

## Research Article

# The Effect of Impulsive Diffusion on Dynamics of a Stage-Structured Predator-Prey System

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Received 6 June 2010; Accepted 13 September 2010

Academic Editor: Antonia Vecchio

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We investigate a predator-prey model with impulsive diffusion on predator and stage structure on prey. The globally attractive condition of prey-extinction periodic solution of the system is obtained by the stroboscopic map of the discrete dynamical system. The permanent condition of the system is also obtained by the theory of impulsive delay differential equation. The results indicate that the discrete time delay has influence on the dynamical behaviors of the system. Finally, some numerical simulations are carried out to support the analytic results.

## 1. Introduction

The dispersal is a ubiquitous phenomenon in the natural world. It is well recognized that the spatial distribution of populations and population dynamics are much affected by spatial heterogeneity and population mobility [1]. The fragmented landscapes are common because the populations of most species occupy mosaic habitats and because of rapid destruction of natural habitats. Briggs and Hoopes [2] identify three mechanisms whereby limited dispersal of hosts and parasitoids combined with other features, such as spatial and temporal heterogeneity, can promote persistence and stability of populations. It is important for us to understand the ecological and evolutionary dynamics of populations mirrored by the large number of mathematical models devoting to it in the scientific literatures [3–6]. In recent years, the analysis of these models focuses on the coexistence of population and local (or global) stability of equilibria [7–13]. Spatial factors play a fundamental role on the persistence and stability of the population, although the complete results have not yet been obtained even

in the simplest one-species case. Most previous papers focused on the population dynamical system modeled by the ordinary differential equations; if the population dynamics with the effects of spatial heterogeneity are modeled by a diffusion process, it will be very interesting. While in practice, it is often the case that diffusion occurs in regular pulse. For example, when winter comes, birds will migrate between patches in search for a better environment, whereas they do not diffuse in other seasons, and the excursion of foliage seeds occurs at fixed period of time every year. Thus, impulsive diffusion provides a more natural description. Lately theories of impulsive differential equations [14] have been introduced into population dynamics. Impulsive differential equations are found in almost a domain of applied science [15–24]. Newly, persistence and stability of population dynamical system involving time delay have been discussed by some authors; see, for example, [8, 25, 26] and references cited therein. They obtained some sufficient conditions that guarantee permanence of population or stability of positive equilibria or positive periodic solutions.

The organization of this paper is as follows. In the next section, we introduce the model and background concepts. In Section 3, some important lemmas are presented. In Section 4, we give the conditions of global attractivity and permanence for system (2.3). In Section 5, A brief discussion is given in the last section to conclude this work.

## 2. The Model

Stage-structured models were analyzed in many literatures [7, 27–33]. The following stage-structured Holling mass defence predator-prey model with impulsive perturbations on predators was introduced by Jiao et al. [31]:

$$\begin{aligned}
 x_1'(t) &= rx_2(t) - re^{-w\tau_1}x_2(t - \tau_1) - wx_1(t), \\
 x_2'(t) &= re^{-w\tau_1}x_2(t - \tau_1) - \frac{\beta x_2(t)}{1 + ax_2 + bx_2^2}x_3(t) - d_3x_2(t) - d_4x_2^2(t), \quad t \neq n\tau, \\
 x_3'(t) &= \frac{k\beta x_2(t)}{1 + ax_2 + bx_2^2}x_3(t) - dx_3(t), \\
 \Delta x_1(t) &= 0, \\
 \Delta x_2(t) &= 0, \quad t = n\tau, \quad n = 1, 2, \dots, \\
 \Delta x_3(t) &= \mu,
 \end{aligned} \tag{2.1}$$

$$(\varphi_1(\xi), \varphi_2(\xi), \varphi_3(\xi)) \in C_+ = C\left([-\tau_1, 0], \mathbb{R}_+^3\right), \quad \varphi_i(0) > 0, \quad i = 1, 2, 3,$$

where  $x_1(t)$ ,  $x_2(t)$  represent the immature and mature pest densities, respectively,  $x_3(t)$  denotes the density of nature enemy,  $\tau_1$  represents a constant time to maturity, and  $r, w, d_3, d_4, d, k, a, b$  and  $\beta$  are positive constants. This model is derived as follows. We assume that at any time  $t > 0$ , birth into the immature population is proportional to the existing mature population with proportionality constant  $r$ . We then assume that the death rate of immature population is proportional to the existing immature population with proportionality constant  $w$ .  $w$  ( $w > d$ ),  $d_3$ , and  $d$  ( $d > d_3$ ) are called the death coefficient of  $x_1(t)$ ,  $x_2(t)$ , and  $x_3(t)$ , respectively. We assume that the death rate of mature populations are

of a logistic nature, that is, proportional to the square of the population with proportionality constant  $d_4$ .  $k > 0$  is the rate of conversing prey into predator.  $\Delta x_3(t) = x_3(t^+) - x_3(t)$ ,  $\mu \geq 0$  is the releasing amount of natural enemies at  $t = n\tau$ ,  $n \in Z_+$  and  $Z_+ = \{1, 2, \dots\}$ ,  $\tau$  and is the period of the impulsive immigration of the predator.

