SOME RESULTS ON COINCIDENCE AND FIXED POINT THEOREMS FOR GENERALIZED CONTRACTION TYPE MAPPINGS

D. K. GANGULY and D. BANDYOPADHYAY

(Received 24 April 1996 and in revised form 1 October 1996)

ABSTRACT. Some coincidence and fixed point theorems are proved for certain generalized contraction type single-valued and set-valued compatible mappings.

2000 Mathematics Subject Classification. 47H10, 54H25.

1. Introduction. Jungck [1] generalized the Banach contraction principle using the commuting map concept, which is extended by Sessa [4] giving weakly commuting map concept; this again modified in [2] by compatibility condition. Several authors [3, 5, 6] discussed various results on coincidence and fixed point theorem for compatible single-valued and multi-valued maps. Here we develop some coincidence and fixed point theorems for compatible single-valued and multi-valued maps satisfying some generalized contraction type condition. Henceforth, we denote by \( \mathbb{N} \) and \( \mathbb{R}_+ \), the set of naturals and nonnegative reals, respectively, and \( \omega = \mathbb{N} \cup \{0\} \) and \((X,d)\), a metric space, unless otherwise stated.

2. Preliminaries

DEFINITION 2.1 (see [3]). Two mappings \( f, g : X \to X \) are compatible if and only if \( d(fg{x_n}, gf{x_n}) \to 0 \) whenever \( \{x_n\} \) is a sequence in \( X \) such that \( fx_n \to t, gx_n \to t, t \in X \).

Let \( C(X) = \) class of closed subsets of \( X \), \( CB(X) = \) class of closed bounded subsets of \( X \), \( co(K) = \) convex hull of \( K \subset X \). The Hausdorff metric \( H \) on \( CB(X) \) is defined as \( H(A,B) = \max\{\sup_{x \in A} D(x,B), \sup_{x \in B} D(x,A)\} \), for all \( A,B \in CB(X) \), where \( D(x,A) = \inf_{y \in A} d(x,y) \).

DEFINITION 2.2 (see [3]). The maps \( f : X \to X \) and \( T : X \to CB(X) \) are compatible if and only if \( fTx \in CB(X) \) for all \( x \in X \) and \( H(fTx_n, Tf{x_n}) \to 0 \) whenever \( \{x_n\} \) is a sequence in \( X \) such that \( Tx_n \to M \in CB(X) \), \( fx_n \to t \in M \), where \( H \) is the Hausdorff metric on \( X \).

We now recall the following lemmas.

LEMMA 2.3 (see [7]). Let \( h : \mathbb{R}_+ \to \mathbb{R}_+ \) be a nondecreasing upper semi-continuous (u.s.c.) function. Then \( h(t) < t \) if and only if \( h^n(t) \to 0 \) for each \( t > 0 \) where \( h^n \) denotes the composition of \( h \) with itself \( n \) times.

LEMMA 2.4 (see [3]). Let \( T : X \to CB(X) \) and \( f : X \to X \) be compatible. If \( fz \in Tz \) for some \( z \in X \), then \( fTx = Tfz \).
3. Coincidence and fixed point theorems for single-valued maps

**Theorem 3.1.** Let $X$ be any nonempty set and $(Y,d)$ be a complete metric space. Let $f, g, T : X \to Y$ satisfy

(i) $f(X), g(X) \subseteq T(X)$;
(ii) $T(X)$ is closed in $Y$;
(iii) for all $x, y \in X$,
\[
d(fx, gy) \leq d(Tx, Ty), d(Tx, fx), d(Ty, gy), d(Ty, fy), d(Ty, gy)\]
where $h(t) = \max[\max[t, t, at, bt, t]] < t$, for each $t > 0$, $a, b \in \{0, 1, 2\}$ with $a + b = 2$ and \(\varphi : \mathbb{R}_+ \to \mathbb{R}_+\) is nondecreasing u.s.c function. Then $f, g, T$ have a coincidence point in $X$.

Further if
(iv) $f$ or $g$ is injective, then the coincidence point is unique in $X$.

