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MAGNETO-THERMOELASTIC WAVES INDUCED BY A THERMAL SHOCK IN A FINITELY CONDUCTING ELASTIC HALF SPACE

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ABSTRACT. The propagation of magneto-thermoelastic disturbances produced by a thermal shock in a finitely conducting elastic half-space in contact with vacuum is investigated. The boundary of the half-space is subjected to a normal load. Lord-Shulman theory of thermoelasticity [1] is used to account for the interaction between the elastic and thermal fields. Laplace transform on time is .used to obtain the short-time approximations of the solutions because of the short duration of 'second sound' effects. It is found that in the half-space the displacement is continuous at the modified dilational and thermal wavefronts, whereas the perturbed magnetic field, stress and the temperature suffer discontinuities at these locations. The perturbed magnetic field, is, however, discontinuous at the Alf'ven-acoustic wavefront in vacuum.

KEY WORDS AND PHRASES: Magneto-thermoelastic waves; finite conduction.
1991 AMS SUBJECT CLASSIFICATION CODES:

1. INTRODUCTION

The generation of magneto-thermoelastic waves by a thermal shock in a perfectly conducting half-space in contact with vacuum was investigated by Kaliski and Nowacki [2]. Both media were supposed to be permeated by a primary uniform magnetic field. But the influence of coupling between the temperature and strain fields was neglected. The coupling between temperature and strain fields was taken into account by Massalas and Dalamangas [3]. Then Roychoudhuri and Chatterjee [4,5] extended the problem [3] in generalized thermoelasticity by using the thermal relaxation time of Lord-Shulman theory [1] and the theory of Green and Lindsay [6], involving two relaxation times. Later, Sharma and Dayal Chand [7] studied transient generalized magneto-thermoelastic waves in a perfectly conducting elastic half-space due to a normal load acting on the boundary of the half-space using the generalized theory of thermoelasticity developed by Lord and Shulman [1].

Solutions of more complicated problems than that of ref. [2], [3] and [5] were investigated respectively by Kaliski and Nowacki [8], Massalas and Dalamangas [9] and Roychoudhuri and Chatterjee [10], where they assumed that the elastic half-space had a finite conductivity.

In the present paper we extend the problem [7] assuming that the elastic half-space has a finite conductivity in the case when the boundary of the half-space is subjected to a prescribed normal load and thermal shock. The solutions valid for short-times, for the deformation, stress, temperature distribution and perturbed magnetic field in the half-space as well as in the vacuum are derived.

2. PROBLEM FORMULATION

We assume that a magneto-thermoelastic wave is produced in an elastic half-space $x_1 \geq 0$ due to a normal load and a thermal shock applied on $x_1 = 0$.

The simplified linear equations of electrodynamics of slowly moving bodies having finite conductivity are the following [10]

$$\left.\begin{aligned} \vec{\nabla}\times\vec{E} &= -\frac{\mu_0}{C}\cdot\frac{\partial\vec{h}}{\partial t} \\ \vec{\nabla}\times\vec{h} &= \frac{4\pi}{C}\vec{j} \\ \vec{\nabla}\cdot\vec{h} &= 0 \\ \vec{j} &= \lambda_0\left[\vec{E}+\frac{\mu_0}{C}\left(\dot{\vec{u}}\times\vec{H}_0\right)\right] \end{aligned}\right] \tag{1}$$

where $\vec{E}$ denotes the electric field, $\vec{h}$ is the perturbation of the magnetic field, $\vec{H}_0$ is the initial constant magnetic field, $\vec{j}$ denotes the current density vector, $\vec{u}$ denotes the displacement vector, μ_0 is the magnetic permeability, λ_0 is the electrical conductivity and C is the velocity of light.

The linear form of the displacement equations of motion including electromagnetic effect and the modified form of Fourier's law of heat conduction in the context of Lord-Shulman theory [1] of thermoelasticity are,

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\vec{\nabla}(\vec{\nabla}\cdot\vec{u})+\frac{\mu_0}{4\pi}[(\vec{\nabla}\times\vec{h})\times\vec{H}_0]-\gamma\vec{\nabla}\theta = \rho\ddot{\vec{u}} \tag{2}$$

$$\rho C_v(\dot{\theta}+\tau_0\ddot{\theta})+\gamma T_0(\dot{\Delta}+\tau_0\ddot{\Delta}) = K\nabla^2\theta \tag{3}$$

where λ, μ are the Lamé constants, $\gamma = (3\lambda+2\mu)\alpha_T$, α_T is the coefficient of linear thermal expansion, $\theta = T - T_0$, T is the absolute temperature, T_0 is the uniform temperature of the body in its natural state, $K = \frac{\lambda_T}{C_\epsilon}$, λ_T denotes coefficient of heat conduction, C_ϵ is the specific heat at constant strain, ρ is the mass density, C_v is the sp. heat at constant volume, and τ_0 is the thermal relaxation time, Δ is the dilation.

The equations (1), after elimination of $\vec{E}$ and $\vec{j}$ give,

$$\nabla^2\vec{h}-\beta\dot{\vec{h}} = -\beta\vec{\nabla}\times(\dot{\vec{u}}\times\vec{H}_0) \tag{4}$$

where

$$\beta = \frac{4\pi\lambda_0\mu_0}{C}$$

The magneto-thermoelastic wave propagated in the medium $x_1 \geq 0$ is assumed to depend on x_1 and time t. Furthermore it is assumed that the initial magnetic field vector is directed along the x_3-axis

i.e. $\vec{H}_0 = (0,0,H_3)$, where H_3 is a constant.

