THE KREPS-YAN THEOREM FOR L^∞

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We prove the following version of the Kreps-Yan theorem. For any norm-closed convex cone $C \subset L^\infty$ such that $C \cap L^\infty_+ = \{0\}$ and $C \supset -L^\infty_+$, there exists a strictly positive continuous linear functional, whose restriction on C is nonpositive. The technique of the proof differs from the usual approach, applicable to a weakly Lindelöf Banach space.

1. The Kreps-Yan theorem

Let $\langle X, Y\rangle$ be a pair of Banach spaces in separating duality [18, Chapter IV]. A convex set $M \subset X$ is called cone if $\lambda x \in M$ for any $x \in M$, $\lambda \geq 0$. A cone M is called pointed if $M \cap (-M) = \{0\}$.

Suppose that X is endowed with a locally convex topology τ, which is always assumed to be compatible with the duality $\langle X, Y\rangle$, and $K \subset X$ is a τ-closed pointed cone. An element $\xi \in Y$ is called strictly positive if $\langle x, \xi\rangle > 0$ for all $x \in K \setminus \{0\}$. An element ξ is called nonnegative if $\langle x, \xi\rangle \geq 0$ for all $x \in K$. We only consider cones K such that the set of strictly positive functionals is nonempty.

Following [10], we say that the Kreps-Yan theorem is valid for the ordered space (X, K) with the topology τ if for any τ-closed convex cone C, containing $-K$, the condition $C \cap K = \{0\}$ implies the existence of a strictly positive element $\xi \in Y$ such that its restriction on C is nonpositive: $\langle x, \xi\rangle \leq 0$, $x \in C$. We also refer to [10] for the comments on the papers of Kreps [12] and Yan [20].

If the above statement is true for any τ-closed pointed cone $K \subset X$, we say that the Kreps-Yan theorem is valid for the space (X, τ). It should be mentioned that in this terminology the Kreps-Yan theorem may be valid for (X, τ) even if there exists a τ-closed pointed cone such that the set of strictly positive functionals is empty.

Recall that a space (X, τ') is said to be Lindelöf, or have the Lindelöf property, if every open cover of X has a countable subcover [11]. As usual, we denote the weak topology by $\sigma(X, Y)$.

The next theorem is, in fact, a partial case of [10, Theorem 3.1].

Theorem 1.1. *Let $(X, \sigma(X, Y))$ be a Lindelöf space. Then the Kreps-Yan theorem is valid for the space (X, τ).*

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Proof. Let $x \in K \setminus \{0\}$, then $x \notin C$ and by the separation theorem [18, Theorem II.9.2] there exists an element $\xi_x \in Y$ such that

$$\langle y, \xi_x \rangle < \langle x, \xi_x \rangle, \quad y \in C. \tag{1.1}$$

But C is a cone, hence we get the inequality $\langle y, \xi_x \rangle \leq 0$, $y \in C$. In addition, $-K \subset C$. Consequently,

$$\langle x, \xi_x \rangle > 0, \qquad \langle z, \xi_x \rangle \geq 0, \quad z \in K. \tag{1.2}$$

Consider the family of sets

$$A_x = \{ y \in X : \langle y, \xi_x \rangle > 0 \}, \quad x \in K \setminus \{0\} \tag{1.3}$$

and let $A_0 = \{ y \in X : |\langle y, \eta \rangle| < 1 \}$, where η is a strictly positive functional. The sets A_x, $x \in K$, are open in the topology $\sigma(X,Y)$ and constitute an open cover of K. Moreover, the cone K is closed in $\sigma(X,Y)$, because all topologies compatible with the duality $\langle X, Y \rangle$ have the identical collection of closed convex sets. In view of Lindelöf property, this implies the existence of the following countable subcover: $K \subset \cup_{i=0}^{\infty} A_{x_i}$, where $x_0 = 0$.