**Proof.** Choose any $x_0 \in X$. From (i), we define an iteration $y_{2n} = fx_{2n} = Tx_{2n+1}$, $y_{2n+1} = gx_{2n+1} = Tx_{2n+2}$. Let $d_n = d(Tx_n, Tx_{n+1})$. Then from (iii), we have
\[
d_{2n+1} = d(Tx_{2n+1}, Tx_{2n+2}) = d(y_{2n}, y_{2n+1}) = d(fx_{2n}, gx_{2n+1})
\leq \varphi \left[ \max \left\{ d(Tx_{2n}, Tx_{2n+1}), d(Tx_{2n}, fx_{2n}), d(Tx_{2n+1}, gx_{2n+1}) \right\} \right]
\leq \varphi \left[ \max \left\{ d_{2n}, d_{2n}, (d_{2n+2} + d_{2n+1}), 0, d_{2n} \right\} \right].
\]

If $d_{2n+1} > d_{2n}$, then contradiction arises; so taking $d_{2n+1} = d_{2n}$, we have $d_{2n+1} \leq h(d_{2n})$. Similarly, $d_{2n+2} \leq d_{2n+1}, d_{2n+2} \leq h(d_{2n+1})$. Hence $d_{n+1} \leq d_n$ and $d_n \leq h(d_{n-1}) \leq \cdots \leq h^n(d_0)$, for all $n \in \omega$.

This yields, by Lemma 2.3, $\lim_n d_n = 0 = \lim_n d(y_n, y_{n+1})$. Now, the sequence $\{y_n\}$ is a Cauchy sequence in $f(X)$, which can be proved using the same technique as used in [6, Theorem 2.1] so from (ii), $\exists u \in X \ni \lim_n y_n = Tu$, that is, $\lim_n Tx_n = Tu$ and $\lim_n fx_{2n} = Tu = \lim_n gx_{2n+1}$. Suppose that $fu = Tu = gu$. Then
\[
d(fu, Tu) \leq d(fu, gx_{2n+1}) + d(gx_{2n+1}, Tu)
\leq \varphi \left[ \max \left\{ d(Tu, Tx_{2n+1}), d(Tu, fu), d(Tu, gx_{2n+1}), d(Tx_{2n+1}, fu), d(Tx_{2n+1}, gx_{2n+1}) \right\} \right] + d(gx_{2n+1}, Tu)
\leq \varphi \left[ \max \left\{ 0, d(Tu, fu), 0, d(Tu, fu), 0 \right\} \right].
\]

as $n \to \infty$; hence $d(fu, Tu) < d(fu, Tu)$ which is absurd. Hence $fu = Tu$. Similarly, $gu = Tu$. Thus, $fu = Tu = gu$ and uniqueness of $u$ follows from (iii) and (iv). \(\square\)
**Lemma 3.2.** Let \( f, g : X \rightarrow X \) be compatible. If \( fz = gz \) for some \( z \in X \), then \( fgz = gfz \).

**Proof.** The proof is similar to that of Kaneko and Sessa [3].

**Theorem 3.3.** Let \((X, d, \delta)\) be a bimetric space such that \( X \) is complete with respect to \( \delta \). Let \( f, g, T : X \rightarrow X \) satisfy conditions (i)-(iii) of Theorem 3.1 with respect to \( d \), and (v) \((f, T)\) and \((g, T)\) are compatible pairs; (vi) \( \delta(x, y) \leq k(d(x, y)) \) for all \( x, y \in X \), where \( k : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) is continuous with \( k(0) = 0 \). Then \( f, g, T \) have a unique common fixed point in \( X \).

**Proof.** By Theorem 3.1, \( \{TX_n\} \) is Cauchy with respect to \( d \) and hence from (vi) it is Cauchy with respect to \( \delta \). Since \( X \) is complete with respect to \( \delta \), from Theorem 3.1(ii), there exists \( z \in X \exists fz = Tz = gz \). Thus, by Lemma 3.2 and (v), \( Tfz = fTz \) and \( gTz = Tgz \). So \( T^2z = fTz = f^2z = ggz = ggz = Tgz = Tgz \). Now, from Theorem 3.1(iii) it is easy to show that \( fz = gfz \). Thus, \( fz = gfz = T^2z = f^2z \) is a common fixed point of \( f, T \) and \( g \) in \( X \). The uniqueness part follows from Theorem 3.1(iii).