Under these assumptions, equations (1) lead to

$$\vec{j} = \frac{C}{4\pi}\left(0, -\frac{\partial h_3}{\partial x_1}, 0\right)$$

$$\dot{\vec{h}} = -\frac{C}{\mu_0}\left(0, 0, \frac{\partial E_2}{\partial x_1}\right)$$

$$\vec{j} = \lambda_0\left[0, \left(E_2 - \frac{\mu_0 H_3 \dot{u}_1}{C}\right), 0\right]$$

Consequently

$$\vec{E} = (0, E_2, 0),$$

where

$$E_2 = -\frac{C}{4\pi\lambda_0}\cdot\frac{\partial h_3}{\partial x_1} + \frac{\mu_0 H_3 \dot{u}_1}{C}$$

The equations (2), (3) and (4) then reduce to

$$(\lambda + 2\mu)\frac{\partial^2 u_1}{\partial x_1^2} - \frac{\mu_0 H_3}{4\pi}\cdot\frac{\partial h_3}{\partial x_1} - \gamma\frac{\partial \theta}{\partial x_1} = \rho\cdot\frac{\partial^2 u_1}{\partial t^2} \tag{5}$$

$$\rho C_v\left(\frac{\partial \theta}{\partial t} + \tau_0\frac{\partial^2 \theta}{\partial t^2}\right) + \gamma T_0\left(\frac{\partial^2 u_1}{\partial x_1 \partial t} + \tau_0\frac{\partial^3 u_1}{\partial x_1 \partial t^2}\right) = K\frac{\partial^2 \theta}{\partial x_1^2} \tag{6}$$

$$\frac{\partial^2 h_3}{\partial x_1^2} - \beta\frac{\partial h_3}{\partial t} = \beta H_3\frac{\partial^2 u_1}{\partial x_1 \partial t} \tag{7}$$

For clarity, we shall use the notations, $u_1 = u, x_1 = x$ in the following.

Since, the elastic medium is in contact with the vacuum, equations (5)-(7) have to be supplemented by the electrodynamic equations in vacuum.

In vacuum, the system of equations of electrodynamics reduce to the following

$$\left.\begin{aligned}
&\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{C^2}\frac{\partial^2}{\partial t^2}\right)\mathring{h}_3 = 0\\
&\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{C^2}\frac{\partial^2}{\partial t^2}\right)\mathring{E}_2 = 0\\
&\dot{\mathring{\vec{h}}} = C\left(0, 0, \frac{\partial \mathring{E}_2}{\partial x'}\right)\\
&\dot{\mathring{\vec{E}}} = C\left(0, \frac{\partial \mathring{h}_3}{\partial x'}, 0\right)\\
&\text{where} \quad x' = -x
\end{aligned}\right| \tag{8}$$

3. BOUNDARY CONDITIONS

The components of Maxwell's stress tensor in the elastic medium T_{11} and in vacuum $\mathring{T}_{11}$ are given by

$$\left.\begin{aligned}
T_{11} &= -\frac{\mu_0}{4\pi}h_3 H_3\\
\mathring{T}_{11} &= -\frac{\mathring{h}_3 H_3}{4\pi}
\end{aligned}\right| \tag{9}$$

The normal stress in the elastic medium is obtained as

$$\sigma_{11} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma\theta \tag{10}$$

The boundary conditions are assumed as

$$\sigma_{11} + T_{11} - \overset{\circ}{T}_{11} = \sigma_0 H(t) \quad \text{on} \quad x = x' = 0 \tag{11}$$

$$E_2 = \overset{\circ}{E}_2 , h_3 = \overset{\circ}{h}_3 \quad \text{on} \quad x = x' = 0 \tag{12}$$

and the thermal boundary condition is assumed as

$$\theta(0,t) = \theta_0 H(t) \quad \text{on} \quad x = x' = 0 \tag{13}$$

where $H(t)$ is the Heaviside unit function.

The initial conditions are,

$$\left.\begin{aligned} u(x,0) &= 0 \\ \theta(x,0) &= 0 \\ \frac{\partial u(x,0)}{\partial t} &= 0 \end{aligned}\right] \tag{14}$$

4. SOLUTION OF THE PROBLEM

To find the solution of the problem we introduce the following notations and non-dimensional variables

$$\zeta = \frac{C_0 x}{\kappa}, \quad \tau = \frac{C_0^2}{\kappa} t, \quad z = \frac{\theta}{T_0},$$

$$U = \frac{C_0(\lambda + 2\mu + a_0^2\rho)u}{\kappa\gamma T_0}, \quad \kappa = \frac{K}{\rho C_v},$$

$$\varepsilon = \frac{\gamma^2 T_0}{C_\varepsilon(\lambda + 2\mu + a_0^2\rho)}, \quad h_3 = h, \quad C_\varepsilon = pC_v,$$

$$\beta_1 = \frac{\mu_0 H_3}{4\pi\gamma T_0}, \quad \beta_2 = \frac{1}{\kappa\beta}, \quad \beta_3 = \frac{H_3\gamma T_0}{\rho C_0^2}$$

$$\beta_4 = \frac{C_0^2}{4\pi\lambda_0\kappa}, \quad \beta_5 = \frac{\mu_0 H_3 \gamma T_0}{\rho C^2}, C_1^2 = \frac{\lambda + 2\mu}{\rho},$$

$$C_0^2 = C_1^2 + a_0^2, \quad a_0^2 = \frac{\mu_0 H_3^2}{4\pi\rho}, \quad \alpha = \frac{C_0}{C}, \quad \tau_0' = \frac{C_0^2}{\kappa} \cdot \tau_0 .$$

The equations (5), (6), (7) then reduce to

$$\frac{C_1^2}{C_0^2} U_{,\zeta\zeta} - \beta_1 h_{,\zeta} - Z_{,\zeta} - U_{,\tau\tau} = 0, \quad \zeta > 0 \tag{15}$$

$$Z_{,\zeta\zeta} - Z_{,\tau} - \tau_0' Z_{,\tau\tau} - \varepsilon U_{,\zeta\tau} - \varepsilon\tau_0' U_{,\zeta\tau\tau} = 0, \quad \zeta > 0 \tag{16}$$

$$\beta_2 h_{,\zeta\zeta} - h_{,\tau} = \beta_3 U_{,\zeta\tau}, \quad \zeta > 0 . \tag{17}$$

