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A NOTE ON QUASI-MONOTONE OPERATORS

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ABSTRACT. The treatment of nonlinear problems using a general framework is often a delicate issue. This is illustrated by the fact that the quasi-monotone operators of M. A. Noor are constant operators.

KEY WORDS AND PHRASES. Quasi-monotone operators, mildly nonlinear, variational inequalities.

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1. INTRODUCTION.

In [1], Noor seeked to establish error estimates for the finite element approximations of variational inequalities associated with certain mildly nonlinear elliptic boundary value problems. The main result of his paper relies on the use of "quasi-monotone operator" and a restricted choice of test functions. One important point that seems to escape the attention of many people is that quasi-monotone operators are in fact constant operators. This of course implies that Noor's result is only valid for linear problems, rather than nonlinear problems, and is actually more restrictive than the result of Mosco and Strang [2] quoted in his paper. It also illustrates the difficulty and delicacy in the treatment of nonlinear problems under a general framework.

To see that a quasi-monotone operator is a constant operator, we first quote its definition from [1],

DEFINITION. **An operator T on H_0^1 is said to be quasi-monotone, if for all z, u, v, w, in H_0^1,**

$$\langle Tu - Tv, w-z\rangle \geq 0. \tag{1.1}$$

One may easily generalize this definition to a Banach space $(V, ||\cdot||)$ with dual $(V^*, ||\cdot||^*)$ and pairing $\langle,\rangle$ on $V^* \times V$. The operator T then maps V to V^*. Here is an easy proof of the nature of quasi-monotonicity.

Let z = 0. Setting w = ± w, we have

$$\langle Tu - Tv, w\rangle = 0 \quad \text{for all} \quad u,v,w \text{ in } V$$

and so

$$||Tu-Tv||^* = \sup_{w \in V} \frac{|\langle Tu - Tv, w\rangle|}{||w||} = 0$$

which shows that Tu = Tv for all u, v in V and T is therefore a constant operator.

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