**Corollary 3.4.** Let \((X, d)\) be a complete metric space \( f, g, T : X \rightarrow X \) satisfying (i)-(iv) of Theorem 3.1 and (v) of Theorem 3.3. Then \( f, g, \) and \( T \) have a unique common fixed point in \( X \).

**Corollary 3.5.** Let \((X, d)\) be a complete metric space and let \( S \) be a family of self maps of \( X \). If there is a map \( T \) in \( S \) such that for each pair \( f, g \) in \( S \) satisfying (i)-(iv) of Theorem 3.1 and (v) of Theorem 3.3, then each member of \( S \) has a unique common fixed point in \( X \) which is a unique common fixed point of the family \( S \).

**Theorem 3.6.** Let \((X, d)\) be a complete metric space. Then \( f, g, T : X \rightarrow X \) satisfying Theorem 3.1(iii) have a unique common fixed point if and only if there is \( u \in X \) such that \( fu = gu = Tu \) and \( f^2u = g^2u = T^2u \).

**Proof.** The necessary part is trivial. To prove the sufficient part, let there be a \( u \in X \exists (a) \ fu = gu = Tu, (b) \ f^2u = g^2u = T^2u \). Let \( y = fu = gu = Tu \). Then from Theorem 3.1(iii) and (b), we can show that \( y = fy = Ty = gy \), that is, \( y \) is a common fixed point of \( f, g, T \) in \( X \). Further, from (iii) of Theorem 3.1, the uniqueness of \( y \) follows at once.

**Theorem 3.7.** Let \( X \) be a set and \( Y \) a Banach space. Let \( f, g : X \rightarrow Y \) be such that

(i) \( \text{co}(f(X)) \subset g(X) \);
(ii) \( g(X) \) is closed in \( Y \);
(iii) \( \|fx - fy\| \leq \varphi[\max\{\|gx - gy\|, \|gx - fx\|, \|gy - fy\|\}] \) for all \( x, y \in X \) where \( \varphi : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) is nondecreasing u.s.c. function with \( \varphi(qt) < t, 1 \leq q \leq 2 \). Then there is a \( u \in X \) such that \( fu = gu \). Further, if \( f \) or \( g \) is injective, then \( u \) is unique.

**Proof.** Choose \( x \in X \). From (i) of Theorem 3.7, we define \( \{x_n\} \) in \( X \) as \( fx_n = gx_{n+1} \), for all \( n \in \omega \). Writing \( d_n = \|fx_n - fx_{n+1}\| \) and using (iii) of Theorem 3.7, we get

\[
d_n < d_{n-1}, \quad d_n \leq \varphi(d_{n-1}) \leq \cdots \leq \varphi^n(d_0), \quad \forall n \in \omega.
\] (3.4)
Now, for each \( p \in \mathbb{N} \),

\[
\|f x_n - f x_{n+p}\| \leq \sum_{i=1}^{p-1} \|f x_{n+i} - f x_{n+1+i}\| \leq \sum_{i=1}^{p-1} \varphi^{n+i}(d_0) = \varphi^n(d_0) \cdot \left( \frac{\varphi^p(d_0) - 1}{\varphi(d_0) - 1} \right) \rightarrow 0
\]  

(3.5)

as \( n \rightarrow \infty \) by Lemma 2.3 implies \( \{f x_n\} \) is Cauchy in \( Y \) and by assumption, \( \lim_n f x_n \) exists finitely in \( Y \). From (i), define \( g y_n = a f x_n + (1 - a) f x_{n+1}, 0 \leq a \leq 1 \) in \( g(X) \). We have

\[
\|g y_n - g y_n\| \leq a \|f x_n - f y_n\| + (1 - a) \|f x_{n+1} - f y_n\|
\]

\[
\leq a \varphi \left[ \max \left\{ \|g x_n - g y_n\|, \|g x_n - f x_n\|, \|g y_n - f y_n\| \right\} \right]
\]

\[
+ (1 - a) \varphi \left[ \max \left\{ \|g x_{n+1} - g y_n\|, \|g x_{n+1} - f x_{n+1}\|, \|g y_n - f y_n\| \right\} \right].
\]