The equations (8) reduce to

$$\left.\begin{aligned} \overset{\circ}{h}_{,\zeta'\zeta'} - \alpha^2 \overset{\circ}{h}_{,\tau\tau} &= 0 \\ \overset{\circ}{E}_{2,\zeta'\zeta'} - \alpha^2 \overset{\circ}{E}_{2,\tau\tau} &= 0 \end{aligned}\right]_{,\zeta' > 0} \tag{18}$$

where $\zeta' = -\zeta$

$\overset{\circ}{h}_3 = \overset{\circ}{h}$

Boundary conditions (11)-(13) reduce to

$$\frac{C_1^2}{C_0^2}U_{,\zeta} - Z + \beta'\overset{\circ}{h} = \frac{\sigma_0}{\gamma T_0}H(\tau) \quad \text{on} \quad \zeta = \zeta' = 0 \tag{19}$$

$$h = \overset{\circ}{h} \quad \text{on} \quad \zeta = \zeta' = 0 \tag{20}$$

$$-\beta_4 h_{,\zeta\tau} + \beta_5 U_{,\tau\tau} + \overset{\circ}{h}_{,\zeta} = 0 \quad \text{on} \quad \zeta = \zeta' = 0 \tag{21}$$

$$Z(0,\tau) = \frac{\theta_0}{T_0}H(\tau) \quad \text{on} \quad \zeta = \zeta' = 0 \tag{22}$$

where

$$\beta' = \frac{(1-\mu_0)H_3}{4\pi\gamma T_0}.$$

The total stress σ_{11}^* in the half-space is given by

$$\sigma_{11}^* = \sigma_{11} + T_{11} \tag{23}$$

where

$$\sigma_{11} = (\lambda + 2\mu)\frac{\partial U}{\partial \zeta} - \gamma\theta$$

$$T_{11} = -\frac{\mu_0 H_3 h_3}{4\pi}.$$

We observe that the equations (15), (16), (17) and (18) are in agreement with the equations (24), (25), (26) and (27) of [9] respectively, on setting $\tau_0' = 0$ (with some change of notations). Also on setting $\sigma_0 = 0$, equation (19) is in agreement with the 1st equation of (14) of [10], on setting $\alpha' = 0$ except possibly a factor due to some erroneous calculations in [10]. Equations (20), (21) and (22) are also in agreement with the 3rd, 2nd and 4th equn. of (14) of [10] respectively.

The initial conditions (14) reduce to

$$U(\zeta,0) = 0, \quad \frac{\partial U}{\partial \tau}(\zeta,0) = 0, \quad Z(\zeta,0) = 0. \tag{24}$$

Usually it is very difficult to find the solution of the above equations, which constitute a set of coupled partial differential equations in three variables $U, Z,$ and h with coupled boundary condition. For this, a simplification is made assuming that the perturbed magnetic field h in $\zeta > 0$ is such that $\frac{\partial^2 h}{\partial \zeta^2} \cong 0$ which implies that the perturbed magnetic field in the half-space varies very slowly with distance so that $\frac{\partial^2 h}{\partial \zeta^2} \cong 0$. For this assumption equation (17) gives

$$h = -\beta_3 U_{,\zeta}, \quad \zeta > 0. \tag{25}$$

Equation (15) then reduces to

$$U_{,\zeta\zeta} - Z_{,\zeta} - U_{,\tau\tau} = 0, \quad \zeta > 0.$$

On taking Laplace transform, this equation reduces to

$$\left(\frac{\partial^2}{\partial \zeta^2} - s^2\right)\overline{U} - \frac{\partial \overline{Z}}{\partial \zeta} = 0, \quad \zeta > 0. \tag{26}$$

On taking Laplace transform, equations (16) and (19)-(22) reduce to

$$\left(\frac{\partial^2}{\partial \zeta^2} - s - \tau_0' s^2\right)\overline{Z} = \varepsilon s(1 + \tau_0' s)\frac{\partial \overline{U}}{\partial \zeta}, \quad \zeta > 0 \tag{27}$$

$$\frac{C_1^2}{C_0^2}\cdot\frac{\partial\overline{U}}{\partial\zeta}-\overline{Z}+\beta'\overline{\overset{\circ}{h}}=\frac{\sigma_0}{\gamma T_0}\cdot\frac{1}{s},\quad \text{on}\quad \zeta=\zeta'=0 \tag{28}$$

$$\overline{h}=\overline{\overset{\circ}{h}},\quad \text{on}\quad \zeta=\zeta'=0 \tag{29}$$

$$\beta_5 s^2\overline{U}+(1-\beta_4 s)\frac{\partial\overline{h}}{\partial\zeta}=0\quad \text{on}\quad \zeta=\zeta'=0 \tag{30}$$

$$\overline{Z}=\frac{\theta_0}{T_0}\cdot\frac{1}{s},\quad \text{on}\quad \zeta=\zeta'=0\,. \tag{31}$$

With the help of equation (27), elimination of $\overline{U}$ from equation (26) and (27) yields

$$\left[\frac{\partial^4}{\partial\zeta^4}-s\{\varepsilon+1+(1+\tau_0'+\varepsilon\tau_0')s\}\frac{\partial^2}{\partial\zeta^2}+s^3(1+\tau_0's)\right]\overline{Z}=0\,. \tag{32}$$

The general solution of the above equation, vanishing at $\zeta=\infty$ is given by

$$\overline{Z}=A_1\exp(-\lambda_1\zeta)+A_2\exp(-\lambda_2\zeta) \tag{33}$$

where λ_1^2, λ_2^2 are the roots of the equation

$$\lambda^4-s\{\varepsilon+1+(1+\tau_0'+\varepsilon\tau_0')s\}\lambda^2+s^3(1+\tau_0's)=0\,. \tag{34}$$

