OPTIMAL PROBLEM OF COST FUNCTION
FOR THE LINEAR NEUTRAL SYSTEMS

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ABSTRACT. We study the optimal control problem of a system governed by linear neutral type in Hilbert space $X$. We investigate optimal condition for quadratic cost function and as applications, we give some examples.

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1. Introduction. Our main concern in this paper is to study the optimal control problem of the linear neutral type:

$$
\frac{d}{dt} \left[ x(t) - \sum_{j=1}^{m} B_j x(t-h_j) \right] = A_0 x(t) + \sum_{j=1}^{m} A_j x(t-h_j), \quad t \geq 0,
$$

$$
x(0) = g^0, \quad x(t) = g^1(t), \quad \text{a.e. } t \in [-h,0),
$$

where $(g^0, g^1) \in X \times C([-h,0];X)$.

The optimal control problem of this type has been extensively studied by many authors (see [1, 2] and the references therein). In [2], Darko studied the Laplace transform and fundamental solution in (1.1). Chukwu [1] handled time optimal control, bang-bang control and stability for the neutral type. In fact, in the case of $B_j = 0, j = 1,2,...,m$ in (1.1), Nakagiri [6] studied structural properties of the linear retarded system and dealt with control problems in a Banach spaces.

In this paper, we obtain the necessary and sufficient condition for the optimal control problems of the quadratic cost function and deal with the properties of the fundamental solution and the adjoint state equations in (1.1). As applications, we will give some examples.

2. Preliminaries. (1) $\mathbb{C}$ denotes the complex plane. $\mathbb{R}$ denotes the real numbers, $\mathbb{R}^+$ the nonnegative real numbers, and the real interval $[0,T] = I$.

(2) The symbol $X$ denotes a given Banach space over the real. However, in some instances when dealing with Laplace transforms, we have to consider the complex extension of $X$ which will again be denoted by $X$. If $[-h,0]$ is an interval in $\mathbb{R}$, the Banach space of all continuous mapping $\phi$ from $[-h,0]$ into $X$ will be denoted by $C([-h,0];X)$. The norm in $C([-h,0];X)$ is defined to be $\|\phi\| = \sup\{|\phi(t)|; t \in [-h,0]\}$, where $\|\cdot\|$ denotes the norm in $X$. If $\phi$ in $C([-h,0];X)$ has a derivative which is also in $C([-h,0];X)$, this will be denoted by $(d/dt)\phi(t) = \dot{\phi}(t)$. 

(3) If $Y$ is a given Banach space, the space of continuous linear mappings from $Y$ into itself will be denoted by $LC(Y)$. If $Z$ is also Banach space, the space of all continuous linear mappings from $Y$ into $Z$ will be denoted by $LC(Y,Z)$.

(4) If $X$ and $Y$ be a given Banach space, the space of bounded mappings from $X$ into $Y$ will be denoted by $B(X,Y)$ and if $X = Y$, by $B(X)$.

(5) Let $A_0 : X \to X$ be a closed linear operator which is the infinitesimal generator of a semigroup, $T(t) = e^{A_0t}$, of class $C_0$ on $X$. The domain, $D(A_0)$, of $A_0$ is dense in $X$ (cf. [3]) and since $T(t)$ is of class $C_0$ there exist constants $M > 1$ and $\alpha$ such that $\|T(t)\| \leq Me^{\alpha t}$ (again see [3]).

(6) \{A_j\} and \{B_j\}, $1 \leq j \leq m$, are operators in $LC(X)$, for each $j$ we assume range($B_j$) is in $D(A_0)$ for each $j$ and $B \in L_\infty([0,T];X)$.

(7) The numbers $0 < h_1 < h_2 < h_3 < \cdots < h_m = h$ are fixed in $\mathbb{R}$.

Under the above hypotheses, we consider an “integrated” form described by the equations in (1.1);

\[
\begin{align*}
x(t,0, (g^0,g^1)) &= \sum_{j=1}^{m} B_j x(t-h_j,0,g) + T(t) \left[ g^0 - \sum_{j=1}^{m} B_j g^1 (-h_j) \right] \\
&\quad + \int_{0}^{t} T(t-\sigma) \left[ \sum_{j=1}^{m} (A_j + A_0 B_j)x(\sigma-h_j,0,g) \right] d\sigma, \quad t \geq 0, \quad (2.1)
\end{align*}
\]

where $g = (g^0, g^1) \in X \times C([-h,0];X)$.