(3.6)

Also,

\[
\|g x_n - g y_n\| \leq \|f x_{n-1} f x_n\| + (1 - a) \|f x_{n-1} - f x_{n+1}\|
\]

\[
\leq \varphi^{n-1}(d_0) + (1 - a) \varphi^n(d_0) \leq (2 - a) \varphi^{n-1}(d_0) \quad \text{(using \( \varphi(d_0) < d_0 \)),}
\]

\[
\|g x_{n+1} - g y_n\| = (1 - a) \|f x_n - f x_{n+1}\| \leq (1 - a) \varphi^n(d_0).
\]

(3.7)

Thus, from (3.4) and (3.7), (3.6) reduces to

\[
\|g y_n - g y_n\| \leq a \varphi \left[ \max \left\{ (2 - a) \varphi^{n-1}(d_0), \varphi^{n-1}(d_0), \|f y_n - g y_n\| \right\} \right]
\]

\[
+ (1 - a) \varphi \left[ \max \left\{ (1 - a) \varphi^n(d_0), \varphi^n(d_0), \|f y_n - g y_n\| \right\} \right]
\]

\[
\leq a \varphi \left[ \max \left\{ (2 - a) \varphi^{n-1}(d_0), \|f y_n - g y_n\| \right\} \right]
\]

\[
+ (1 - a) \varphi \left[ \max \left\{ (2 - a) \varphi^{n-1}(d_0), \|f y_n - g y_n\| \right\} \right],
\]

(3.8)

\[
\leq \varphi \left[ \max \left\{ (2 - a) \varphi^{n-1}(d_0), \|f y_n - g y_n\| \right\} \right]
\]

\[
\leq \varphi \left[ (2 - a) \varphi^{n-1}(d_0) \right] \leq \varphi^{n-1}(d_0),
\]

otherwise, if \( \|f y_n - g y_n\| \) is maximum then a contradiction arises.

Now, for any \( p \in \mathbb{N} \), writing \( K_p = (\varphi^p(d_0) - 1) / (\varphi(d_0) - 1) \) we get

\[
\|g y_n - g y_{n+p}\| \leq a \|f x_n - f x_{n+p}\| + (1 - a) \|f x_{n+1} - f x_{n+1+p}\|
\]

\[
\leq a \varphi^n(d_0) + (1 - a) \varphi^{n-1}(d_0) \] \( K_p \rightarrow 0 \quad \text{as} \ n \rightarrow \infty \Rightarrow \{g y_n\}
\]

(3.9)
is Cauchy in \(g(X) \subset Y\), and from (ii) of Theorem 3.7 there exists \(u \in X \ni \lim_n g y_n = g u\). So, from (3.4), (3.7), and (3.8) we have, \(\|fx_n - fy_n\| \leq \|fx_n - g x_n\| + \|g x_n - g y_n\| + \|g y_n - fy_n\| \to 0\) as \(n \to \infty\). Hence, \(\lim_n fx_n = \lim_n fy_n = \lim_n g y_n = \lim_n g x_n = g u\).