Diffusive predator-prey models were analyzed in literatures [16, 34, 35]. In [16], Hui and Chen considered the single species model with impulsive diffusion as follows:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= x_1(t)(a_1 - b_1x_1(t)), \\ \frac{dx_2(t)}{dt} &= x_2(t)(a_2 - b_2x_2(t)), \end{aligned} \quad t \neq n\tau, \quad (2.2)$$

$$\begin{aligned} \Delta x_1(t) &= d_1(x_2(t) - x_1(t)), \\ \Delta x_2(t) &= d_2(x_1(t) - x_2(t)), \end{aligned} \quad t = n\tau, \quad n = 1, 2, \dots,$$

where we suppose that the system is composed of two patches connected by diffusion and  $x_i$  ( $i = 1, 2$ ) is the density of species in the  $i$ th patch. Intrinsic rate of natural increase and density dependence rate of prey population in the first habitat are denoted by  $a_i$  ( $i = 1, 2$ ), and  $d_i$  ( $i = 1, 2$ ) is the dispersal rate in the  $i$ th patch. It is assumed here that the net exchange from the  $j$ th patch to  $i$ th patch is proportional to the difference  $x_j - x_i$  of population densities. The pulse diffusion occurs every  $\tau$  period ( $\tau > 0$ ). The system evolves from its initial state without being further affected by diffusion until the next pulse appears;  $\Delta x_i = x_i(n\tau^+) - x_i(n\tau)$ , and  $x_i(n\tau^+)$  represents the density of population in the  $i$ th patch immediately after the  $n$ th diffusion pulse at time  $t = n\tau$ , while  $x_i(n\tau)$  represents the density of population in the  $i$ th patch before the  $n$ th diffusion pulse at time  $t = n\tau$ ,  $n = 0, 1, 2, \dots$ ;  $r_i, k_i$ , and  $d_i$  ( $i = 1, 2$ ) are positive constants.

Motivated by these ideals from all of the above, we assume that the predator population diffuses between the two patches, and the prey population exists only in one patch. Then, we consider a delayed predator-prey model with impulsive diffusion on predator and stage structure on prey as follows:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= rx_2(t) - re^{-w\tau_1}x_2(t - \tau_1) - wx_1(t), \\ \frac{dx_2(t)}{dt} &= re^{-w\tau_1}x_2(t - \tau_1) - dx_2(t) - \beta x_2(t)y_1(t), \\ \frac{dy_1(t)}{dt} &= -a_1y_1(t) + k\beta x_2(t)y_1(t), \\ \frac{dy_2(t)}{dt} &= y_2(t)(a_2 - b_2y_2(t)), \end{aligned} \quad t \neq n\tau, \quad (2.3)$$

$$\begin{aligned} \Delta x_1(t) &= 0, \\ \Delta x_2(t) &= 0, \\ \Delta y_1(t) &= D(y_2(t) - y_1(t)), \\ \Delta y_2(t) &= D(y_1(t) - y_2(t)), \end{aligned} \quad t = n\tau, \quad n = 1, 2, \dots,$$

with initial condition

$$(\varphi_1(\zeta), \varphi_2(\zeta), \varphi_3(\zeta), \varphi_4(\zeta)) \in C_+ = C([- \tau_1, 0], R_+^4), \quad \varphi_i(0) > 0, \quad i = 1, 2, 3, 4, \quad (2.4)$$

where  $x_1(t), x_2(t)$  represent the immature and mature prey population densities, respectively.  $\tau_1$  represents a constant time to maturity, that is, immature individuals and mature individuals are divided by age  $\tau_1$ . It is assumed that the system is composed of two patches connected by diffusion and occupied by a single species and  $y_i(t)$  ( $i = 1, 2$ ) is the density of predator species in the  $i$ th patch. Death rate of the predator population in the first patch is denoted by  $a_1 > 0$ . Intrinsic rate of natural increase and density dependence rate of predator population in the second patch are denoted by  $a_2 > 0$  and  $b_2 > 0$ ;  $a_2/b_2$  denotes the carrying capacity in the second patch.  $0 < D < 1$  is dispersal rate of the predator population between two patches. It is assumed here that the net exchange from the  $j$ th patch to  $i$ th patch is proportional to the difference  $y_j - y_i$  of predator population densities.  $\beta$  is the prey capture rate by mature predator.  $k$  is the rate of conversion of nutrients into the reproduction rate of the mature predator.  $w$  is the death rate of immature prey population in the first patch.  $d$  is the death rate of mature prey population in first patch. The pulse diffusion occurs every  $\tau > 0$  period. The system evolves from its initial state without being further affected by diffusion until the next pulse appears.  $\Delta y_i(n\tau) = y_i(n\tau^+) - y_i(n\tau)$ , where  $y_i(n\tau^+)$  represents the density of population in the  $i$ th patch immediately after the  $n$ th diffusion pulse at time  $t = n\tau$ , while  $y_i(n\tau)$  represents the density of population in the  $i$ th patch before the  $n$ th diffusion pulse at time  $t = n\tau$  ( $n = 0, 1, 2, \dots$ ).

Obviously, (2.3) can be simplified as follows:

$$\begin{aligned} \frac{dx_2(t)}{dt} &= re^{-w\tau_1} x_2(t - \tau_1) - dx_2(t) - \beta x_2(t) y_1(t), \\ \frac{dy_1(t)}{dt} &= -a_1 y_1(t) + k\beta x_2(t) y_1(t), \quad t \neq n\tau, \\ \frac{dy_2(t)}{dt} &= y_2(t)(a_2 - b_2 y_2(t)), \end{aligned} \quad (2.5)$$

$$\Delta x_2(t) = 0,$$

$$\Delta y_1(t) = D(y_2(t) - y_1(t)), \quad t = n\tau, \quad n = 1, 2, \dots,$$

$$\Delta y_2(t) = D(y_1(t) - y_2(t)),$$

with initial condition

$$(\varphi_2(\zeta), \varphi_3(\zeta), \varphi_4(\zeta)) \in C'_+ = C([- \tau_1, 0], R_+^3), \quad \varphi_i(0) > 0, \quad i = 2, 3, 4. \quad (2.6)$$