Hence,

$$\lambda_{1,2}=\left[\frac{s}{2}\{(s+1+\varepsilon+\tau_0'\varepsilon s+\tau_0's)\pm[(1+\varepsilon^2\tau_0'^2+\tau_0'^2+2\varepsilon\tau_0'+2\varepsilon\tau_0'^2-2\tau_0')s^2\right.$$
$$\left.+2(\varepsilon-1+2\varepsilon\tau_0'+\tau_0'+\varepsilon^2\tau_0')s+(1+\varepsilon)^2]^{1/2}\}\right]^{1/2}. \tag{35}$$

The equation (34) is in agreement with the equation (3.18) of [4] and with equation (34) of [7]. Also the equation (34) agrees with equation (33) of [5] for $\alpha'=\alpha^{*\prime}=\tau_0'$.

Again on setting $\tau_0'=0$, the equation (35) agrees with equation (41) of [9]. Thus equation (34) is more general in the sense that it incorporates the effect of thermal relaxation time τ_0' of Lord-Shulman theory. From (27), (25) and (33), we obtain

$$\overline{U}(\zeta,s)=\frac{1}{\varepsilon s(1+\tau_0's)}\left[A_1\frac{(s+\tau_0's^2-\lambda_1^2)}{\lambda_1}\exp(-\lambda_1\zeta)+A_2\frac{(s+\tau_0's^2-\lambda_2^2)}{\lambda_2}\exp(-\lambda_2\zeta)\right] \tag{36}$$

$$\overline{h}(\zeta,s)=-\frac{\beta_3}{\varepsilon s(1+\tau_0's)}[A_1(\lambda_1^2-s-\tau_0's^2)\exp(-\lambda_1\zeta)+A_2(\lambda_2^2-s-\tau_0's^2)\exp(-\lambda_2\zeta)] \tag{37}$$

Also from (18)

$$\overline{\overset{\circ}{h}}(\zeta',s)=A_3\exp(-\alpha s\zeta')\,. \tag{38}$$

Where the constants A_1, A_2, A_3 are to be determined from the boundary conditions (28), (29) and (31). Hence,

$$\overline{Z}(\zeta,s)=\frac{1}{\beta''s(\lambda_2^2-\lambda_1^2)}[(K_1\lambda_2^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_1\zeta)-(K_1\lambda_1^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_2\zeta)] \tag{39}$$

$$\overline{U}(\zeta,s)=\frac{1}{\beta''\varepsilon s^2(1+\tau_0's)(\lambda_2^2-\lambda_1^2)}\left[\frac{(s+\tau_0's^2-\lambda_1^2)}{\lambda_1}(K_1\lambda_2^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_1\zeta)\right.$$
$$\left.-\frac{(s+\tau_0's^2-\lambda_2^2)(K_1\lambda_1^2-K_2s-K_2\tau_0's^2)}{\lambda_2}\exp(-\lambda_2\zeta)\right] \tag{40}$$

$$\overline{h}(\zeta,s)=\frac{\beta_3}{\beta''\varepsilon s^2(1+\tau_0's)(\lambda_1^2-\lambda_2^2)}[(s+\tau_0's^2-\lambda_1^2)(K_1\lambda_2^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_1\zeta)$$

$$-(s+\tau_0's^2-\lambda_2^2)(K_1\lambda_1^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_2\zeta)] \tag{41}$$

$$\overline{\mathring{h}}(\zeta',s)=\frac{\beta_3\exp(-\alpha s\zeta')}{\beta''\varepsilon s^2(1+\tau_0's)(\lambda_2^2-\lambda_1^2)}[(s+\tau_0's^2-\lambda_1^2)(K_1\lambda_2^2-K_2s-K_2\tau_0's^2)$$

$$-(s+\tau_0's^2-\lambda_2^2)(K_1\lambda_1^2-K_2s-K_2\tau_0's^2)] \tag{42}$$

where

$$\beta''=1-\frac{H_3^2}{4\pi\rho C_0^2}$$

$$K_1=\frac{\theta_0\beta''}{T_0}$$

$$K_2=\frac{\theta_0\beta''}{T_0}+\frac{\theta_0\varepsilon}{T_0}+\frac{\sigma_0\varepsilon}{\gamma T_0}.$$

The non-dimensional total stress in the half-space in the transformed domain is given by

$$\overline{\sigma}_{11}'=\frac{1}{\beta''\varepsilon s^2(1+\tau_0's)(\lambda_2^2-\lambda_1^2)}[(\lambda_1^2-s-\tau_0's^2)(K_1\lambda_2^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_1\zeta)$$

$$-(\lambda_2^2-s-\tau_0's^2)(K_1\lambda_1^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_2\zeta)]$$

$$-\frac{1}{\beta''s(\lambda_2^2-\lambda_1^2)}[(K_1\lambda_2^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_1\zeta)$$

$$-(K_1\lambda_1^2-K_2s-K_2\tau_0's^2)\exp(-\lambda_2\zeta)] \tag{43}$$

where

$$\sigma_{11}'=\frac{\overset{\bullet}{\sigma}_{11}}{\gamma T_0}.$$

5. SHORT TIME APPROXIMATION

The inversion of the Laplace transform is very difficult because of the dependency of λ_1, λ_2 on s. To reduce these difficulties, we use some approximate methods. The thermal relaxation effects are short-lived. Accordingly we concentrate our attention on small time approximations.