Now, let \(fu \neq g u\). Then from (iii) of Theorem 3.7, we have \(\|fu - fx_n\| \leq \varphi[\max\{\|gu - g x_n\|, \|gu - fu\|, \|g x_n - fx_n\|\}]\); taking limit as \(n \to \infty\), we have \(\|fu - gu\| \leq \varphi[\max\{0, \|fu - gu\|, 0\}] < \|fu - gu\|\) which is a contradiction. Hence \(fu = gu\). The second part follows from (iii) of Theorem 3.7 and injectiveness of \(f\) or \(g\).

\[\square\]

4. Coincidence point for multivalued mappings

**Theorem 4.1.** Let \(X\) be a Banach space; and let \(S, T : X \to \text{CB}(X)\) and \(f : X \to X\) be such that

(i) \(S(X) \cup T(X) \subseteq f(X) \subset C(X)\),

(ii) for all \(x, y \in X\), \(H(Sx, Ty) \leq \varphi[\max\{|\langle \|Sx - Ty\|, D(fx, Sx), D(fy, Ty)\rangle, D(fx, Ty), D(fy, Sx)\}\} \subseteq \mathbb{R}_+\) is u.s.c. and nondecreasing in each coordinate variable with \(\gamma(t) = \max[\gamma(t), t, t, at, bt] : a + b = 2, a, b \in (0, 1, 2)] \subseteq qt, 0 \leq q < 1, t > 0\). Then \(f, S\) and \(T\) have a coincidence point in \(X\).

**Proof.** Choose \(a \in (0, 1)\) such that \(q^{1-a} < 1\). Let \(x_0 \in X\). Form (i), we define a sequence \(\{x_n\}\) in \(X\) as \(fx_{2n+1} = Sx_{2n}, fx_{2n+2} = Tx_{2n+1}\) such that

\[
\begin{align*}
\|fx_{2n+1} - fx_{2n+2}\| &< q^{-a} H(Sx_{2n}, Tx_{2n+1}), \\
\|fx_{2n+2} - fx_{2n+3}\| &< q^{-a} H(Tx_{2n+1}, Sx_{2n+2}),
\end{align*}
\]

(4.1)

for all \(n \in \omega\), writing \(d_n = \|fx_n - fx_{n+1}\|\), we have from (ii) by routine calculations that \(d_{2n+1} \leq d_{2n}\) and \(d_{2n+1} \leq q^{1-a}d_{2n}\). Similarly, \(d_{2n+2} \leq d_{2n+1}\) and \(d_{2n+2} \leq q^{1-a}d_{2n+1}\). Thus, combining these we can write

\[d_{n+1} \leq d_n, \quad d_n \leq q^{1-a}d_{n-1} \leq \cdots \leq q^{(1-a)n}d_0, \quad \forall n \in \omega, 0 \leq q^{1-a} < 1.\]

(4.2)

This shows that \(\{fx_n\}\) is Cauchy in \(f(X)\) and from (i) of Theorem 4.1, there exists \(z \in X \ni \lim fx_n = f z,\)

\[
D(fz, Sz) \leq \|fz - fx_{2n+2}\| + D(fx_{2n+2}, Sz) \leq \|fz - fx_{2n+2}\| + H(Sz, Tx_{2n+1})
\]

\[
\leq \varphi\left\{\|fz - fx_{2n+1}\|, D(fz, Sz), D(fx_{2n+1}, Tx_{2n+1}), D(fz, Tx_{2n+1})\right\},
\]

\[
D(fx_{2n+1}, Sz) + \|fz - fx_{2n+2}\|
\]

(4.3)

\[
\leq \varphi\left\{\|fz - fx_{2n+1}\|, D(fz, Sz), \|fx_{2n+1} - fx_{2n+2}\|, \|fz - fx_{2n+2}\|, \right\} + \|fz - fx_{2n+2}\|.
\]

As \(n \to \infty\), we have \(D(fz, Sz) \leq \varphi\{0, D(fz, Sz), 0, 0, D(fz, Sz)\} \leq \varphi\{t, t, t, t, t\} \leq qt\) (where \(t = D(fz, Sz)\)) which implies that \(fz \in Sz = Sz\). Similarly \(fz \in Tx\).

Hence \(z\) is a coincidence point of \(f, S\) and \(T\) in \(X\).
In [3, Theorem 2] the continuity of the involved maps are taken; but in Theorem 4.1 instead of the continuity condition of the maps we take only \( f(X) \in C(X) \) for the existence of a coincidence point; to support this we give the following example.

