FUZZY ASSOCIATIVE $\mathcal{H}$-IDEALS OF IS-ALGEBRAS

EUN HWAN ROH, YOUNG BAE JUN, and WOOK HWAN SHIM

(Received 20 November 1999)

ABSTRACT. We fuzzify the concept of an associative $\mathcal{H}$-ideal in an IS-algebra. We give a relation between a fuzzy $\mathcal{H}$-ideal and a fuzzy associative $\mathcal{H}$-ideal, and we investigate some related properties.

Keywords and phrases. IS-algebra, (fuzzy) $\mathcal{H}$-ideal, (fuzzy) associative $\mathcal{H}$-ideal.

2000 Mathematics Subject Classification. Primary 06F35, 03G25, 94D05.

1. Introduction. The notion of BCK-algebras was proposed by Imai and Iséki in 1966. In the same year, Iséki [2] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. For the general development of BCK/BCI-algebras, the ideal theory plays an important role. In 1993, Jun et al. [4] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup/BCI-monoid/BCI-group. In 1998, for the convenience of study, Jun et al. [7] renamed the BCI-semigroup (respectively, BCI-monoid and BCI-group) as the IS-algebra (respectively, IM-algebra and IG-algebra) and studied further properties of these algebras (see [6, 7]). In [8] Roh et al. introduced the concept of an associative $\mathcal{H}$-ideal and a strong $\mathcal{H}$-ideal in an IS-algebra. They gave necessary and sufficient conditions for an $\mathcal{H}$-ideal to be an associative $\mathcal{H}$-ideal and established a characterization of a strong $\mathcal{H}$-ideal of an IS-algebras. Jun et al. [3] established the fuzzification of $\mathcal{H}$-ideals in IS-algebras.

In this paper, we consider the fuzzification of an associative $\mathcal{H}$-ideal of an IS-algebra. We prove that every fuzzy associative $\mathcal{H}$-ideal is a fuzzy $\mathcal{H}$-ideal. By giving an appropriate example, we verify that a fuzzy $\mathcal{H}$-ideal may not be a fuzzy associative $\mathcal{H}$-ideal. We give a condition for a fuzzy $\mathcal{H}$-ideal to be a fuzzy associative $\mathcal{H}$-ideal, and we investigate some related properties.

2. Preliminaries. We review some definitions and properties that will be useful in our results.

By a $BCI$-algebra we mean an algebra $(X, \ast, 0)$ of type $(2, 0)$ satisfying the following conditions:

(I) $((x \ast y) \ast (x \ast z)) \ast (z \ast y) = 0$,

(II) $(x \ast (x \ast y)) \ast y = 0$,

(III) $x \ast x = 0$,

(IV) $x \ast y = 0$ and $y \ast x = 0$ imply $x = y$.

A BCI-algebra $X$ satisfying $0 \leq x$ for all $x \in X$ is called a $BCK$-algebra. In any BCI-algebra $X$ one can define a partial order “$\leq$” by putting $x \leq y$ if and only if $x \ast y = 0$. 
A BCI-algebra $X$ has the following properties for any $x, y, z \in X$:

1. $x \ast 0 = x,$
2. $(x \ast y) \ast z = (x \ast z) \ast y,$
3. $x \leq y$ implies that $x \ast z \leq y \ast z$ and $z \ast y \leq z \ast x,$
4. $(x \ast z) \ast (y \ast z) \leq x \ast y,$
5. $x \ast (x \ast (x \ast y)) = x \ast y,$
6. $0 \ast (x \ast y) = (0 \ast x) \ast (0 \ast y),$ 
7. $0 \ast (0 \ast ((x \ast z) \ast (y \ast z))) = (0 \ast y) \ast (0 \ast x).$

A nonempty subset $I$ of a BCK/BCI-algebra $X$ is called an ideal of $X$ if it satisfies:

(i) $0 \in I,$
(ii) $x \ast y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X.$

Any ideal $I$ has the property: $y \in I$ and $x \leq y$ imply $x \in I.$

For a BCI-algebra $X$, the set $X_+ := \{x \in X \mid 0 \leq x\}$ is called the BCK-part of $X.$ If $X_+ = \{0\}$, then we say that $X$ is a $p$-semisimple BCI-algebra. Note that a BCI-algebra $X$ is $p$-semisimple if and only if $0 \ast (0 \ast x) = x$ for all $x \in X.$

In [4], Jun et al. introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup, and in [7] they renamed it as an IS-algebra for the convenience of study.