### 3. The Lemmas

The solution of (2.3), denoted by  $x(t) = (x_1(t), x_2(t), y_1(t), y_2(t))^T$ , is a piecewise continuous function  $x : R_+ \rightarrow R_+^4$ .  $x(t)$  is continuous on  $(n\tau, (n+1)\tau]$ ,  $n \in Z_+$ , and  $x(n\tau^+) = \lim_{t \rightarrow n\tau^+} x(t)$  exists. Obviously, the global existence and uniqueness of solutions of (2.3) is guaranteed by

the smoothness of properties of  $f$ , which denotes the mapping defined by right-side of system (2.3) (see Lakshmikantham et al. [36]). Before having the main results, we need some lemmas which will be used next.

According to the biological meanings, it is assumed that  $x_1(t) \geq 0$ ,  $x_2(t) \geq 0$ ,  $y_1(t) \geq 0$ , and  $y_2(t) \geq 0$ . Now, we will show that all solutions of (2.3) are uniformly ultimately bounded.

**Lemma 3.1.** *There exists a constant  $M > 0$  such that  $x_1(t) \leq M/k$ ,  $x_2(t) \leq M/k$ ,  $y_1(t) \leq M$ , and  $y_2(t) \leq M$ , for each solution  $(x_1(t), x_2(t), y_1(t), y_2(t))$  of (2.3) with all large enough  $t$ .*

*Proof.* Define  $V(t) = kx_1(t) + kx_2(t) + y_1(t) + y_2(t)$ . Denote  $\bar{d} = \min\{a_1, r - d, w\}$ . When  $t \neq n\tau$ , we have

$$\begin{aligned} D^+V(t) + \bar{d}V(t) &= -k(w - \bar{d})x_1(t) - k[(r - d) - \bar{d}]x_2(t) \\ &\quad - (a_1 - \bar{d})y_1(t) + y_2(t)[(a_2 + \bar{d}) - b_2y_2(t)] \\ &\leq y_2(t)((a_2 + \bar{d}) - b_2y_2(t)) \leq M_0, \end{aligned} \quad (3.1)$$

where  $M_0 = (a_2 + \bar{d})^2/4b_2$ . When  $t = n\tau$ , we also have  $V(n\tau^+) = V(n\tau)$ . By Lemma 2.3 in [14], for  $t \in (n\tau, (n+1)\tau]$ , we obtain

$$\begin{aligned} V(t) &\leq V(0) \exp(-\bar{d}t) + \int_0^t M_0 \exp(-\bar{d}(t-s)) ds \\ &= V(0) \exp(-\bar{d}t) + \frac{M_0}{\bar{d}} (1 - \exp(-\bar{d}t)) \\ &< V(0) \exp(-\bar{d}t) + \frac{M_0}{\bar{d}} (1 - \exp(-\bar{d}t)) \\ &\rightarrow \frac{M_0}{\bar{d}}, \quad \text{as } t \rightarrow \infty. \end{aligned} \quad (3.2)$$

So,  $V(t)$  is uniformly ultimately bounded. Hence, by the definition of  $V(t)$ , there exists a constant  $M > 0$  such that  $x_1(t) \leq M/k$ ,  $x_2(t) \leq M/k$ ,  $y_1(t) \leq M$ ,  $y_2(t) \leq M$  for all  $t$  large enough. The proof is complete.  $\square$

If  $x_2(t) = 0$ , the subsystem of (2.3) is obtained as follows:

$$\begin{aligned} \frac{dy_1(t)}{dt} &= -a_1y_1(t), \\ \frac{dy_2(t)}{dt} &= y_2(t)(a_2 - b_2y_2(t)), \end{aligned} \quad t \neq n\tau, \quad (3.3)$$

$$\begin{aligned} \Delta y_1(t) &= D(y_2(t) - y_1(t)), \\ \Delta y_2(t) &= D(y_1(t) - y_2(t)), \end{aligned} \quad t = n\tau, \quad n = 1, 2, \dots$$

It is easy to solve the first two equations of system (3.3) between pulses

$$\begin{aligned} y_1(t) &= y_1(n\tau^+)e^{-a_1(t-n\tau)}, \quad n\tau < t \leq (n+1)\tau, \\ y_2(t) &= \frac{a_2e^{a_2(t-n\tau)}y_1(n\tau^+)}{a_2 + b_2(e^{a_2(t-n\tau)} - 1)y_1(n\tau^+)}, \quad n\tau < t \leq (n+1)\tau. \end{aligned} \quad (3.4)$$

By considering the last two equations of System (3.3), we obtain the following stroboscopic map of system (3.3):

$$\begin{aligned} y_1((n+1)\tau^+) &= (1-D)e^{-a_1\tau}y_1(n\tau^+) + D\frac{a_2e^{a_2\tau}y_2(n\tau^+)}{a_2 + b_2(e^{a_2\tau} - 1)y_2(n\tau^+)}, \\ y_2((n+1)\tau^+) &= (1-D)\frac{a_2e^{a_2\tau}y_2(n\tau^+)}{a_2 + b_2(e^{a_2\tau} - 1)y_2(n\tau^+)} + De^{-a_1\tau}y_1(n\tau^+). \end{aligned} \quad (3.5)$$

Then, two lemmas are obtained as follows.