Let $\alpha_i = 1/(\|\xi_{x_i}\| 2^i)$, then $\sum_{i=1}^{\infty} \alpha_i \xi_{x_i}$ converges in the norm topology to some element $\xi \in Y$. Evidently, $\xi \leq 0$ on C. Moreover, ξ is strictly positive. Indeed, for any element $x \in K \setminus \{0\}$ there exists a $\lambda > 0$ such that $\lambda x \notin A_0$. Consequently, $\lambda x \in A_{x_k}$ for some $k \geq 1$ and

$$\langle \lambda x, \xi \rangle = \sum_{i=1}^{\infty} \alpha_i \langle \lambda x, \xi_{x_i} \rangle \geq \alpha_k \langle \lambda x, \xi_{x_k} \rangle > 0. \tag{1.4}$$

This completes the proof. □

In [10, Theorem 3.1] the following condition was used, conceptually connected with the Halmos-Savage theorem [8, Lemma 7]. For any family of nonnegative functionals $\{\xi_\beta\}_{\beta \in I} \subset Y$, there exists a countable subset $\{\xi_{\beta_i}\}_{i=1}^{\infty}$ with the following property: if for $x \in K \setminus \{0\}$ there exists a $\beta \in I$ such that $\langle x, \xi_\beta \rangle > 0$, then $\langle x, \xi_{\beta_i} \rangle > 0$ for some i.

We prefer to require that the space $(X, \sigma(X,Y))$ verifies the more standard Lindelöf condition. Clearly, this condition is satisfied if any topology of the space X, compatible with the duality $\langle X, Y \rangle$, has the Lindelöf property.

Denote by X^* the topological dual of X. Evidently, the space X is Lindelöf if it may be represented as the union of a countable collection of compact sets. Hence, a reflexive space X is Lindelöf in the weak topology $\sigma(X, X^*)$ (shortly, weakly Lindelöf) in view of the weak compactness of the unit ball, and the space X^* is Lindelöf in the $*$-weak topology $\sigma(X^*, X)$ by the Banach-Alaoglu theorem. So, the Kreps-Yan theorem is valid for any reflexive space with the norm topology and for the space $(X^*, \sigma(X^*, X))$.

A Banach space X is called weakly compactly generated (shortly, WCG), if X contains a weakly compact subset whose linear span is dense in X. Corson conjectured that the notions of weakly Lindelöf and WCG spaces are equivalent [3]. The one half of this conjecture was confirmed in [19] (see also [7, Theorem 12.35]): every WCG space is weakly Lindelöf (the converse implication appeared to be false in general as follows from [14, 16]). Therefore, the Kreps-Yan theorem is valid for any WCG space, endowed with the norm topology.

Let $(\Omega,\mathcal{F},\mathbf{P})$ be a probability space. Denote by $L^p = L^p(\Omega,\mathcal{F},\mathbf{P})$, $1 \le p \le \infty$, the Banach spaces of equivalence classes of measurable functions, whose p's power is integrable if $1 \le p < \infty$ (resp., which are essentially bounded if $p = \infty$). The above arguments imply the following result (compare with [10], [17, Theorem 1.4]): the Kreps-Yan theorem is valid for the spaces (L^p,τ_p), $1 \le p \le \infty$, where τ_p is the norm topology for $1 \le p < \infty$, and $\tau_\infty = \sigma(L^\infty,L^1)$.

Indeed, the spaces L^p, $1 < p < \infty$, are reflexive, the topology $\sigma(L^\infty,L^1)$ of the space L^∞ coincides with the weak-star one, and L^1 is a WCG space [5, page 143].

On the other hand, it is shown in [10, Example 2.1] that the Kreps-Yan theorem may fail even if (X,K) is a Banach lattice (with the norm topology). So, the imposed Lindelöf condition is not superfluous. Note also, that Theorem 1.1 does not imply the validity of the Kreps-Yan theorem for the space L^∞ with the norm topology: it is known that even the space of bounded sequences is not weakly Lindelöf [3, Example 4.1(i)].

2. The case of L^∞

Let L^∞_+ be the cone, generating the natural order structure on L^∞. Our main result is the following.