By an IS-algebra [7] we mean a nonempty set $X$ with two binary operations “$\ast$” and “$\cdot$” and constant 0 satisfying the axioms:

(V) $I(X) := (X, \ast, 0)$ is a BCI-algebra.

(VI) $S(X) := (X, \cdot)$ is a semigroup.

(VII) The operation “$\cdot$” is distributive (on both sides) over the operation “$\ast$,” that is, $x \cdot (y \ast z) = (x \cdot y) \ast (x \cdot z)$ and $(x \ast y) \cdot z = (x \cdot z) \ast (y \cdot z)$ for all $x, y, z \in X.$

Especially, if $I(X) := (X, \ast, 0)$ is a $p$-semisimple BCI-algebra in the definition of IS-algebras, we say that $X$ is a PS-algebra. We write the multiplication $x \cdot y$ by $xy$, for convenience.

**Example 2.1** (see [8]). Let $X = \{0, a, b, c\}$ be a set with Cayley tables:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>0</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>c</td>
<td>b</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
<td>0</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>0</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>

Then $X$ is an IS-algebra.

Every $p$-semisimple BCI-algebra gives an abelian group by defining $x + y := x \ast (0 \ast y),$ and hence a PS-algebra leads to the ring structure. On the while, every ring gives a BCI-algebra by defining $x \ast y := x - y$ and so we can construct an IS-algebra. This means that the category of PS-algebras is equivalent to the category of rings. In Example 2.1, we can see that $(a + b) + c = 0 + a = a + (b + c)$ if we define $x + y := x \ast (0 \ast y).$ Hence the IS-algebra is a generalization of the ring.
Lemma 2.2 [4, Proposition 1]. Let $X$ be an IS-algebra. Then for any $x, y, z \in X$, we have

(i) $0x = x0 = 0$,
(ii) $x \leq y$ implies that $xz \leq yz$ and $zx \leq zy$.

A nonempty subset $A$ of a semigroup $S(X) := (X, \cdot)$ is said to be left (respectively, right) stable [1] if $xa \in A$ (respectively, $ax \in A$) whenever $x \in S(X)$ and $a \in A$.

A nonempty subset $A$ of an IS-algebra $X$ is called a left (respectively, right) $\mathcal{J}$-ideal of $X$ [7] if

(a1) $A$ is a left (respectively, right) stable subset of $S(X)$,
(a2) for any $x, y \in I(X), x \ast y \in A$ and $y \in A$ imply that $x \in A$.

Note that $\{0\}$ and $X$ are left (respectively, right) $\mathcal{J}$-ideals. If $A$ is a left (respectively, right) $\mathcal{J}$-ideal of an IS-algebra $X$, then $0 \in A$. Thus $A$ is an ideal of $I(X)$.

We now review some fuzzy logic concepts.

A fuzzy set in a set $X$ is a function $\mu : X \to [0, 1]$. For $t \in [0, 1]$ the set $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is called a level subset of $\mu$.

A fuzzy set $\mu$ in a BCI-algebra $X$ is called a fuzzy ideal of $X$ if

(b1) $\mu(0) \geq \mu(x)$ for all $x \in X$,
(b2) $\mu(x) \geq \min\{\mu(x \ast y), \mu(y)\}$ for all $x, y \in X$.

A fuzzy set $\mu$ in a semigroup $S(X) := (X, \cdot)$ is said to be fuzzy left (respectively, fuzzy right) stable [5] if $\mu(xy) \geq \mu(y)$ (respectively, $\mu(xy) \geq \mu(x)$) for all $x, y \in X$.

A fuzzy set $\mu$ in an IS-algebra $X$ is called a fuzzy left (respectively, fuzzy right) $\mathcal{J}$-ideal of $X$ if

(b3) $\mu$ is a fuzzy left (respectively, fuzzy right) stable set in $S(X)$,
(b4) $\mu$ is a fuzzy ideal of a BCI-algebra $X$.

From now on, a (fuzzy) $\mathcal{J}$-ideal shall mean a (fuzzy) left $\mathcal{J}$-ideal.

3. Fuzzy associative $\mathcal{J}$-ideals

Definition 3.1 (see [8]). A nonempty subset $A$ of an IS-algebra $X$ is called a left (respectively, right) associative $\mathcal{J}$-ideal of $X$ if

(a1) $A$ is a left (respectively, right) stable subset of $S(X)$,
(a2) for any $x, y, z \in I(X), (x \ast y) \ast z \in A$ and $y \ast z \in A$ imply that $x \in A$.