Here, $x(t,0,g)$ is the solution with initial condition $t = 0$.

Note that if $f \in C(t;X)$, then

\[
\frac{d}{dt} \int_{0}^{t} T(t-\sigma)f(\sigma) d\sigma = A_0 \int_{0}^{t} T(t-\sigma)f(\sigma) d\sigma + f(t) \quad (2.2)
\]

(cf. [2]). Thus we can differentiate (2.1) to obtain

\[
\begin{align*}
\frac{d}{dt} x(t,0,g) &= \sum_{j=1}^{m} B_j \frac{d}{dt} x(t-h_j,0,g) + \frac{d}{dt} \left[ T(t) \left( g^0 - \sum_{j=1}^{m} B_j g^1 (-h_j) \right) \right] \\
&\quad + \sum_{j=1}^{m} (A_j + A_0 B_j)x(t-h_j,0,g) \\
&\quad + A_0 \int_{0}^{t} T(t-\sigma) \left[ \sum_{j=1}^{m} (A_j + A_0 B_j)x(\sigma-h_j,0,g) \right] d\sigma. \quad (2.3)
\end{align*}
\]

Since $(d/dt)(T(t)g^0) = A_0 T(t)g^0$, if the derivative exists, and making use of (2.1) we again obtain from (2.3) the equation

\[
\begin{align*}
\dot{x}(t,0,g) - \sum_{j=1}^{m} B_j \dot{x}(t-h_j,0,g) &= A_0 \left( x(t,0,g) - \sum_{j=1}^{m} B_j x(t-h_j,0,g) \right) \\
&\quad + \sum_{j=1}^{m} A_j x(t-h_j,0,g) + A_0 \sum_{j=1}^{m} B_j x(t-h_j,0,g) \\
&= A_0 x(t,0,g) + \sum_{j=1}^{m} A_j x(t-h_j,0,g) \quad (2.4)
\end{align*}
\]

(cf. [2, Theorem 2]).
3. Optimality conditions for quadratic cost function. First of all, we consider the construction of the solution in the following type:

\[ \frac{d}{dt} \left[ x(t) - \sum_{j=1}^{m} B_j x(t-h_j) \right] = A_0 x(t) + \sum_{j=1}^{m} A_j x(t-h_j) + B(t)u(t), \quad t \geq 0 \]

\[ x(0) = g^0, \quad x(t) = g^1(t), \quad \text{a.e. } t \in [-h,0), \]

where \((g^0, g^1) \in X \times C([-h,0];X).

Define the fundamental solution \(W(t)\) of (3.1) by

\[ W(t) = \begin{cases} x(t;0,(g^0,0)), & t \geq 0, \\ 0, & t < 0, \end{cases} \]

(3.2)

where \(x(t;u,(g^0,g^1))\) is the general solution of (3.1) (see [1]).

Hence \(W(t)\) is the unique solution of

\[ W(t) = T(t) + \sum_{j=1}^{m} \chi(t-h_j)B_j W(t-h_j) + \int_{0}^{t} T(t-\sigma) \sum_{j=1}^{m} \chi(\sigma-h_j)(A_j+A_0B_j) W(\sigma-h_j) \, d\sigma, \]

(3.3)

where \(\chi(\sigma) = 0\) if \(\sigma < 0, \chi(\sigma) = I\) if \(\sigma \geq 0, I\) identity (cf. [1]).

Note that if \(g^1 \in C([-h,0];X)\) is absolutely continuous, then the solution of (3.1) can be written as

\[ x(t;u,(g^0,g^1)) = \left[ W(t) - \sum_{j=1}^{m} W(t-h_j)B_j \right] g^0 \]

\[ + \sum_{j=1}^{m} \int_{0}^{t-h_j} W(t-s-h_j) \left[ A_j g^1(s) + B_j g^1(s) \right] \, ds \]

\[ + \int_{0}^{t} W(t-s)B(s)u(s) \, ds \]

\[ = x(t;0,(g^0,g^1)) + \int_{0}^{t} W(t-s)B(s)u(s) \, ds. \]

(Cf. [4, page 400]).