**Lemma 3.2** (see [37, Theorem 3.1]). *If  $a_2\tau < \ln((1 - (1-D)e^{-a_1\tau}) / (1 - (1-2D)e^{-a_1\tau}))$ , the trivial equilibrium  $(0, 0)$  of (3.5) is globally asymptotically stable. If  $a_2\tau > \ln((1 - (1-D)e^{-a_1\tau}) / (1 - (1-2D)e^{-a_1\tau}))$ , there exists a unique positive equilibrium  $(y_1^*, y_2^*)$ , which is globally asymptotically stable, where  $y_1^* = De^{a_2\tau}y_2^* / ((1-D)e^{a_2\tau} - (1-2D)e^{(a_2-a_1)\tau})$  and  $y_2^* = a_2 [(1-D)(e^{a_2\tau} + e^{-a_1\tau}) - (1-2D)e^{(a_2-a_1)\tau} - 1] / b_2(e^{a_2\tau} - 1)(1 - (1-D)e^{(a_2-a_1)\tau})$ .*

**Lemma 3.3** (see [37, Theorem 3.2]). *If  $a_2\tau > \ln((1 - (1-D)e^{-a_1\tau}) / (1 - (1-2D)e^{-a_1\tau}))$ , then (3.3) has a  $\tau$ -periodic positive solution  $(\widetilde{y}_1(t), \widetilde{y}_2(t))$ , which is globally asymptotically stable. Here  $(\widetilde{y}_1(t), \widetilde{y}_2(t))$  can be expressed as*

$$\begin{aligned} \widetilde{y}_1(t) &= y_1^*e^{-a_1(t-n\tau)}, \quad n\tau < t \leq (n+1)\tau, \\ \widetilde{y}_2(t) &= \frac{a_2e^{a_2(t-n\tau)}y_2^*}{a_1 + b_2(e^{a_2(t-n\tau)} - 1)y_2^*}, \quad n\tau < t \leq (n+1)\tau, \end{aligned} \quad (3.6)$$

where  $y_1^*$  and  $y_2^*$  are defined as in Lemma 3.2.

**Lemma 3.4** (see [38]). *Consider the following delay equation:*

$$\frac{dx(t)}{dt} = c_1x(t-\tau) - c_2x(t). \quad (3.7)$$

one assumes that  $c_1, c_2, \tau > 0$ ,  $x(t) > 0$  for  $-\tau \leq t \leq 0$ . Assuming that  $c_1 < c_2$ , then

$$\lim_{t \rightarrow \infty} x(t) = 0. \quad (3.8)$$

#### 4. The Dynamics

From the above discussion, we can easily know that there exists a prey-extinction boundary periodic solution  $(0, 0, \widetilde{y}_1(t), \widetilde{y}_2(t))$  of system (2.3). In this section, we will prove that the prey-extinction boundary periodic solution  $(0, 0, \widetilde{y}_1(t), \widetilde{y}_2(t))$  of system (2.3) is globally attractive.

**Theorem 4.1.** *If*

$$\begin{aligned} a_2\tau &> \ln \frac{1 - (1 - D)e^{-a_1\tau}}{1 - (1 - 2D)e^{-a_1\tau}}, \\ re^{-w\tau_1} &< d + \frac{D\beta e^{a_2\tau} y_2^*}{(1 - D)e^{a_2\tau} - (1 - 2D)e^{(a_2 - a_1)\tau}}, \end{aligned} \quad (4.1)$$

hold, the prey-extinction boundary periodic solution  $(0, 0, \widetilde{y}_1(t), \widetilde{y}_2(t))$  of (2.3) is globally attractive. Here,  $y_2^* = a_2 [(1 - D)(e^{a_2\tau} + e^{-a_1\tau}) - (1 - 2D)e^{(a_2 - a_1)\tau} - 1] / b_2(e^{a_2\tau} - 1)[1 - (1 - D)e^{(a_2 - a_1)\tau}]$ .

*Proof.* It is clear that the global attractivity of the predator-extinction boundary periodic solution  $(0, 0, \widetilde{y}_1(t), \widetilde{y}_2(t))$  of system (2.3) is equivalent to the global attractivity of predator-extinction boundary periodic solution  $(0, \widetilde{y}_1(t), \widetilde{y}_2(t))$  of system (2.5). So, we devote ourselves to investigate system (2.5). Since  $Dk\beta y_2^* e^{a_2\tau - w\tau_1} < d[(1 - D)e^{a_2\tau} - (1 - 2D)e^{(a_2 - a_1)\tau}]$ , we can choose  $\varepsilon_0$  sufficiently small such that

$$k\beta e^{-w\tau_1} \left[ \frac{De^{a_2\tau} y_2^*}{(1 - D)e^{a_2\tau} - (1 - 2D)e^{(a_2 - a_1)\tau}} + \varepsilon_0 \right] < d. \quad (4.2)$$

Since we know from the second equation of system (2.5) that  $dy_1(t)/dt \leq -a_1 y_1(t)$ , we consider the following comparison an impulsive differential system:

$$\begin{aligned} \frac{dy_1'(t)}{dt} &= -a_1 y_1'(t), & t \neq n\tau, \\ \frac{dy_2'(t)}{dt} &= y_2'(t)(a_2 - b_2 y_2'(t)), & \\ \Delta y_1'(t) &= D(y_2'(t) - y_1'(t)), & t = n\tau, \quad n = 1, 2, \dots \\ \Delta y_2'(t) &= D(y_1'(t) - y_2'(t)), & \end{aligned} \quad (4.3)$$