For large s,

$$\lambda_{1,2}=\frac{s}{V_{1,2}}+B_{1,2}+D_{1,2}\left(\frac{1}{s}\right)+O(s^{-2}) \tag{44}$$

where

$$V_{1,2}^{-1}=(P_2\pm\Gamma^{1/2})^{1/2}/\sqrt{2} \tag{45}$$

$$B_{1,2}=[P_1\pm(P_1P_2-2)/\Gamma^{1/2}]/2\sqrt{2}(P_2\pm\Gamma^{1/2})^{1/2} \tag{46}$$

$$D_{1,2}=\left[\pm P^2/\Gamma^{1/2}\mp(P_1P_2-2)^2/\Gamma^{3/2}-(P_1\pm(P_1P_2-2)/\Gamma^{1/2})^2/2(P_2\pm\Gamma^{1/2})\right]/4\sqrt{2}(P_2\pm\Gamma^{1/2})^{1/2} \tag{47}$$

$$\Gamma=P_2^2-4\tau_0',\quad P_1=1+\varepsilon,\quad P_2=1+\tau_0'+\varepsilon\tau_0' \tag{48}$$

$$\Gamma = (1 + \tau_0' + \varepsilon\tau_0')^2 - 4\tau_0' \tag{49}$$

The expressions for λ_1, λ_2 imply that the solution given by eqs. (39)-(43) consists of two types of waves in the half-space propagating with speeds V_1 and V_2 given by (45). From (49), $(1 + \tau_0' + \varepsilon\tau_0')^2 > \Gamma$, which implies $V_1 < V_2$. Thus the wave propagating with the speed V_1 is the slowest wave which is called modified elastic dilational wave and the wave propagating with the speed V_2 corresponds to the fastest wave which is called modified thermal wave. Also from equations (45)-(48) we notice that as $\tau_0' = 0, V_1 = 1, V_2 \to \infty$ which corresponds to the case of conventional coupled theory of thermoelasticity. As $V_1 < V_2$, the modified elastic wave follows the modified thermal wave. The third wave travelling with velocity $\frac{1}{\alpha}$ is the Alf'ven acoustic wave.

We now expand $\overline{Z}, \overline{U}, \overline{h}, \overline{\overset{\circ}{h}}$ and $\overline{\sigma}_{11}'$ in ascending powers of $\frac{1}{s}$ and retain terms up to $\frac{1}{s^2}$, neglecting higher order terms, to obtain

$$\begin{aligned}
\overline{Z}(\zeta, s) = & \frac{V_1^2 V_2^2}{\beta''(V_1^2 - V_2^2)}\left[\left(\frac{K_1}{V_2^2} - K_2\tau_0'\right)\frac{1}{s} + \left\{\left(\frac{2K_1B_2}{V_2} - K_2\right) - \frac{2V_1V_2(V_1B_2 - V_2B_1)}{(V_1^2 - V_2^2)}\right.\right.\\
& \left.\left.\times\left(\frac{K_1}{V_2^2} - K_2\tau_0'\right)\right\}\frac{1}{s^2}\right]\exp\left[-\left(\frac{s}{V_1} + B_1\right)\zeta\right] - \frac{V_1^2V_2^2}{\beta''(V_1^2 - V_2^2)}\left[\left(\frac{K_1}{V_1^2} - K_2\tau_0'\right)\frac{1}{s}\right.\\
& \left. + \left\{\left(\frac{2K_1B_1}{V_1} - K_2\right) - \frac{2V_1V_2(V_1B_2 - V_2B_1)}{(V_1^2 - V_2^2)}\left(\frac{K_1}{V_1^2} - K_2\tau_0'\right)\right\}\frac{1}{s^2}\right]\exp\left[-\left(\frac{s}{V_2} + B_2\right)\zeta\right]
\end{aligned}$$

$$\begin{aligned}
\overline{U}(\zeta, s) = & \frac{V_1^3V_2^2}{\beta''\varepsilon\tau_0'(V_1^2 - V_2^2)}\left[\left\{\left(\frac{K_1}{V_2^2} + \frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right\}\frac{1}{s^2} + \left\{2\left(\frac{K_1B_2}{V_2} + \frac{K_2B_1}{V_1}\right)\tau_0'\right.\right.\\
& - K_2\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2} + \frac{B_2}{V_1}\right) + \frac{K_1}{V_1^2V_2^2\tau_0'} - \left(B_1V_1 + \frac{2V_1V_2(B_2V_1 - B_1V_2)}{(V_1^2 - V_2^2)}\right)\left(\left(\frac{K_1}{V_2^2} + \frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2}\right.\\
& \left.\left.\left.- K_2\tau_0'^2\right)\right\}\cdot\frac{1}{s^3}\right]\exp\left[-\left(\frac{s}{V_1} + B_1\right)\zeta\right] - \frac{V_1^2V_2^3}{\beta''\varepsilon\tau_0'(V_1^2 - V_2^2)}\left[\left\{\left(\frac{K_1}{V_1^2} + \frac{K_2}{V_2^2}\right)\tau_0'\right.\right.\\
& \left.- \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right\}\frac{1}{s^2} + \left\{2\left(\frac{K_1B_1}{V_1} + \frac{K_2B_2}{V_2}\right)\tau_0' - K_2\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2} + \frac{B_2}{V_1}\right) + \frac{K_1}{V_1^2V_2^2\tau_0'}\right.\\
& \left.\left.- \left(B_2V_2 + \frac{2V_1V_2(B_2V_1 - B_1V_2)}{(V_1^2 - V_2^2)}\right)\left(\left(\frac{K_1}{V_1^2} + \frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right)\right\}\frac{1}{s^3}\right]\exp\left[-\left(\frac{s}{V_2} + B_2\right)\zeta\right]
\end{aligned}$$

$$\bar{h}(\zeta,s)=\frac{\beta_3V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Big[\Big\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'-\frac{K_1}{V_1^2V_2^2}-K_2\tau_0'^2\Big\}\frac{1}{s}+\Big\{2\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)\tau_0'$$