In the following, we obtain the properties of the fundamental solution.

**Lemma 3.1.** Let \(W(t)\) be fundamental solution of (3.1). Then we have the following:

\[ \frac{d}{dt} \left[ W(t) - \sum_{j=1}^{m} B_j W(t-h_j) \right] = A_0 W(t) + \sum_{j=1}^{m} A_j W(t-h_j). \]

(3.5)
**Proof.** From (3.3) and \((d/dt)T(t) = A_0T(t)\),

\[
\frac{d}{dt} \left[ W(t) - \sum_{j=1}^{m} \chi(t-h_j)B_jW(t-h_j) \right] = \frac{d}{dt} \left[ T(t) + \int_{0}^{t} T(t-s) \sum_{j=1}^{m} \chi(s-h_j)(A_j + A_0B_j)W(s-h_j) \, ds \right]
\]

\[
= A_0T(t) + \int_{0}^{t} A_0T(t-s) \sum_{j=1}^{m} \chi(s-h_j)(A_j + A_0B_j)W(s-h_j) \, ds
\]

\[
+ \sum_{j=1}^{m} \chi(t-h_j)(A_j + A_0B_j)W(t-h_j)
\]

\[
= A_0 \left[ T(t) + \int_{0}^{t} T(t-s) \sum_{j=1}^{m} \chi(s-h_j)(A_j + A_0B_j)W(s-h_j) \, ds \right]
\]

\[
+ \sum_{j=1}^{m} \chi(t-h_j)(A_j + A_0B_j)W(t-h_j)
\]

\[
= A_0W(t) - A_0 \sum_{j=1}^{m} \chi(t-h_j)B_jW(t-h_j) + \sum_{j=1}^{m} \chi(t-h_j)(A_j + A_0B_j)W(t-h_j)
\]

\[
= A_0W(t) + \sum_{j=1}^{m} \chi(t-h_j)A_jW(t-h_j).
\]

(3.6)

Since definition of \(\chi(\cdot)\),

\[
\sum_{j=1}^{m} \chi(t-h_j)B_jW(t-h_j) = \sum_{j=1}^{m} B_jW(t-h_j),
\]

(3.7)

\[
\sum_{j=1}^{m} \chi(t-h_j)A_jW(t-h_j) = \sum_{j=1}^{m} A_jW(t-h_j).
\]

Hence this proof is complete. □

\(W^*(t), A^*_j, A^*_0,\) and \(B^*_j\) denote the adjoint operators of \(W(t), A_j, A_0,\) and \(B_j,\) respectively.

A similar method as in Lemma 3.1, we consider Lemma 3.2.

**Lemma 3.2.** Let \(W(t)\) be fundamental solution of (3.1). Then

\[
\frac{d}{dt} \left( W^*(t) - \sum_{j=1}^{m} W^*(t-h_j)B^*_j \right) = A^*_0 W^*(t) + \sum_{j=1}^{m} \chi(t-h_j)A^*_j W^*(t-h_j).
\]

(3.8)

**Proof.** From (3.3) and \(A^*_0 W^*(t) = W^*(t)A^*_0, A^*_j W^*(t) = W^*(t)A^*_j, B^*_j W^*(t) = W^*(t)B^*_j,\) we have

\[
W^*(t) - \sum_{j=1}^{m} \chi(t-h_j)B^*_j W^*(t-h_j) = T^*(t) + \int_{0}^{t} T^*(t-\sigma) \sum_{j=1}^{m} (A^*_j + A^*_0 B^*_j) W^*(\sigma-h_j) \, d\sigma.
\]