In view of Lemma 3.3 and (3.6), we obtain the  $\tau$ -periodic solution of system (4.3)

$$\begin{aligned} \widetilde{y}_1'(t) &= y_1^{*} e^{-a_1(t - n\tau)}, \quad n\tau < t \leq (n + 1)\tau, \\ \widetilde{y}_2'(t) &= \frac{a_2 e^{a_2(t - n\tau)} y_2^{*}}{a_2 + b_2(e^{a_2(t - n\tau)} - 1) y_2^{*}}, \quad n\tau < t \leq (n + 1)\tau, \end{aligned} \quad (4.4)$$

which is globally asymptotically stable, where  $y_1^{*} = De^{a_2\tau} y_2^{*} / ((1 - D)e^{a_2\tau} - (1 - 2D)e^{(a_2 - a_1)\tau})$  and  $y_2^{*} = a_2((1 - D)(e^{a_2\tau} + e^{-a_1\tau}) - (1 - 2D)e^{(a_2 - a_1)\tau} - 1) / b_2(e^{a_2\tau} - 1)(1 - (1 - D)e^{(a_2 - a_1)\tau})$ .

From Lemma 3.3 and comparison theorem of impulsive equation [36], we have  $y_1(t) \leq y_1'(t)$  and  $y_1'(t) \rightarrow \widetilde{y_1(t)}$  as  $t \rightarrow \infty$ . Then there exists an integer  $k_2 > k_1, t > k_2$  such that

$$y_1(t) \leq y_1'(t) \leq \widetilde{y_1(t)} + \varepsilon_0, \quad n\tau < t \leq (n+1)\tau, \quad n > k_2, \quad (4.5)$$

that is,

$$y_1(t) < \widetilde{y_1(t)} + \varepsilon_0 \leq y_1^* + \varepsilon_0 \stackrel{\Delta}{=} Q, \quad n\tau < t \leq (n+1)\tau, \quad n > k_2. \quad (4.6)$$

From (2.5), we get

$$\frac{dx_2(t)}{dt} \leq re^{-w\tau} x_2(t - \tau_1) - (d + \beta\varrho)x_2(t), \quad t > n\tau + \tau_1, \quad n > k_2. \quad (4.7)$$

Consider the following comparison differential system:

$$\frac{dx_2'(t)}{dt} = re^{-w\tau} x_2'(t - \tau_1) - (d + \beta\varrho)x_2'(t), \quad t > n\tau + \tau_1, \quad n > k_2. \quad (4.8)$$

So we have  $re^{-w\tau} < d + \beta\varrho$ . According to Lemma 3.4, we have  $\lim_{t \rightarrow \infty} x_2'(t) = 0$ .

Let  $(x_2(t), y_1(t), y_2(t))$  be the solution of system (2.5) with its initial conditions and  $x_2(\zeta) = \varphi_2(\zeta)$  ( $\zeta \in [-\tau_1, 0]$ ),  $y_2'(t)$  is the solution of system (4.8) with initial condition  $x_2'(\zeta) = \varphi_2'(\zeta)$  ( $\zeta \in [-\tau_1, 0]$ ). By the comparison theorem, we have

$$\lim_{t \rightarrow \infty} x_2(t) < \lim_{t \rightarrow \infty} x_2'(t) = 0. \quad (4.9)$$

Incorporating into the positivity of  $x_2(t)$ , we know that  $\lim_{t \rightarrow \infty} x_2(t) = 0$ . Therefore, for any  $\varepsilon_1 > 0$  (sufficiently small), there exists an integer  $k_3$  ( $k_3\tau > k_2\tau + \tau_1$ ) such that  $x_2(t) < \varepsilon_1$  for all  $t > k_3\tau$ .

For system (2.5), we have

$$-a_2 y_1(t) \leq \frac{dy_1(t)}{dt} \leq -(a_2 - k\beta\varepsilon_1) y_1(t). \quad (4.10)$$

Then,  $z_2(t) \leq y_1(t) \leq z'_2(t)$  and  $z_2(t) \rightarrow \widetilde{y_1(t)}$ ,  $z'_2(t) \rightarrow \widetilde{y_1(t)}$  as  $t \rightarrow \infty$ , where  $(z_1(t), z_2(t))$  and  $(z'_1(t), z'_2(t))$  are the solutions of

$$\begin{aligned} \frac{dz_1(t)}{dt} &= -a_1 z_1(t), & t \neq n\tau, \\ \frac{dz_2(t)}{dt} &= z_2(t)[a_2 - b_2 z_2(t)], & t \neq n\tau, \end{aligned} \quad (4.11)$$

$$\begin{aligned} \Delta z_1(t) &= D(z_2(t) - z_1(t)), \\ \Delta z_2(t) &= D(z_1(t) - z_2(t)), \end{aligned} \quad t = n\tau, n = 1, 2, \dots,$$

$$\begin{aligned} \frac{dz'_1(t)}{dt} &= -(a_1 - \beta \varepsilon_1) z'_1(t), & t \neq n\tau, \\ \frac{dz'_2(t)}{dt} &= z'_2(t)(a_2 - b_2 z'_2(t)), & t \neq n\tau, \end{aligned} \quad (4.12)$$

$$\begin{aligned} \Delta z'_1(t) &= D(z'_2(t) - z'_1(t)), \\ \Delta z'_2(t) &= D(z'_1(t) - z'_2(t)), \end{aligned} \quad t = n\tau, n = 1, 2, \dots,$$

respectively. Here

$$\begin{aligned} \widetilde{z'_1(t)} &= z_1^* e^{-(a_1 - k\beta\varepsilon_1)(t - n\tau)}, \quad n\tau < t \leq (n+1)\tau, \\ \widetilde{z'_2(t)} &= \frac{a_2 e^{a_2(t - n\tau)} z_2^*}{a_2 + b_2(e^{a_2(t - n\tau)} - 1) z_2^*}, \quad n\tau < t \leq (n+1)\tau, \end{aligned} \quad (4.13)$$