Theorem 2.1. *The Kreps-Yan theorem is valid for the ordered space (L^∞,L^∞_+) with the norm topology.*

Recall that the dual of L^∞ (with the norm topology) coincides with the Banach space $\mathrm{ba} = \mathrm{ba}(\Omega,\mathcal{F},\mathbf{P})$ of all bounded finitely additive measures μ on $(\Omega,\mathcal{F})$ with the property that $\mathbf{P}(A) = 0$ implies $\mu(A) = 0$ [6]. Let

$$\mathrm{ba}_+ = \{\mu \in \mathrm{ba} : \langle x,\mu\rangle \ge 0,\ x \in L^\infty_+\} \tag{2.1}$$

be the set of nonnegative elements of ba. A probability measure $\mathbf{Q}$ is identified with the continuous functional on L^∞ by the formula

$$\langle x,\mathbf{Q}\rangle = \int_\Omega x\,d\mathbf{Q} = \mathbf{E}_\mathbf{Q}x. \tag{2.2}$$

For the convenience of the reader, we recall here Yan's theorem [20, Theorem 1], [15, Lemma 3, page 145].

Theorem 2.2 (Yan). *Let M be a convex subset of $L^1(\mathbf{P})$, $0 \in M$. Assume that for any $\varepsilon > 0$ there exists $c > 0$ such that $\mathbf{P}(x \ge c) \le \varepsilon$ for all $x \in M$. Then there exists a probability measure $\mathbf{Q}$ equivalent to $\mathbf{P}$ (with a bounded density $d\mathbf{Q}/d\mathbf{P}$) such that $\sup_{x\in M}\mathbf{E}_\mathbf{Q}x < \infty$.*

Let $C \subset L^\infty$ be a norm-closed convex cone, satisfying the conditions

$$C \cap L^\infty_+ = \{0\}, \quad -L^\infty_+ \subset C. \tag{2.3}$$

Put $C_\varepsilon = \{x \in C : \operatorname{ess\,inf} x \ge -\varepsilon\}$.

LEMMA 2.3. *For any norm-closed convex cone $C \subset L^\infty$ satisfying (2.3), there exists a probability measure $\mathbf{Q}$ equivalent to $\mathbf{P}$ such that*

$$\sup_{x \in C_1} \langle x, \mathbf{Q} \rangle < \infty. \tag{2.4}$$

Proof. It suffice to show that the set C_1 satisfies the conditions of Yan's theorem. We literally follow the argumentation of [4, Proposition 3.1], where a somewhat more special set is considered.

Clearly, C_1 is convex and $0 \in C_1$. Assume that there exist a sequence of elements $x_n \in C_1$, $n \geq 1$, and a number $\alpha > 0$ such that $\mathbf{P}(x_n \geq n) > \alpha$. The elements $y_n = \min\{x_n/n, 1\}$ belong to $C_{1/n} \subset C_1$ and

$$\mathbf{P}(y_n = 1) = \mathbf{P}\left(\frac{x_n}{n} \geq 1\right) > \alpha. \tag{2.5}$$

Denote by $\operatorname{conv} A$ the convex hull of the set A. If $D \subset \Omega$ we put

$$I_D(\omega) = 1, \quad \omega \in D; \qquad I_D(\omega) = 0, \quad \omega \notin D. \tag{2.6}$$

By [4, Lemma A1.1] there exists a sequence

$$z_n \in \operatorname{conv}(y_n, y_{n+1}, \dots) \subset C_{1/n}, \tag{2.7}$$

converging a.s. to $z : \Omega \mapsto [0,1]$. Furthermore, the inequality

$$\mathbf{E_P} y_n \geq \mathbf{E_P}(I_{\{y_n = 1\}}) - \mathbf{E_P}\left(\frac{I_{\{y_n < 1\}}}{n}\right) \geq \alpha - 1/n, \tag{2.8}$$

implies that $\mathbf{E_P} z_n \geq \alpha - 1/n$ and by Lebesgue's dominated convergence theorem,

$$\mathbf{E_P} z = \lim_{n \to \infty} \mathbf{E_P} z_n \geq \alpha. \tag{2.9}$$

Hence

$$\mathbf{P}(z > 0) = \beta \geq \mathbf{E_P}(z I_{\{z > 0\}}) = \mathbf{E_P} z \geq \alpha. \tag{2.10}$$