(3.9)
Differenting (3.9) and using \((d/dt)\tau_t^* = A_0^* T^*(t)\), we have

\[
\frac{d}{dt} \left[ W^*(t) - \sum_{j=1}^m \chi(t-h_j) B_j^* W^*(t-h_j) \right] = A_0^* T^*(t) + \sum_{j=1}^m \chi(t-h_j) (A_j^* + A_0^* B_j^*) W^*(t-h_j)
\]

\[
+ \int_0^t A_0^* T^*(t-\sigma) \sum_{j=1}^m \chi(\sigma-h_j) (A_j^* + A_0^* B_j^*) W^*(\sigma-h_j) d\sigma
\]

\[
= A_0^* \left[ T^*(t) + \int_0^t T^*(t-\sigma) \sum_{j=1}^m \chi(\sigma-h_j) (A_j^* + A_0^* B_j^*) W^*(\sigma-h_j) d\sigma \right]
\]

\[
+ \sum_{j=1}^m \chi(t-h_j) (A_j^* + A_0^* B_j^*) W^*(t-h_j)
\]

\[
= A_0^* \left[ W^*(t) - \sum_{j=1}^m \chi(t-h_j) B_j^* W^*(t-h_j) \right]
\]

\[
+ \sum_{j=1}^m \chi(t-h_j) (A_j^* + A_0^* B_j^*) W^*(t-h_j)
\]

\[
= A_0^* W^*(t) + \sum_{j=1}^m \chi(t-h_j) A_j^* W^*(t-h_j).
\]

Since definition of \(\chi(\cdot)\),

\[
\sum_{j=1}^m \chi(t-h_j) B_j^* W^*(t-h_j) = \sum_{j=1}^m B_j^* W^*(t-h_j),
\]

\[
\sum_{j=1}^m \chi(t-h_j) A_j^* W^*(t-h_j) = \sum_{j=1}^m A_j^* W^*(t-h_j).
\]

Hence this proof is complete. \(\square\)

Secondly, we consider the following cost function:

\[
J(u) = \int_0^T \| C x_u(t) - z_d \|_X^2 dt + \int_0^T (N u(t), u(t)) dt,
\]

where the observation operator \(C\) is bounded from \(H\) to another Hilbert space \(X\), every control \(u \in \mathcal{L}^2(0,T;U)\) and \(z_d \in \mathcal{L}^2(I;X)\), \(I = [0,T]\).

Finally, we assume that \(N\) is a selfadjoint operator in \(B(X)\) such that

\[
(Nu, u) \geq c \| u \|^2, \quad c > 0,
\]

where \(B(X)\) denotes the space of bounded operators on \(X\). Let \(x_u(t)\) stands for a solution of (3.1) associated with the control \(u \in \mathcal{L}^2(0,T;U)\). Let \(\mathcal{U}_{ad}\) be a closed convex subset of \(\mathcal{L}^2(0,T;U)\).
THEOREM 3.3. Let the operators $C$ and $N$ satisfy conditions (3.12) and (3.13). Then there exists a unique element $u \in U_{ad}$ such that

$$\bar{f}(u) = \inf_{v \in U_{ad}} \bar{f}(v).$$  \hspace{1cm} (3.14)

Furthermore, it is hold the following inequality:

$$\int_{0}^{T} (-\Lambda^{-1} B(t)^* p(s) + N(s), v(s) - u(s)) \, ds \geq 0,$$  \hspace{1cm} (3.15)

where $p(s)$ is a solution of adjoint state equation for (3.1) and with the initial condition $p(s) = 0$ for $s \in [T, T + h]$ substituting $q_u$ by $-C^* \Lambda (Cx_u(t) - z_d)$. That is, $p(t)$ satisfies the following transposed system:

$$\frac{d}{dt} p(t) + \sum_{j=1}^{m} B_j^* \frac{d}{dt} p(t + h_j) + A_0^* p(t) + \sum_{j=1}^{m} A_j^* p(t + h_j) + C^* \Lambda (z_d - Cx_u(t)) = 0, \quad a.e. \ t \in I,$$  \hspace{1cm} (3.16)

$$p(s) = 0 \quad a.e. \ s \in [T, T + h]$$  \hspace{1cm} (3.17)

in the weak sense. Here, the operator $\Lambda$ is the canonical isomorphism of $U$ onto $U^*$.  