where

$$\begin{aligned} z_1^* &= \frac{D e^{a_2 \tau} z_2^*}{(1 - D) e^{a_2 \tau} - (1 - 2D) e^{(a_2 - (a_1 - k\beta\varepsilon_1))\tau}}, \\ z_2^* &= \frac{a_2 ((1 - D)(e^{a_2 \tau} + e^{-(a_1 - k\beta\varepsilon_1)\tau}) - (1 - 2D) e^{(a_2 - (a_1 - k\beta\varepsilon_1))\tau} - 1)}{b_2 (e^{a_2 \tau} - 1) (1 - (1 - D) e^{(a_2 - (a_1 - k\beta\varepsilon_1))\tau})}. \end{aligned} \quad (4.14)$$

Therefore, for any  $\varepsilon_2 > 0$ , there exists an integer  $k_4$ ,  $n > k_4$  such that  $\widetilde{z_2(t)} - \varepsilon_2 < y_1(t) < \widetilde{z'_2(t)} + \varepsilon_2$ ,  $\widetilde{z_1(t)} - \varepsilon_2 < y_2(t) < \widetilde{z'_1(t)} + \varepsilon_2$ . Let  $\varepsilon_1 \rightarrow 0$ . So we have  $\widetilde{y_1(t)} - \varepsilon_2 < y_1(t) < \widetilde{y_1(t)} + \varepsilon_2$ ,  $\widetilde{y_2(t)} - \varepsilon_2 < y_2(t) < \widetilde{y_2(t)} + \varepsilon_2$ , for  $t$  large enough, which implies  $y_1(t) \rightarrow \widetilde{y_1(t)}$  and  $y_2(t) \rightarrow \widetilde{y_2(t)}$  as  $t \rightarrow \infty$ . This completes the proof.  $\square$

The next work is to investigate the permanence of the system (2.3). Before starting the following theorem, we give the following definition.

*Definition 4.2.* System (2.3) is said to be permanent if there are constants  $m, M > 0$  (independent of initial value) and a finite time  $T_0$ , such that for all solutions  $(x_1(t), x_2(t), y_1(t), y_2(t))$  with all initial values  $x_1(0^+) > 0, x_2(0^+) > 0, y_1(0^+) > 0, y_2(0^+) > 0, m \leq x_1(t) \leq M/k, m \leq x_2(t) \leq M/k, m \leq y_1(t) \leq M, m \leq y_2(t) \leq M$ , holds for all  $t \geq T_0$ . Here  $T_0$  may depend on the initial values  $(x_1(0^+), x_2(0^+), y_1(0^+), y_2(0^+))$ .

**Theorem 4.3.** *If*

$$\begin{aligned} a_2\tau &> \ln \frac{1 - (1 - D)e^{-(a_1 - \beta x_2^*)\tau}}{1 - (1 - 2D)e^{-(a_1 - \beta x_2^*)\tau}}, \\ re^{-w\tau_1} &> d + \frac{D\beta v_1^* e^{(a_1\tau - w\tau_1)}}{(1 - D)e^{a_1\tau} - (1 - 2D)e^{(a_1 - (a_2 + \beta x_2^*))\tau}}, \end{aligned} \quad (4.15)$$

*hold, there is a positive constant  $g$  such that each positive solution  $(x_2(t), y_1(t), y_2(t))$  of (2.5) satisfies  $x_2(t) \geq g$  for large enough  $t$ .  $x_2^*$  can be confirmed by the equation*

$$(re^{-w\tau_1} - d) \left[ (1 - D)e^{a_2\tau} - (1 - 2D)e^{(a_2 - (a_1 + \beta x_2^*))\tau} \right] = D\beta v_1^* e^{a_2\tau - w\tau_1}. \quad (4.16)$$

*Here*

$$v_1^* = \frac{a_2 \left( (1 - D)(e^{a_2\tau} + e^{-(a_1 + \beta x_2^*)\tau}) - (1 - 2D)e^{(a_2 - (a_1 + \beta x_2^*))\tau} - 1 \right)}{b_2(e^{a_2\tau} - 1)(1 - (1 - D)e^{(a_2 - (a_1 + \beta x_2^*))\tau})}. \quad (4.17)$$

*Proof.* The fourth equation of (2.5) can be rewritten as

$$\frac{dx_2(t)}{dt} = [re^{-w\tau_1} - (d + \beta y_1(t))]x_2(t) - re^{-w\tau_1} \frac{d}{dt} \int_{t-\tau_1}^t x_2(u) du. \quad (4.18)$$

Let us consider any positive solution  $(x_2(t), y_1(t), y_2(t))$  of system (2.5). According to (4.18),  $Q(t)$  is defined as

$$Q(t) = x_2(t) + re^{-w\tau_1} \int_{t-\tau_1}^t x_2(u) du. \quad (4.19)$$

We calculate the derivative of  $Q(t)$  along the solution of (2.5)

$$\frac{dQ(t)}{dt} = [re^{-w\tau_1} - (d + \beta y_1(t))]x_2(t). \quad (4.20)$$

Since  $Dk\beta v_1^* e^{(a_1\tau - w\tau_1)} / ((1 - D)e^{a_1\tau} - (1 - 2D)e^{(a_1 - (a_2 + \beta y_2^*))\tau}) > d$ , we can easily know that there exists sufficiently small  $\varepsilon > 0$  such that

$$k\beta e^{-w\tau_1} \left( \frac{Dv_1^* e^{a_1\tau}}{(1 - D)e^{a_1\tau} - (1 - 2D)e^{(a_1 - (a_2 + \beta y_2^*))\tau}} - \varepsilon \right) > d. \quad (4.21)$$