We start with the fuzzification of a left (respectively, right) associative $\mathcal{J}$-ideal.

Definition 3.2. A fuzzy set $\mu$ in an IS-algebra $X$ is called a fuzzy left (respectively, fuzzy right) associative $\mathcal{J}$-ideal of $X$ if

(b3) $\mu$ is a fuzzy left (respectively, fuzzy right) stable set in $S(X)$,
(b5) $\mu(x) \geq \min\{\mu((x \ast y) \ast z), \mu(y \ast z)\}$ for all $x, y, z \in X$.

In what follows, a (fuzzy) associative $\mathcal{J}$-ideal shall mean a (fuzzy) left associative $\mathcal{J}$-ideal.
**Example 3.3.** Consider an IS-algebra $X = \{0, a, b, c\}$ with the following Cayley tables:

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>0</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>0</td>
<td>b</td>
<td>c</td>
</tr>
</tbody>
</table>

Define a fuzzy set $\mu$ in $X$ by $\mu(0) = 0.7$ and $\mu(b) = 0.5$. Then $\mu$ is a fuzzy associative $\mathcal{J}$-ideal of $X$.

**Example 3.4.** Consider an IS-algebra $X = \{0, a, b, c\}$ with Cayley tables as follows:

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>c</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>0</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Let $\mu$ be a fuzzy set in $X$ defined by $\mu(0) = t_0$, $\mu(a) = t_1$, $\mu(b) = t_2$, where $t_0 > t_1 > t_2$ in $[0, 1]$. Then $\mu$ is a fuzzy associative $\mathcal{J}$-ideal of $X$.

We give a relation between a fuzzy associative $\mathcal{J}$-ideal and a fuzzy $\mathcal{J}$-ideal. To do this, we need the following lemma.

**Lemma 3.5** (see [3]). A fuzzy set $\mu$ in an IS-algebra $X$ is a fuzzy $\mathcal{J}$-ideal of $X$ if and only if it satisfies:

(i) $\mu(x) \geq \min\{\mu(x \ast y), \mu(y)\}$ for all $x, y \in X$,

(ii) $\mu(xy) \geq \mu(y)$ for all $x, y \in X$.

**Theorem 3.6.** Every fuzzy associative $\mathcal{J}$-ideal is a fuzzy $\mathcal{J}$-ideal.

**Proof.** Let $\mu$ be a fuzzy associative $\mathcal{J}$-ideal of an IS-algebra $X$ and let $x, y \in X$. Then

\[
\mu(x) \geq \min\{\mu((x \ast y) \ast 0), \mu(y \ast 0)\} \quad \text{(by (b))}
\]

\[
= \min\{\mu(x \ast y), \mu(y)\} \quad \text{(by (1))}.
\]

It follows from Lemma 3.5 that $\mu$ is a fuzzy $\mathcal{J}$-ideal of $X$. □

The following example shows that the converse of Theorem 3.6 may not be true.

**Example 3.7.** Let $X$ be an IS-algebra in Example 3.3 and let $\mu$ be a fuzzy set in $X$ defined by $\mu(0) = \mu(b) = 0.6$ and $\mu(a) = \mu(c) = 0.2$. It is routine to check that $\mu$ is a fuzzy $\mathcal{J}$-ideal of $X$. But $\mu$ is not a fuzzy associative $\mathcal{J}$-ideal of $X$, since

\[
\mu(a) < \min\{\mu((a \ast b) \ast c), \mu(b \ast c)\}.
\]

Now we find a condition for a fuzzy $\mathcal{J}$-ideal to be a fuzzy associative $\mathcal{J}$-ideal. Let $\mu$ be a fuzzy set in an IS-algebra $X$ and consider the following inequality:
(b₆) \( \mu(x) \geq \mu((x \ast y) \ast y) \) for all \( x, y \in X \).

We know that, in general, a fuzzy \( J \)-ideal of an IS-algebra \( X \) may not satisfy the condition (b₆). In fact, if we take the fuzzy \( J \)-ideal \( \mu \) in Example 3.7, then \( \mu(a) = 0.2 < 0.6 = \mu((a \ast c) \ast c) \). But we have the following theorem.

**Theorem 3.8.** Every fuzzy associative \( J \)-ideal of an IS-algebra satisfies inequality (b₆).

**Proof.** Let \( \mu \) be a fuzzy associative \( J \)-ideal of an IS-algebra \( X \) and let \( x, y \in X \). Using (III) and (b₅), we get

\[
\mu(x) \geq \min \{ \mu((x \ast y) \ast y), \mu(y \ast y) \}
= \min \{ \mu((x \ast y) \ast y), \mu(0) \}
= \mu((x \ast y) \ast y).
\]

(3.3)

This completes the proof.