$$-K_2\tau_0'-\frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right)+\frac{K_1}{V_1^2V_2^2\tau_0'}-\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'+\frac{2K_1}{V_1V_2}\left(\frac{V_1B_2-V_2B_1}{V_1^2-V_2^2}\right)$$

$$+\frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\Big\}\frac{1}{s^2}\Big]\exp\left[-\left(\frac{s}{V_1}+B_1\right)\zeta\right]-\frac{\beta_3V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Big[\Big\{\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0'-\frac{K_1}{V_1^2V_2^2}$$

$$-K_2\tau_0'^2\Big\}\frac{1}{s}+\Big\{2\left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right)\tau_0'-K_2\tau_0'-\frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right)+\frac{K_1}{V_1^2V_2^2\tau_0'}-\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}$$

$$\times\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0'+\frac{2K_1(V_1B_2-V_2B_1)}{V_1V_2(V_1^2-V_2^2)}+\frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\Big\}\frac{1}{s^2}\Big]\exp\left[-\left(\frac{s}{V_2}+B_2\right)\zeta\right]$$

$$\bar{\mathring{h}}(\zeta,s)=\frac{\beta_3V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Big[\Big\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)-\left(\frac{K_1}{V_1^2}\right)-\left(\frac{K_2}{V_2^2}\right)\Big\}\tau_0'\frac{1}{s}+\Big\{2\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)\tau_0'-2\left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right)\tau_0'$$

$$-\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'+\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0'\Big\}\frac{1}{s^2}\Big]\exp[-\alpha s\zeta']$$

$$\bar{\sigma}_{11}'(\zeta,s)=-\frac{V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Big[\Big\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'-\frac{K_1}{V_1^2V_2^2}-K_2\tau_0'^2+\varepsilon\tau_0'\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)\Big\}\frac{1}{s}$$

$$+\Big\{2\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)\tau_0'-\frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right)-K_2\tau_0'+\frac{K_1}{V_1^2V_2^2\tau_0'}-\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'$$

$$+\frac{2K_1}{V_1V_2}\left(\frac{V_1B_2-V_2B_1}{V_1^2-V_2^2}\right)+\frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}+\varepsilon\tau_0'\left(\frac{2K_1B_2}{V_2}-K_2\right)-\frac{2\varepsilon\tau_0'V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}$$

$$\times\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)\Big\}\frac{1}{s^2}\Big]\exp\left[-\left(\frac{s}{V_1}+B_1\right)\zeta\right]+\frac{V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Big[\Big\{\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0'-\frac{K_1}{V_1^2V_2^2}-K_2\tau_0'^2$$

$$+\varepsilon\tau_0'\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right)\Big\}\frac{1}{s}+\Big\{2\left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right)\tau_0'-K_2\tau_0'-\frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right)+\frac{K_1}{V_1^2V_2^2\tau_0'}$$

$$-\frac{2V_1V_2\tau_0'(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)+\frac{2K_1(V_1B_2-V_2B_1)}{V_1V_2(V_1^2-V_2^2)}+\frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}$$

$$+\varepsilon\tau_0'\left(\frac{2K_1B_1}{V_1}-K_2\right)-\frac{2\varepsilon\tau_0'V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right)\Big\}\frac{1}{s^2}\Big]\exp\left[-\left(\frac{s}{V_2}+B_2\right)\zeta\right]$$

The inversion of the above expression yields the following short-time solutions

$$Z(\zeta,\tau)\approx\frac{V_1^2V_2^2\exp(-B_1\zeta)}{\beta''(V_1^2-V_2^2)}\Big[\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)H\left(\tau-\frac{\zeta}{V_1}\right)+\Big\{\left(\frac{2K_1B_2}{V_2}-K_2\right)-\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}$$

$$\times\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)\Big\}\left(\tau-\frac{\zeta}{V_1}\right)H\left(\tau-\frac{\zeta}{V_1}\right)\Big]-\frac{V_1^2V_2^2\exp(-B_2\zeta)}{\beta''(V_1^2-V_2^2)}\Big[\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right)H\left(\tau-\frac{\zeta}{V_2}\right)$$

$$+\Big\{\left(\frac{2K_1B_1}{V_1}-K_2\right)-\frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right)\Big\}\left(\tau-\frac{\zeta}{V_2}\right)H\left(\tau-\frac{\zeta}{V_2}\right)\Big]$$

$$U(\zeta,\tau) \cong \frac{V_1^3V_2^2\exp(-B_1\zeta)}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Bigg[\Bigg[\left\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0' - K_2\tau_0'^2 - \frac{K_1}{V_1^2V_2^2}\right\}\left(\tau-\frac{\zeta}{V_1}\right)H\left(\tau-\frac{\zeta}{V_1}\right) + \Bigg\{2\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)\tau_0'$$

$$- K_2\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right) + \frac{K_1}{V_1^2V_2^2\tau_0'} - \left(B_1V_1 + \frac{2V_1V_2(B_2V_1-B_1V_2)}{(V_1^2-V_2^2)}\right)\left(\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2}\right.$$

$$\left.\left. - K_2\tau_0'^2\right)\Bigg\}\frac{\left(\tau-\frac{\zeta}{V_1}\right)^2}{2}H\left(\tau-\frac{\zeta}{V_1}\right)\Bigg] - \frac{V_1^2V_2^3\exp(-B_2\zeta)}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Bigg[\Bigg[\left\{\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2}\right.$$

$$\left. - K_2\tau_0'^2\right\}\left(\tau-\frac{\zeta}{V_2}\right)H\left(\tau-\frac{\zeta}{V_2}\right) + \Bigg\{2\left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right)\tau_0' - K_2\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right) + \frac{K_1}{V_1^2V_2^2\tau_0'}$$