We claim that for any  $t_0 > 0$ , it is impossible that  $x_2(t) < x_2^*$  for all  $t > t_0$ . Suppose that the claim is not valid. Then there is a  $t_0 > 0$  such that  $x_2(t) < x_2^*$  for all  $t > t_0$ . It follows from the first equation of (2.5) that for all  $t > t_0$ ,

$$\frac{dy_1(t)}{dt} < -(a_2 - k\beta x_2^*)y_1(t). \quad (4.22)$$

Consider the following comparison an impulsive system for all  $t > t_0$

$$\begin{aligned} \frac{dv_1(t)}{dt} &= -(a_1 - k\beta x_2^*)v_1(t), \\ \frac{dv_2(t)}{dt} &= v_2(t)(a_2 - b_2v_2(t)), \end{aligned} \quad t \neq n\tau, \quad (4.23)$$

$$\begin{aligned} \Delta v_1(t) &= D(v_2(t) - v_1(t)), \\ \Delta v_2(t) &= D(v_1(t) - v_2(t)), \end{aligned} \quad t = n\tau, \quad n = 1, 2, \dots$$

By Lemma 3.2 and the condition  $a_2\tau > \ln((1 - (1 - D)e^{-(a_1 + \beta x_2^*)\tau}) / (1 - (1 - 2D)e^{-(a_1 + \beta x_2^*)\tau}))$ , we obtain

$$\begin{aligned} \widetilde{v}_1(t) &= v_1^* e^{-(a_1 + \beta x_2^*)(t - n\tau)}, \quad n\tau < t \leq (n + 1)\tau, \\ \widetilde{v}_2(t) &= \frac{a_2 e^{a_2(t - n\tau)} v_2^*}{a_2 + b_2(e^{a_2(t - n\tau)} - 1)v_2^*}, \quad n\tau < t \leq (n + 1)\tau, \end{aligned} \quad (4.24)$$

where

$$\begin{aligned} v_1^* &= \frac{De^{a_2\tau}v_2^*}{(1 - D)e^{a_2\tau} - (1 - 2D)e^{(a_2 - (a_1 - \beta x_2^*)\tau)}}, \\ v_2^* &= \frac{a_2((1 - D)(e^{a_2\tau} + e^{-(a_1 - \beta x_2^*)\tau}) - (1 - 2D)e^{(a_2 - (a_1 - \beta x_2^*)\tau)} - 1)}{b_2(e^{a_2\tau} - 1)(1 - (1 - D)e^{(a_2 - (a_1 - \beta x_2^*)\tau)})}, \end{aligned} \quad (4.25)$$

is the unique positive periodic solution of (4.23), which is globally asymptotically stable. By the comparison theorem for impulsive differential equation [36], we know that there exists  $t_1 (> t_0 + \tau_1)$  such that the inequalities  $y_1(t) \leq \widetilde{v}_1(t) + \varepsilon$  and  $y_2(t) \leq \widetilde{v}_2(t) + \varepsilon$  hold for  $t \geq t_1$ . Thus  $y_1(t) \leq v_1^* + \varepsilon$  for all  $t \geq t_1$ . We make notation as  $\sigma \triangleq v_1^* + \varepsilon$  for convenience. Also

$$re^{-w\tau_1} > d + \beta\sigma. \quad (4.26)$$

Then

$$\frac{dQ(t)}{dt} > x_2(t)[re^{-w\tau_1} - (d + \beta\sigma)] \quad (4.27)$$

for all  $t > t_1$ . Setting  $x_2^m = \min_{t \in [t_1, t_1 + \tau_1]} x_2(t)$ , we will show that  $x_2(t) \geq x_2^m$  for all  $t \geq t_1$ . Supposing the contrary, then there is a  $T_0 > 0$  such that  $x_2(t) \geq x_2^m$  for  $t_1 \leq t \leq t_1 + \tau_1 + T_0$ ,  $x_2(t_1 + \tau_1 + T_0) = x_2^m$  and  $x_2'(t_1 + \tau_1 + T_0) < 0$ . Hence, the first equation of system (2.5) implies that

$$\begin{aligned} x_2'(t_1 + \tau_1 + T_0) &= re^{-w\tau_1} x_2(t_1 + T_0) - dx_2(t_1 + \tau_1 + T_0) - \beta x_2(t_1 + \tau_1 + T_0) y_1(t_1 + \tau_1 + T_0) \\ &\geq [re^{-w\tau_1} - (d + \beta\sigma)] x_2^m > 0. \end{aligned} \quad (4.28)$$

This is a contradiction. Thus,  $x_2(t) \geq x_2^m$  for all  $t > t_1$ . As a consequence,  $Q'(t) > x_2^m [re^{-w\tau_1} - (d + \beta\sigma)] > 0$  for all  $t > t_1$ . This implies that as  $t \rightarrow \infty$ ,  $Q(t) \rightarrow \infty$ . It is a contradiction to  $Q(t) \leq M(1 + \tau_1 re^{-w\tau_1})$ . Hence, the claim is complete.

By the claim, we are left to consider two case. First,  $x_2(t) \geq x_2^*$  for all  $t$  large enough. Second,  $x_2(t)$  oscillates about  $x_2^*$  for large enough  $t$ .