It is natural to have the question: is a fuzzy set satisfying (b₆) a fuzzy \( J \)-ideal? The following example provides a negative answer, and hence we know that the converse of Theorem 3.8 may not be true.

**Example 3.9.** In Example 3.4, define a fuzzy set \( \mu \) in \( X \) by \( \mu(0) = \mu(a) = \mu(b) = 0.8 \) and \( \mu(c) = 0.5 \). Then \( \mu \) satisfies the condition (b₆), but \( \mu \) is not a fuzzy \( J \)-ideal and hence not a fuzzy associative \( J \)-ideal of \( X \).

**Theorem 3.10.** If a fuzzy \( J \)-ideal of an IS-algebra satisfies condition (b₆), then it is a fuzzy associative \( J \)-ideal.

**Proof.** Let \( \mu \) be a fuzzy \( J \)-ideal of an IS-algebra \( X \) satisfying condition (b₆). It is sufficient to show that \( \mu \) satisfies condition (b₅). Notice that

\[
((x \ast z) \ast z) \ast (y \ast z) = ((x \ast z) \ast (y \ast z)) \ast z \leq (x \ast y) \ast z
\]

(3.4)

for all \( x, y, z \in X \). It follows from (b₆) and Lemma 3.5(i) that

\[
\mu(x) \geq \mu((x \ast z) \ast z)
\geq \min \{ \mu(((x \ast z) \ast z) \ast (y \ast z)), \mu(y \ast z) \}
\geq \min \{ \mu((x \ast y) \ast z), \mu(y \ast z) \}
\]

(3.5)

for all \( x, y, z \in X \). This completes the proof.

By Theorems 3.8 and 3.10, we have the following corollary.

**Corollary 3.11.** Let \( \mu \) be a fuzzy \( J \)-ideal of an IS-algebra \( X \). Then \( \mu \) is a fuzzy associative \( J \)-ideal of \( X \) if and only if it satisfies condition (b₆).

**Proposition 3.12.** Let \( \mu \) be a fuzzy set in an IS-algebra. Then \( \mu \) is a fuzzy associative \( J \)-ideal of \( X \) if and only if the nonempty level set \( U(\mu; t) \) of \( \mu \) is an associative \( J \)-ideal of \( X \) for every \( t \in [0,1] \).

We then call \( U(\mu; t) \) a level associative \( J \)-ideal of \( \mu \).
Suppose that $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$. Let $x \in S(X)$ and $y \in U(\mu; t)$. Then $\mu(y) \geq t$ and so $\mu(xy) \geq \mu(y) \geq t$, which implies that $xy \in U(\mu; t)$. Hence $U(\mu; t)$ is a stable subset of $S(X)$. Let $x, y, z \in I(X)$ be such that $(x \ast y) \ast z \in U(\mu; t)$ and $y \ast z \in U(\mu; t)$. Then $\mu((x \ast y) \ast z) \geq t$ and $\mu(y \ast z) \geq t$. It follows from (b5) that

$$\mu(x) \geq \min \{\mu((x \ast y) \ast z), \mu(y \ast z)\} \geq t$$

(3.6)

so that $x \in U(\mu; t)$. Hence $U(\mu; t)$ is an associative $\mathcal{I}$-ideal of $X$. Conversely, assume that the nonempty level set $U(\mu; t)$ of $\mu$ is an associative $\mathcal{I}$-ideal of $X$ for every $t \in [0, 1]$. If there are $x, y, z \in S(X)$ such that $\mu(x_0y_0) < \mu(y_0)$, then by taking $t_0 := (1/2)(\mu(x_0y_0) + \mu(y_0))$ we have $\mu(x_0y_0) < t_0 < \mu(y_0)$. It follows that $y_0 \in U(\mu; t_0)$ and $x_0y_0 \notin U(\mu; t_0)$. This is a contradiction. Therefore $\mu$ is a fuzzy stable set in $S(X)$. Suppose that $\mu(x_0) < \min \{\mu((x_0 \ast y_0) \ast z_0), \mu(y_0 \ast z_0)\}$ for some $x_0, y_0, z_0 \in X$. Putting $s_0 := (1/2)(\mu(x_0) + \min \{\mu((x_0 \ast y_0) \ast z_0), \mu(y_0 \ast z_0)\})$, then $\mu(x_0) < s_0 < \min \{\mu((x_0 \ast y_0) \ast z_0), \mu(y_0 \ast z_0)\}$, which shows that $(x_0 \ast y_0) \ast z_0 \in U(\mu; s_0), y_0 \ast z_0 \in U(\mu; s_0)$ and $x_0 \notin U(\mu; s_0)$. This is impossible. Thus $\mu$ satisfies the condition (b5). This completes the proof. \qed

Using Proposition 3.12, we can consider a generalization of Example 3.3 as follows.