$$- \left(B_2V_2 + \frac{2V_1V_2(B_2V_1-B_1V_2)}{(V_1^2-V_2^2)}\right)\left(\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right)\Bigg\}\frac{\left(\tau-\frac{\zeta}{V_2}\right)^2}{2}H\left(\tau-\frac{\zeta}{V_2}\right)\Bigg]$$

$$h(\zeta,\tau) \cong \frac{\beta_3V_1^2V_2^2\exp(-B_1\zeta)}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Bigg[\Bigg[\left\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right\}H\left(\tau-\frac{\zeta}{V_1}\right) + \Bigg\{2\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)\tau_0'$$

$$- K_2\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right) + \frac{K_1}{V_1^2V_2^2\tau_0'} - \frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'$$

$$+ \frac{2K_1}{V_1V_2}\left(\frac{V_1B_2-V_2B_1}{V_1^2-V_2^2}\right) + \frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\Bigg\}\left(\tau-\frac{\zeta}{V_1}\right)H\left(\tau-\frac{\zeta}{V_1}\right)\Bigg]$$

$$- \frac{\beta_3V_1^2V_2^2\exp(-B_2\zeta)}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Bigg[\Bigg[\left\{\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right\}H\left(\tau-\frac{\zeta}{V_2}\right) + \Bigg\{2\left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right)\tau_0'$$

$$- \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right) - K_2\tau_0' + \frac{K_1}{V_1^2V_2^2\tau_0'} - \frac{2V_1V_2\tau_0'(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)$$

$$+ \frac{2K_1}{V_1V_2}\left(\frac{B_1V_2-B_2V_1}{V_1^2-V_2^2}\right) + \frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\Bigg\}\left(\tau-\frac{\zeta}{V_2}\right)H\left(\tau-\frac{\zeta}{V_2}\right)\Bigg]$$

$$\dot{h}(\zeta',\tau) = \frac{\beta_3V_1^2V_2^2}{\beta''\varepsilon(V_1^2-V_2^2)}\Bigg[\Bigg[\left\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right) - \left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\right\}H(\tau-\alpha\zeta') + 2\Bigg\{\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)$$

$$- \left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right) + \frac{V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}-\frac{K_1}{V_2^2}-\frac{K_2}{V_1^2}\right)\Bigg\}(\tau-\alpha\zeta')H(\tau-\alpha\zeta')\Bigg]$$

$$\sigma_{11}'(\zeta,\tau) \simeq -\frac{V_1^2V_2^2\exp(-B_1\zeta)}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Bigg[\Bigg\{\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2 + \varepsilon\tau_0'\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)\Bigg\}H\left(\tau-\frac{\zeta}{V_1}\right)$$

$$+\Bigg\{2\left(\frac{K_1B_2}{V_2}+\frac{K_2B_1}{V_1}\right)\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right) - K_2\tau_0' + \frac{K_1}{V_1^2V_2^2\tau_0'} - \frac{2V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0'$$

$$+\frac{2K_1}{V_1V_2}\left(\frac{V_1B_2-V_2B_1}{V_1^2-V_2^2}\right) + \frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)} + \varepsilon\tau_0'\left(\frac{2K_1B_2}{V_2}-K_2\right) - \frac{2\varepsilon\tau_0'V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}$$

$$\times\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)\Bigg\}\left(\tau-\frac{\zeta}{V_1}\right)H\left(\tau-\frac{\zeta}{V_1}\right)\Bigg] + \frac{V_1^2V_2^2\exp(-B_2\zeta)}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\Bigg[\Bigg\{\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2}$$

$$-K_2\tau_0'^2 + \varepsilon\tau_0'\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right)\Bigg\}H\left(\tau-\frac{\zeta}{V_2}\right) + \Bigg\{2\left(\frac{K_1B_1}{V_1}+\frac{K_2B_2}{V_2}\right)\tau_0' - \frac{2K_1}{V_1V_2}\left(\frac{B_1}{V_2}+\frac{B_2}{V_1}\right)$$

$$-K_2\tau_0' + \frac{K_1}{V_1^2V_2^2\tau_0'} + \frac{2K_2V_1V_2\tau_0'^2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)} + \varepsilon\tau_0'\left(\frac{2K_1B_1}{V_1}-K_2\right) - \frac{2\varepsilon\tau_0'V_1V_2(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}$$

$$\times\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right) + \frac{2K_1(V_1B_2-V_2B_1)}{V_1V_2(V_1^2-V_2^2)} - \frac{2V_1V_2\tau_0'(V_1B_2-V_2B_1)}{(V_1^2-V_2^2)}\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\Bigg\}\left(\tau-\frac{\zeta}{V_2}\right)H\left(\tau-\frac{\zeta}{V_2}\right)\Bigg]$$

6. DISCUSSION

From the short-time solutions it is observed that the solution consists of three waves—the modified elastic wave travelling with velocity V_1, the modified thermal wave travelling with velocity V_2 and Alf'ven acoustic wave moving with velocity $\frac{1}{\alpha}$.

The terms containing $H\left(\tau-\frac{\zeta}{V_1}\right)$ represent the contribution of the elastic wave in the vicinity of the wave front $\zeta = V_1\tau$, the terms with $H(\tau-\frac{\zeta}{V_2})$ represent the contribution of the thermal wave in the vicinity of the wavefront $\zeta = V_2\tau$ and the terms with $H(\tau-\alpha\zeta')$ represent the contribution of the Alf'ven acoustic wave in the vicinity of the wavefront $\zeta' = \frac{\tau}{\alpha}$.

We observe that, in the solid the displacement is continuous at the modified elastic and thermal wavefronts, but the temperature, total stress and perturbed magnetic field suffer discontinuities at the two wave-fronts, whereas in the vacuum the perturbed magnetic field suffers discontinuity at the Alf'ven acoustic wavefront.