By Egorov's theorem $z_n \to z$ uniformly on a set Ω': $\mathbf{P}(\Omega') \geq 1 - \beta/2$. The functions $w_n = \min\{z_n, I_{\Omega'}\}$ belong to C and $w_n = z_n I_{\Omega'} \to z I_{\Omega'}$ in the norm topology of L^∞. We obtain a contradiction, since

$$\mathbf{P}(z I_{\Omega'} > 0) = \mathbf{P}(\Omega') + \mathbf{P}(z > 0) - \mathbf{P}(\Omega' \cup \{z > 0\}) \geq \frac{\beta}{2}. \tag{2.11}$$

This completes the proof. □

Now we need some additional notation, used in convex analysis (e.g., [13]). Let again $\langle X, Y \rangle$ be a pair of Banach spaces in duality. The indicator and support functions of a convex set $A \subset X$ are defined by the formulas

$$\delta A(x) = 0, \quad x \in A, \qquad \delta A(x) = +\infty, \quad x \notin A; \qquad sA(\xi) = \sup_{x \in A} \langle x, \xi \rangle. \tag{2.12}$$

The same notation is used if $A \subset Y$. The sets

$$A^\circ = \{\xi \in Y : \langle x,\xi\rangle \le 1,\, x \in A\}, \qquad A^{\circ\circ} = \{x \in X : \langle x,\xi\rangle \le 1,\, \xi \in A^\circ\} \tag{2.13}$$

are called polar and bipolar of A.

The Young-Fenchel transform of a function $f : X \mapsto [-\infty,+\infty]$ is defined as follows:

$$f^*(\xi) = \sup_{x\in X}\left(\langle x,\xi\rangle - f(x)\right). \tag{2.14}$$

The function

$$(f_1 \oplus f_2)(x) = \inf\{f_1(x_1) + f_2(x_2) : x_1 + x_2 = x\} \tag{2.15}$$

is called an infimal convolution of f_1, f_2.

Note, that the support function of a set A is equal to the Minkowski function μA° of the polar A°:

$$sA(\xi) = \mu A^\circ(\xi), \quad \mu A^\circ(\xi) = \inf\{\lambda > 0 : \xi \in \lambda A^\circ\}. \tag{2.16}$$

We will use the next formula for its Young-Fenchel transform:

$$\begin{aligned}(\mu A^\circ)^*(x) &= \sup_{\xi\in Y}\left(\langle x,\xi\rangle - \inf\{\lambda > 0 : \xi \in \lambda A^\circ\}\right) = \sup_{\lambda>0}\sup_{\xi\in\lambda A^\circ}\left(\langle x,\xi\rangle - \lambda\right)\\ &= \sup_{\lambda>0}\lambda\left(\sup_{\eta\in A^\circ}\langle x,\eta\rangle - 1\right) = \delta A^{\circ\circ}(x).\end{aligned} \tag{2.17}$$

Proof of Theorem 2.1. Let $\mathbf{Q}$ be a measure, introduced in Lemma 2.3. Put

$$\varphi(\varepsilon) = -\sup_{x\in C_\varepsilon}\langle x,\mathbf{Q}\rangle. \tag{2.18}$$

Note, that $C_\varepsilon = \varnothing$ for $\varepsilon < 0$, $C_0 = \{0\}$, and $C_\varepsilon = \varepsilon C_1$ for $\varepsilon > 0$. Since the support function of an empty set is equal to $-\infty$, we get

$$\varphi(\varepsilon) = \varepsilon\varphi(1) + \delta[0,+\infty)(\varepsilon), \quad \varphi(1) \le 0. \tag{2.19}$$

