_{(\alpha, \beta)}) \) since \( B \) is an IFOS. It follows from (iii) that there exists an IFPOS \( A \) in \( X \) such that \( p_{(\alpha, \beta)} \in A \) and \( f(A) \subseteq B \) so that

\[
P_{(\alpha, \beta)} \subseteq f^{-1}(A) \subseteq f^{-1}(B). \quad (4.5)
\]

Applying Theorem 3.5 induces that \( f^{-1}(B) \) is an IFPOS in \( X \). Therefore, \( f \) is intuitionistic fuzzy precontinuous.

\[\square\]

**Definition 4.5** [7]. Let \( f \) be a mapping from an IFTS \( (X, \tau) \) to an IFTS \( (Y, \kappa) \). Then \( f \) is called an **intuitionistic fuzzy \( \alpha \)-continuous mapping** if \( f^{-1}(B) \) is an IF\( \alpha \)OS in \( X \) for every IFOS \( B \) in \( Y \).

**Theorem 4.6.** Let \( f \) be a mapping from an IFTS \( (X, \tau) \) to an IFTS \( (Y, \kappa) \) that satisfies

\[
\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(\text{cl}(B)) \quad (4.6)
\]

for every IFS \( B \) in \( Y \). Then \( f \) is intuitionistic fuzzy \( \alpha \)-continuous.

**Proof.** Let \( B \) be an IFOS in \( Y \). Then \( \overline{B} \) is an IFCS in \( Y \), which implies from hypothesis that

\[
\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(\text{cl}(B)) = f^{-1}(\overline{B}). \quad (4.7)
\]

It follows that

\[
\overline{\text{int}(\text{cl}(f^{-1}(B)))) = \text{cl}(\overline{\text{int}(f^{-1}(B)))) = \text{cl}(\text{int}(\overline{\text{cl}(f^{-1}(B)))) = \text{cl}(\text{int}(\text{cl}(f^{-1}(\overline{B})))) = \text{cl}(\text{int}(\text{cl}(f^{-1}(\overline{B})))) \subseteq f^{-1}(\overline{B})
\]

so that \( f^{-1}(B) \subseteq \text{int}(\text{cl}(\text{int}(f^{-1}(B)))) \). This shows that \( f^{-1}(B) \) is an IF\( \alpha \)OS in \( X \). Hence, \( f \) is intuitionistic fuzzy \( \alpha \)-continuous. \[\square\]
Theorem 4.7. Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then the following assertions are equivalent.

(i) $f$ is intuitionistic fuzzy $\alpha$-continuous.

(ii) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN $A$ of $f(p_{(\alpha,\beta)})$, there exists an IFOS $B$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

(iii) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN $A$ of $f(p_{(\alpha,\beta)})$, there exists an IFOS $B$ such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii). Let $p_{(\alpha,\beta)}$ be an IFP in $X$ and let $A$ be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS $C$ in $Y$ such that $f(p_{(\alpha,\beta)}) \subseteq C \subseteq A$. Since $f$ is intuitionistic fuzzy $\alpha$-continuous, $B := f^{-1}(C)$ is an IFOS and

$$p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(C) = B \subseteq f^{-1}(A). \quad (4.9)$$

Thus (ii) is valid.

(ii) $\Rightarrow$ (iii). Let $p_{(\alpha,\beta)}$ be an IFP in $X$ and let $A$ be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS $B$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$ by (ii). Thus, we have $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (iii) is valid.

(iii) $\Rightarrow$ (i). Let $B$ be an IFOS in $Y$ and take $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in f(f^{-1}(B)) \subseteq B$. Since $B$ is an IFOS, it follows that $B$ is an IFN of $f(p_{(\alpha,\beta)})$ so from (iii), there exists an IFOS $A$ such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. This shows that

$$p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B). \quad (4.10)$$

Using Theorem 3.5, we know that $f^{-1}(B)$ is an IFOS in $X$, and hence $f$ is intuitionistic fuzzy $\alpha$-continuous.

Combining Theorems 4.6, 4.7, and [8, Theorems 3.12 and 3.13], we have the following characterization of an intuitionistic fuzzy $\alpha$-continuous mapping.

Theorem 4.8. Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then the following assertions are equivalent.

(i) $f$ is intuitionistic fuzzy $\alpha$-continuous.

(ii) If $C$ is an IFCS in $Y$, then $f^{-1}(C)$ is an IFCS in $X$.

(iii) $\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(\text{cl}(B))$ for every IFS $B$ in $Y$.

(iv) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN $A$ of $f(p_{(\alpha,\beta)})$, there exists an IFOS $B$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

(v) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN $A$ of $f(p_{(\alpha,\beta)})$, there exists an IFOS $B$ such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Some aspects of intuitionistic fuzzy continuity, intuitionistic fuzzy almost continuity, intuitionistic fuzzy weak continuity, intuitionistic fuzzy $\alpha$-continuity, intuitionistic fuzzy precontinuity, intuitionistic fuzzy semicontinuity, and intuitionistic fuzzy $\beta$-continuity
are studied in [7] as well as in several papers. The relation among these types of intuitionistic fuzzy continuity is given in [7] as follows, where “IF” means “intuitionistic fuzzy”:

\[
\begin{array}{c}
\text{IF weak continuity} \\
\text{IF almost continuity} \\
\text{IF continuity} \\
\text{IF } \alpha\text{-continuity} \\
\text{IF semicontinuity} \\
\text{IF precontinuity} \\
\text{IF } \beta\text{-continuity}
\end{array}
\]

(4.11)

The reverse implications are not true in the above diagram in general (see [7]).

**Theorem 4.9.** Let \( f \) be a mapping from an IFTS \((X, \tau)\) to an IFTS \((Y, \kappa)\). If \( f \) is both intuitionistic fuzzy precontinuous and intuitionistic fuzzy semicontinuous, then it is intuitionistic fuzzy \( \alpha \)-continuous.

**Proof.** Let \( B \) be an IFOS in \( Y \). Since \( f \) is both intuitionistic fuzzy precontinuous and intuitionistic fuzzy semicontinuous, \( f^{-1}(B) \) is both an IFPOS and an IFSOS in \( X \). It follows from Theorem 3.2 that \( f^{-1}(B) \) is an IF\( \alpha \)OS in \( X \) so that \( f \) is intuitionistic fuzzy \( \alpha \)-continuous. \( \square \)

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**References**


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Special Issue on
Decision Support for Intermodal Transport

Call for Papers

Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:

- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
- Loading orders
- Selection of routing and service

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