The discontinuities are given by

$$[z^+-Z^-]_{\zeta=V_1\tau} = \frac{V_1^2V_2^2}{\beta''(V_1^2-V_2^2)}\left[\left(\frac{K_1}{V_2^2}-K_2\tau_0'\right)\exp(-B_1V_1\tau)\right]$$

$$[z^+-Z^-]_{\zeta=V_2\tau} = \frac{-V_1^2V_2^2}{\beta''(V_1^2-V_2^2)}\left[\left(\frac{K_1}{V_1^2}-K_2\tau_0'\right)\exp(-B_2V_2\tau)\right]$$

$$[h^+-h^-]_{\zeta=V_1\tau} = \frac{\beta_3V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\left[\left(\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right]\exp(-B_1V_1\tau)$$

$$[h^+-h^-]_{\zeta=V_2\tau} = \frac{-\beta_3V_1^2V_2^2}{\beta''\varepsilon\tau_0'(V_1^2-V_2^2)}\left[\left(\frac{K_1}{V_1^2}+\frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2V_2^2} - K_2\tau_0'^2\right]\exp(-B_2V_2\tau)$$

$$\left[\mathring{h}^+-\mathring{h}^-\right]_{\zeta'=\frac{\tau}{\alpha}} = \frac{\beta_3V_1^2V_2^2}{\beta''\varepsilon(V_1^2-V_2^2)}\left[\frac{K_1}{V_2^2}+\frac{K_2}{V_1^2}-\frac{K_1}{V_1^2}-\frac{K_2}{V_2^2}\right]$$

$$[\sigma_{11}'^{+} - \sigma_{11}'^{-}]_{\zeta = V_1\tau} = -\frac{V_1^2 V_2^2}{\beta''\varepsilon\tau_0'(V_1^2 - V_2^2)}\left[\left(\frac{K_1}{V_2^2} + \frac{K_2}{V_1^2}\right)\tau_0' - \frac{K_1}{V_1^2 V_2^2} - K_2\tau_0'^2 + \varepsilon\tau_0'\left(\frac{K_1}{V_2^2} - K_2\tau_0'\right)\right]\exp(-B_1 V_1 \tau)$$

$$[\sigma_{11}'^{+} - \sigma_{11}'^{-}]_{\zeta = V_2\tau} = \frac{V_1^2 V_2^2}{\beta''\varepsilon\tau_0'(V_1^2 - V_2^2)}\left[\left(\frac{K_1}{V_1^2} + \frac{K_2}{V_2^2}\right)\tau_0' - \frac{K_1}{V_1^2 V_2^2} - K_2\tau_0'^2 + \varepsilon\tau_0'\left(\frac{K_1}{V_1^2} - K_2\tau_0'\right)\right]\exp(-B_2 V_2 \tau)$$

which clearly depend on the magneto-thermoelastic and thermal relaxation parameters of the medium.

It is to be mentioned that the case $\tau_0' = 0$ corresponds to conventional coupled theory of thermoelasticity where,

$$K_1 = 1 + \varepsilon\,,\quad K_2 = 1\,,\quad V_1 = 1\,,$$

$$V_2 \to \infty\,,\quad \Gamma = 1\,,\quad B_1 = \frac{\varepsilon}{2}\,,$$

$$B_2 \to \infty\,,\quad D_1 = \frac{\varepsilon(4-\varepsilon)}{8}\,,\quad D_2 \to \infty\,.$$

7. NUMERICAL RESULTS

We define

$$[Z_{1,2}] = [\beta'' Z^{+} - \beta'' Z^{-}]_{\zeta = V_{1,2}\tau}$$

$$[h_{1,2}] = \left[\frac{\beta''}{\beta_3}h^{+} - \frac{\beta''}{\beta_3}h^{-}\right]_{\zeta = V_{1,2}\tau}$$

$$\left[\dot{h}\right] = \left[\frac{\beta''}{\beta_3}\dot{h}^{+} - \frac{\beta''}{\beta_3}\dot{h}^{-}\right]_{\zeta' = \tau/\alpha}$$

$$[\sigma_{1,2}] = \left[\frac{\beta''}{\beta_3}\sigma_{11}'^{+} - \frac{\beta''}{\beta_3}\sigma_{11}'^{-}\right]_{\zeta = V_{1,2}\tau}.$$

For numerical work we use the following values for the physical constants (which correspond to copper)

$$\rho = 8.930\, g/cm^3\,,\quad \kappa = 1.14\, cm^2/s$$

$$\lambda = 1.387 \times 10^{12}\, dyn/cm^2\,,$$

$$\mu = 0.448 \times 10^{12}\, dyn/cm^2\,,$$

$$\alpha_t = 16.5 \times 10^{-6} ({}^{\circ}c)^{-1}\,,$$

$$\mu_0 = 1\, g/{}^{\circ}c\,,\quad \varepsilon = 0.0168$$

We take $\theta_0 = 1$, $\sigma_0 = 1$, $H_3 = 1000$, $\tau_0 = 10^{-11}$, so $\beta'' \cong 1$. Finite jumpts of temperature, stress and perturbed magnetic field for different values of time τ are presented in the following table.

Table

Jumps	$\tau = 0.25$	$\tau = 0.50$	$\tau = 0.75$	$\tau = 1.25$
$[Z_1]$	0.926549316	0.877368634	0.830798435	0.744942582
$[Z_2]$	0.021487410	0.021461659	0.021435939	0.021384592
$[h_1]$	-1.634355069	-1.547604483	-1.465458568	-1.314016064
$[h_2]$	0.725098384	0.724229472	0.72336155	0.721628824
$[\mathring{h}]$	-0.999999947	-0.999999947	-0.999999947	-0.999999947
$[\sigma_1]$	0.707805751	0.670235847	0.634660131	0.56907348
$[\sigma_2]$	-0.746585838	-0.745691123	-0.744797481	-0.743013408

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