PROOF. Let $x(t) = x(t; 0, (g^0, g^1))$. Then it holds that

$$\bar{f}(v) = \int_{0}^{T} ||Cx_v(t) - z_d||^2 dt + \int_{0}^{T} (Nv(t), v(t)) dt$$
$$= \int_{0}^{T} ||C(x_v(t) - x(t)) + Cx(t) - z_d||^2 dt + \int_{0}^{T} (Nv(t), v(t)) dt$$
$$= \pi(u, v) - 2L(v) + \int_{0}^{T} ||z_d - Cx(t)||^2 dt,$$  \hspace{1cm} (3.18)

where

$$\pi(u, v) = \int_{0}^{T} (C(x_u(t) - x(t)), C(x_v(t) - x(t))) dt + \int_{0}^{T} (Nu(t), v(t)) dt,$$
$$L(v) = \int_{0}^{T} (z_d - Cx(t), C(x_v(t) - x(t))) dt.$$  \hspace{1cm} (3.19)

The form $\pi(u, v)$ is a continuous bilinear form in $L^2(0, T; U)$ and from the assumption that the operator $N$ is positive definite, we have

$$\pi(v, v) \geq c ||v||^2, \quad v \in L^2(0, T; U).$$  \hspace{1cm} (3.20)

Therefore in virtue of Theorem 1.1 of Chapter 1 in [5], there exists a unique $u \in L^2(0, T; U)$ such that (3.14) holds.

If $u$ is an optimal control (cf. [5, Theorem 1.3. of Chapter 1]), then

$$\bar{f}'(u)(v - u) \geq 0, \quad u \in U_{ad},$$  \hspace{1cm} (3.21)
where \( \mathcal{J}'(u)v \) means the Frechet derivative of \( \mathcal{J} \) at \( u \), applied to \( v \)

\[
\mathcal{J}'(u)(v-u) = \int_0^T \left( Cx_u(t) - z_d, C \int_0^t W(t-s)B(s)(v(s) - u(s)) \right) ds dt \\
+ \int_0^T (Nu(t), v(t) - u(t)) dt \\
= \int_0^T \left( C^* \Lambda(Cx_u(t) - z_d), \int_0^t W(t-s)B(s)(v(s) - u(s)) ds \right) dt.
\]

(3.22)

Note that \( C^* \in B(X^*, H) \) and for \( \phi \) and \( \psi \) in \( H \), we have

\[
(C^* \Lambda C \psi, \phi) = (C \psi, C \phi),
\]

(3.23)

where duality pairing is also denoted by \((\cdot, \cdot)\).

From Fubini’s theorem, we have

\[
\int_0^T \int_0^t \left( C^* \Lambda(Cx_u(t) - z_d), W(t-s)B(s)(v(s) - u(s)) \right) ds dt + \int_0^T (Nu(t), v(t) - u(t)) dt \\
= \int_0^T \left( C^* \Lambda(Cx_u(t) - z_d), W(t-s)B(s)(v(s) - u(s)) \right) ds ds \\
+ \int_0^T (Nu(t), v(t) - u(t)) dt \\
= \int_0^T \left( C^* \Lambda(Cx_u(t) - z_d), \int_0^t W(t-s)B(s)(v(s) - u(s)) ds \right) ds \\
= \int_0^T (\Lambda^{-1}B^*(s)p(s) + Nu(s), v(s) - u(s)) ds \geq 0,
\]

(3.24)

where \( p(s) \) is given by (3.14) and (3.16), that is,

\[
p(s) = - \int_s^T W^*(t-s)C^* \Lambda(Cx_u(t) - z_d) dt.
\]

(3.25)

By using Lemma 3.2 and differentiating (3.25) with respect to \( s \), we get (3.16).

**COROLLARY 3.4** (maximal principle). Let \( u \) be an optimal solution for \( \mathcal{J} \). Then

\[
\max_{v \in \mathcal{U}_{ad}} (v, \Lambda^{-1}B^*(s)p(s)) = (u, \Lambda^{-1}B^*(s)p(s)),
\]

(3.26)

where \( p(s) \) is as in Theorem 3.3.

In application, by using Lemmas 3.1 and 3.2, Theorem 3.3, and differentiating \( p(s) \)
with respect to \( s \), we obtain some examples.