Denote by $\mathcal{P}$ the set of all probability measures $\mathbf{P}'$, absolutely continuous with respect to $\mathbf{P}$. We have

$$\operatorname{ess\,inf} x = \inf_{\mathbf{P}'\in\mathcal{P}}\langle x,\mathbf{P}'\rangle = -s(-\mathcal{P})(x) \tag{2.20}$$

and $C_\varepsilon = C \cap \{x \in L^\infty : s(-\mathcal{P})(x) \le \varepsilon\}$. So, for $\tau < 0$ the function φ^* has the following representation:

$$\begin{aligned}\varphi^*(\tau) &= \sup_{\varepsilon\ge 0}\sup_{x\in C_\varepsilon}\left(\varepsilon\tau + \langle x,\mathbf{Q}\rangle\right) = \sup_{x\in C}\sup_{s(-\mathcal{P})(x)\le\varepsilon}\left(\varepsilon\tau + \langle x,\mathbf{Q}\rangle\right)\\ &= \sup_{x\in C}\left(\tau\cdot s(-\mathcal{P})(x) + \langle x,\mathbf{Q}\rangle\right).\end{aligned} \tag{2.21}$$

For $\lambda = -\tau$ we obtain

$$\begin{aligned}\varphi^*(-\lambda) &= \sup_{x\in L^\infty}\left(\langle x,\mathbf{Q}\rangle - \lambda\cdot s(-\mathcal{P})(x) - \delta C(x)\right) = \left(s(-\lambda\mathcal{P}) + \delta C\right)^*(\mathbf{Q}) \\ &= \left(\left(s(-\lambda\mathcal{P})\right)^* \oplus (\delta C)^*\right)(\mathbf{Q}).\end{aligned} \tag{2.22}$$

The last equality (see, e.g., [9]) is valid, because the function $s(-\lambda\mathcal{P})$ is continuous on the whole space L^∞ in the norm topology.

Using the identities

$$\left(s(-\lambda\mathcal{P})\right)^* = \left(\mu(-\lambda\mathcal{P})^\circ\right)^* = \delta(-\lambda\mathcal{P})^{\circ\circ}, \qquad (\delta C)^* = sC = \delta C^\circ, \tag{2.23}$$

we get

$$\varphi^*(-\lambda) = \left(\delta(-\lambda\mathcal{P})^{\circ\circ} \oplus \delta C^\circ\right)(\mathbf{Q}) = \delta\left((-\lambda\mathcal{P})^{\circ\circ} + C^\circ\right)(\mathbf{Q}). \tag{2.24}$$

On the other hand, directly from the representation (2.19), we obtain

$$\varphi^*(\tau) = \sup_{\varepsilon}\left(\varepsilon\tau - \varphi(\varepsilon)\right) = \sup_{\varepsilon\geq 0}\left(\varepsilon\left(\tau - \varphi(1)\right)\right) = \delta\left(-\infty,\varphi(1)\right](\tau). \tag{2.25}$$

It follows that $\varphi^*(-\lambda) = 0$ for $\lambda > -\varphi(1)$. Thus,

$$\mathbf{Q} \in C^\circ + (-\lambda\mathcal{P})^{\circ\circ}, \quad \lambda \in \left(-\varphi(1),+\infty\right) \tag{2.26}$$

and there exists an element $\mu \in C^\circ$ such that

$$\mu = \mathbf{Q} + \nu, \quad \nu \in -(-\lambda\mathcal{P})^{\circ\circ} = \lambda\mathcal{P}^{\circ\circ}. \tag{2.27}$$

But $\mathcal{P}^{\circ\circ}$ coincides with the $\sigma(\mathrm{ba},L^\infty)$-closed convex hull of the set $\mathcal{P}\cup\{0\}\subset \mathrm{ba}_+$ by the bipolar theorem [18, Theorem IV.1.5]. Hence, $\nu \in \mathrm{ba}_+$ and μ is a desired functional: it is strictly positive and $\langle x,\mu\rangle \leq 0$, $x\in C$. The proof is complete. □

After the paper was submitted, Professor G. Cassese informed the author that he (by another methods) had independently and simultaneously proved a somewhat more general version of Theorem 2.1 [2]. We find it convenient to restate here the main ingredient of this approach together with its simple proof, based on Theorem 2.1. It should be mentioned that the argumentation of [2] goes in the opposite direction.