**Example 4.2.** Let \( X = [0, 1] \). Define \( S, T : X \to CB(X) \) and \( f : X \to X \) as follows:

\[
Sx = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{2}, \\
\{1\}, & \frac{1}{2} < x \leq 1,
\end{cases}
\]

\[
Tx = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{2}, \\
\{1\}, & \frac{1}{2} < x \leq 1,
\end{cases}
\]

\[
fX = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{2}, \\
\frac{1}{4}, & \frac{1}{2} < x < 1,
\end{cases}
\]

Then \( SX = \{0, 1/4\} = TX \), \( fX = \{0, 1/4, 2/3\} \in C(X) \); \( S, T \), and \( f \) are discontinuous.

Let \( \varphi : \mathbb{R}_+ \to \mathbb{R}_+ \) be given by \( \varphi(t_1, t_2, t_3, t_4, t_5) = \sqrt{t_1} / 2 \), \( t_i > 0 \); then \( y(t) = \sqrt{t_1} / 2 \).

Clearly \( S, T, f \) and \( \varphi \), \( y \) satisfy all the conditions of Theorem 4.1 with \( q = 1/2 \) and \( 0 = f0 \in S0 = T0 \), that is, 0 is a coincidence point of \( S, T \), and \( f \).

**Theorem 4.3.** Let \( X \) be a Banach space and \( f : X \to X \), \( S, T : X \to C(X) \) satisfy (i)–(ii) of Theorem 4.1 and (iii) \((f, S)\) and \((f, T)\) are compatible pairs. Then there is a point \( z \in X \) such that \( fz \in Sz \cap Tz \). Suppose that \( \{zn = fnz\} \) is a sequence of iterate in \( X \) for \( z \) and \( \{Sn\}, \{Tn\} \) are sequences of multifunctions on \( X \) where \( Snz = SF^{n-1}z \), \( Tnz = TF^{n-1}z \), \( fnz \in Snz \cap Tnz \), \( \forall n \in \mathbb{N} \).

If \( zn \rightharpoonup z \) and \( \{Sn\}, \{Tn\} \) converge, respectively, to \( S \) and \( T \) on \( X \) pointwise, then \( z \) is a common fixed point of \( S \) and \( T \).

**Proof.** From Theorem 4.1, there is \( z \in X \ni fz \in Sz \cap Tz \). Again from (ii) of Theorem 4.1, it is easy to show that \( Sz = Tz \). Again, from (iii) of Theorem 4.3 and Lemma 2.4, we have \( fz \in Sz = Tz \Rightarrow f^{2}z \in fSz = Sfz \), \( f^{2}z \in fTz = Tfz \), and \( Sfz = Tfz \). Continuing this process, we get \( Snz = SF^{n-1}z = TF^{n-1}z \), \( Tnz = TF^{n-1}z \) where \( zn = fnz \in SF^{n-1}z = TF^{n-1}z \). By hypothesis, \( Snz \rightharpoonup Sz \) and \( Tnz \rightharpoonup Tz \). Then

\[
D(z, Sz) \leq \|z - zn\| + D(zn, Sz) \\
\leq \|z - zn\| + H(Snz, Sz) \rightarrow 0 \quad \text{as} \quad n \rightarrow \infty , \quad \text{which implies that} \quad z \in Sz = Sz.
\]

As \( Sz = Tz \), hence \( z \) is a common fixed point of \( S \) and \( T \) in \( X \).

**Acknowledgement.** The authors convey their gratitudes to the referee for his valuable suggestions.

**References**


D. K. Ganguly and D. Bandyopadhyay: Department of Pure Mathematics, University of Calcutta, 35, B.C. Road, Calcutta-700019, India
Mathematical Problems in Engineering

Special Issue on
Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

<table>
<thead>
<tr>
<th>Manuscript Due</th>
<th>December 1, 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Round of Reviews</td>
<td>March 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>June 1, 2009</td>
</tr>
</tbody>
</table>

Guest Editors

Edson Denis Leonel, Departamento de Estatística, Matemática Aplicada e Computação, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru

Hindawi Publishing Corporation
http://www.hindawi.com