The cost \( \mathcal{J}_1 \) is given by

\[
\mathcal{J}_1 = \langle x(T), \psi_0^* \rangle + \int_I \langle x(t), \psi_1^*(t) \rangle dt,
\]

(3.27)

where \( \psi_0^* \in X^* \) and \( \psi_1^* \in L^1(I; X^*) \).

Then we have the following example.
**Example 3.5** (special linearized Bolza problem). Let \((u, x) \in \text{U}_{\text{ad}} \times C(I; X)\) be an optimal solution for \(\mathcal{J}_1\) in (3.27). Then
\[
\max_{v \in \text{U}(t)} \langle B(t)v, p(t) \rangle = \langle B(t)u(t), p(t) \rangle \quad \text{a.e. } t \in I,
\]
where
\[
p(t) = -W^*(T-t)\psi_0^* - \int_t^T W^*(s-t)\psi^*(s) \, ds, \quad t \in I.
\]
If \(X\) is reflexive, then \(p(t)\) in (3.29) belongs to \(C(I; X^*)\) and satisfies
\[
\frac{d}{dt} \left[ p(t) + \sum_{j=1}^m B_j^* p(t+h_j) \right] + A_0 p(t) + \sum_{j=1}^m A_j^* p(t+h_j) - \psi_j^*(t) = 0 \quad \text{a.e. } t \in I,
\]
\[
p(T) = -\psi_0^*, \quad p(s) = 0, \quad s \in (T, T+h]
\]
in the weak sense.

Let \(X\) be a Hilbert space. As usual we identify \(X\) and \(X^*\). The cost \(\mathcal{J}_2\) is given by
\[
\mathcal{J}_2 = \frac{1}{2} |x(T) - x_d|^2, \quad x_d \in X.
\]

**Example 3.6** (terminal value control problem). Let \((u, x) \in \text{U}_{\text{ad}} \times C(I; X)\) be an optimal solution for \(\mathcal{J}_2\) in (3.31). Then
\[
\max_{v \in \text{U}(t)} \langle B(t)v, p(t) \rangle = \langle B(t)u(t), p(t) \rangle \quad \text{a.e. } t \in I,
\]
where \(p(t)\) is given by
\[
p(t) = W^*(T-t)(x_d - x(T)), \quad t \in I.
\]
The adjoint state \(p \in C(I; X)\) in (3.33) satisfies
\[
\frac{d}{dt} \left[ p(t) + \sum_{j=1}^m B_j^* p(t+h_j) \right] + A_0 p(t) + \sum_{j=1}^m A_j^* p(t+h_j) = 0 \quad \text{a.e. } t \in I,
\]
\[
p(T) = x_d - x(T), \quad p(s) = 0, \quad s \in (T, T+h]
\]
in the weak sense.

Let \(X\) and \(Y\) be Hilbert spaces. The cost \(\mathcal{J}_3\) is given by
\[
\mathcal{J}_3 = \int_I (\lambda^2 |x(t)|^2 + |u(t)|_Y^2) \, dt,
\]
where \(\lambda > 0\). Then we have the following example.

**Example 3.7** (minimum energy problem). Let \((u, x) \in \text{U}_{\text{ad}} \times C(I; X)\) be an optimal solution for \(\mathcal{J}_3\) in (3.35). Then
\[
\max_{v \in \text{U}(t)} \left( \langle B(t)v, p(t) \rangle - |v|_Y^2 \right) = \langle B(t)u(t), p(t) \rangle - |u(t)|_Y^2 \quad \text{a.e. } t \in I,
\]
where

\[ p(t) = -\int_t^T W^*(s-t)(2\lambda^2 x(s)) \, ds, \quad t \in I. \quad (3.37) \]

The adjoint state \( p \in C(I;X) \) in (3.37) satisfies

\[
\frac{d}{dt} \left[ p(t) + \sum_{j=1}^m B_j p(t + h_j) \right] + A_0^* p(t) + \sum_{j=1}^m A_j^* p(t + h_j) - 2\lambda^2 x(t) = 0 \quad \text{a.e. } t \in I,
\]

\[ p(s) = 0, \quad s \in [T, T + h] \quad (3.38) \]

in the weak sense.

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Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from “Qualitative Theory of Differential Equations,” allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the Mathematical Problems in Engineering aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

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