Theorem 2.4 (Cassese). *Let $\mathcal{M}\subset \mathrm{ba}_+$ be a convex $\sigma(\mathrm{ba},L^\infty)$-closed set of finitely additive probabilities, that is, $\langle 1,m\rangle = 1$, $m\in\mathcal{M}$. If for any $x\in L^\infty_+\setminus\{0\}$ there exists $m\in\mathcal{M}$ such that $\langle x,m\rangle > 0$, then $\mathcal{M}$ contains a strictly positive element.*

Proof. Note, that the set $D = \{\lambda x : x\in\mathcal{M},\ \lambda\geq 0\}$ is convex and $\sigma(\mathrm{ba},L^\infty)$-closed [1, Lemma III.2.10, page 116]. Furthermore, its polar $C = D^\circ$, taken in L^∞, is norm-closed and satisfies the conditions (2.3). By Theorem 2.1 there exists a strictly positive element $\xi\in C^\circ$. By the bipolar theorem and the closedness of D, we have $\xi\in D$. It remains to note that ξ can be normalized such that $\xi\in\mathcal{M}$. □

Another interesting comment comes from Professor W. Schachermayer, who in a personal communication pointed out that the above ideas can be transformed in a more direct proof of Theorem 2.1. This proof also is based on Lemma 2.3, but uses only separation arguments and does not appeal to Fenchel duality. We have the pleasure to present it below.

LEMMA 2.5. *Let $C \subset L^\infty$ be a norm-closed convex cone, satisfying (2.3). For any element $f \in \mathrm{ba}$ the following conditions are equivalent:*

(i) $\sup_{x\in C_1}\langle x,f\rangle < +\infty$, $C_1 = \{x \in C : \|x^-\|_{L^\infty} \le 1\}$, $x^- = \max\{0,-x\}$;

(ii) *there exists $g \in \mathrm{ba}$ such that $g \ge f$ and $g \in C^\circ$.*

Proof. (ii)⇒(i). Let $x \in C_1$, then

$$\langle x,f\rangle = \langle x,g\rangle + \langle x,f-g\rangle \le \langle -x,g-f\rangle \le \langle x^-,g-f\rangle \le \|g-f\|_{\mathrm{ba}}. \tag{2.28}$$

(i)⇒(ii). Consider the $\sigma(\mathrm{ba},L^\infty)$-compact convex set $\Pi = \{h \in \mathrm{ba}_+ : \|h\|_{\mathrm{ba}} \le 1\}$ and put

$$\lambda = \sup_{x\in C_1}\langle x,f\rangle. \tag{2.29}$$

If the condition (ii) is false, we may separate the sets $f+\lambda\Pi$ and C° by an element $x \in L^\infty$:

$$\sup_{\eta\in C^\circ}\langle x,\eta\rangle < \inf_{\zeta\in f+\lambda\Pi}\langle x,\zeta\rangle. \tag{2.30}$$

Since C° is a cone, we get $\langle x,\eta\rangle \le 0$, $\eta \in C^\circ$. Thus, $x \in C^{\circ\circ} = C$ by the bipolar theorem and

$$\langle x,f\rangle + \lambda\inf_{h\in\Pi}\langle x,h\rangle > 0. \tag{2.31}$$

Furthermore, since $x \notin L^\infty_+$ and it can be normalized such that $\inf_{h\in\Pi}\langle x,h\rangle = -1$. Hence, $x \in C_1$ and $\langle x,f\rangle > \lambda$. This yields the desired contradiction to (2.29), which completes the proof. □

Clearly, Theorem 2.1 is implied by Lemmas 2.3 and 2.5 (put $f = \mathbf{Q}$). By a more careful analysis it can be shown that Lemma 2.5 still holds true for any convex cone $C \subset L^\infty$ such that $C \cap L^\infty_+ = \{0\}$.

Finally, we mention that the case of L^∞ with the norm topology is of special interest for mathematical finance in view of characterization of the no free lunch with vanishing risk condition [4].

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