**Proposition 3.13.** Let $A$ be an associative $\mathcal{I}$-ideal of an IS-algebra $X$ and let $\mu$ be a fuzzy set in $X$ defined by

$$\mu(x) := \begin{cases} t_0 & \text{if } x \in A, \\ t_1 & \text{otherwise,} \end{cases}$$

(3.7)

where $t_0 > t_1$ in $[0, 1]$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$, and $U(\mu; t_0) = A$.

**Proof.** Notice that

$$U(\mu; t) = \begin{cases} \emptyset & \text{if } t_0 < t, \\ A & \text{if } t_1 < t \leq t_0, \\ X & \text{if } t \leq t_1. \end{cases}$$

(3.8)

It follows from Proposition 3.12 that $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$. Clearly, we have $U(\mu; t_0) = A$. \qed

Proposition 3.13 suggests that any associative $\mathcal{I}$-ideal of an IS-algebra $X$ can be realized as a level associative $\mathcal{I}$-ideal of some fuzzy associative $\mathcal{I}$-ideal of $X$.

We now consider the converse of Proposition 3.13.

**Proposition 3.14.** For a nonempty subset $A$ of an IS-algebra $X$, let $\mu$ be a fuzzy set in $X$ which is given in Proposition 3.13. If $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$, then $A$ is an associative $\mathcal{I}$-ideal of $X$.

**Proof.** Assume that $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$ and let $x \in S(X)$ and $y \in A$. Then $\mu(xy) \geq \mu(y) = t_0$ and so $xy \in U(\mu; t_0) = A$. Hence $A$ is a stable subset
FUZZY ASSOCIATIVE $\mathcal{I}$-IDEALS OF IS-ALGEBRAS

of $S(X)$. Let $x, y, z \in I(X)$ be such that $(x * y) * z \in A$ and $y * z \in A$. From (b$_3$) it follows that

$$\mu(x) \geq \min \{\mu((x * y) * z), \mu(y * z)\} = t_0$$

so that $x \in U(\mu; t_0) = A$. This completes the proof.

The following theorem shows that the concept of a fuzzy associative $\mathcal{I}$-ideal of an IS-algebra is a generalization of an associative $\mathcal{I}$-ideal. The proof is straightforward by using Propositions 3.13 and 3.14.

**Theorem 3.15.** Let $A$ be a nonempty subset of an IS-algebra $X$ and let $\mu$ be a fuzzy set in $X$ such that $\mu$ is into $\{0, 1\}$, so that $\mu$ is the characteristic function of $A$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$ if and only if $A$ is an associative $\mathcal{I}$-ideal of $X$.

**References**


Eun Hwan Roh: Department of Mathematics Education, Chinju National University of Education, Chinju 660-765, Korea

E-mail address: ehrroh@ns.chinju-e.ac.kr

Young Bae Jun: Department of Mathematics Education, Gyeongsang National University, Chinju 660-701, Korea

E-mail address: ybjun@nongae.gsnu.ac.kr

Wook Hwan Shim: Department of Mathematics Education, Gyeongsang National University, Chinju 660-701, Korea
Special Issue on
Intelligent Computational Methods for
Financial Engineering

Call for Papers
As a multidisciplinary field, financial engineering is becoming increasingly important in today’s economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems).

This special issue will include (but not be limited to) the following topics:

- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

Authors should follow the Journal of Applied Mathematics and Decision Sciences manuscript format described at the journal site [http://www.hindawi.com/journals/jamds/](http://www.hindawi.com/journals/jamds/). Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at [http://mts.hindawi.com/](http://mts.hindawi.com/), according to the following timetable:

<table>
<thead>
<tr>
<th>Manuscript Due</th>
<th>December 1, 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Round of Reviews</td>
<td>March 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>June 1, 2009</td>
</tr>
</tbody>
</table>

Guest Editors

**Lean Yu**, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; yulean@amss.ac.cn

**Shouyang Wang**, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; sywang@amss.ac.cn

**K. K. Lai**, Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskklai@cityu.edu.hk