Define

$$g = \min \left\{ \frac{x_2^*}{2}, g_1 \right\}, \quad (4.29)$$

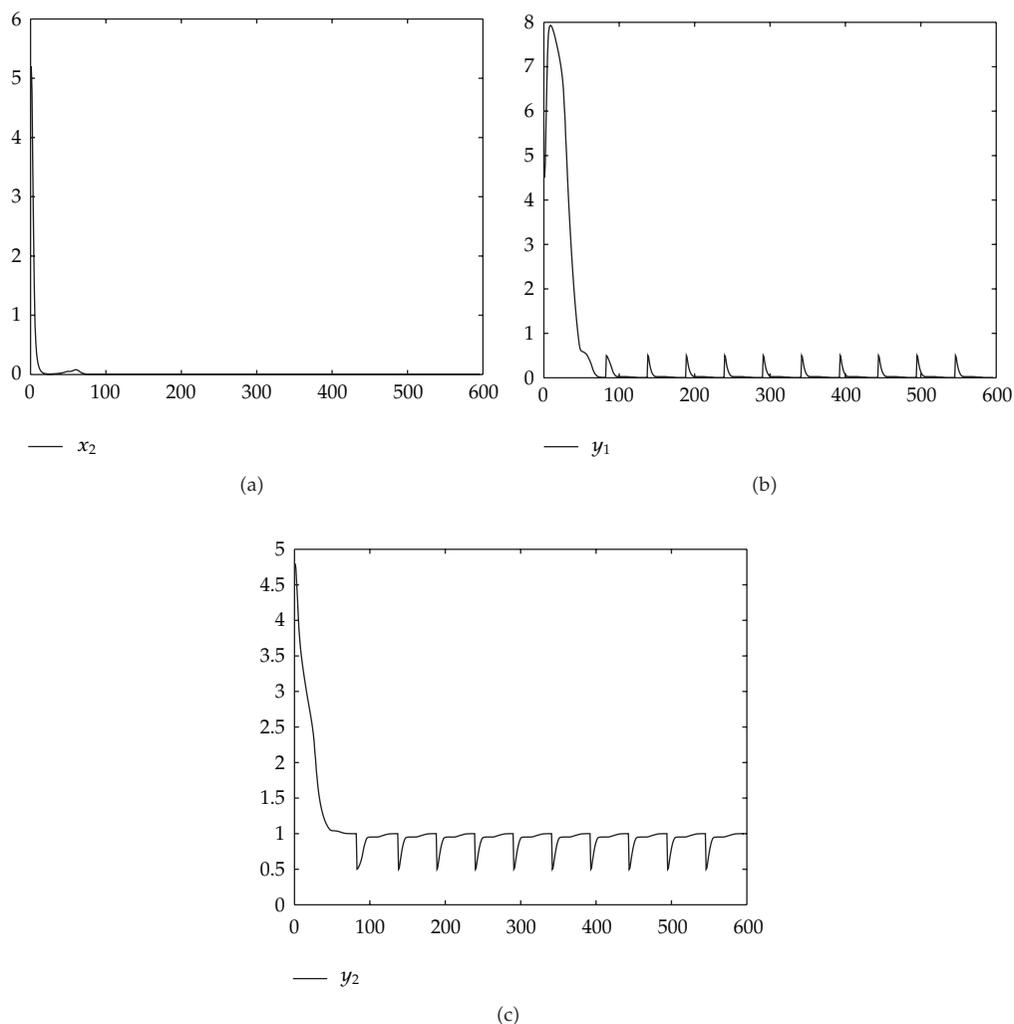
where  $g_1 = x_2^* e^{-(d+\beta M)\tau_1}$ . We hope to show that  $x_2(t) \geq g$  for all  $t$  large enough. The conclusion is evident in the first case. For the second case, let  $t^* > 0$  and  $\xi > 0$  satisfy  $x_2(t^*) = x_2(t^* + \xi) = x_2^*$  and  $x_2(t) < x_2^*$  for all  $t^* < t < t^* + \xi$ , where  $t^*$  is sufficiently large such that  $x_2(t) > \sigma$  for  $t^* < t < t^* + \xi$ ,  $x_2(t)$  is uniformly continuous. The positive solutions of (2.5) are ultimately bounded, and  $x_2(t)$  is not affected by impulses. Hence, there is a  $T$  ( $0 < T < \tau_1$  and  $T$  is dependent on the choice of  $t^*$ ) such that  $x_2(t^*) > x_2^*/3$  for  $t^* < t < t^* + T$ . If  $\xi < T$ , there is nothing to prove. Let us consider the case  $T < \xi < \tau_1$ . Since  $x_2'(t) > -(d + \beta M)x_2(t)$  and  $x_2(t^*) = x_2^*$ , it is clear that  $x_2(t) \geq g_1$  for  $t \in [t^*, t^* + \tau_1]$ . Then, proceeding exactly as the proof for the above claim, we see that  $x_2(t) \geq g_1$  for  $t \in [t^* + \tau_1, t^* + \xi]$ . Because the kind of interval,  $t \in [t^*, t^* + \xi]$  is chosen in an arbitrary way (we only need  $t^*$  to be large). We concluded that  $x_2(t) \geq g$  for all large  $t$ . In the second case, in view of our above discussion, the choice of  $g$  is independent of the positive solution, and we proved that any positive solution of (2.5) satisfies  $x_2(t) \geq g$  for all sufficiently large  $t$ . This completes the proof of this theorem.  $\square$

**Theorem 4.4.** *If*

$$\begin{aligned} a_2\tau &> \ln \frac{1 - (1 - D)e^{-(a_1 - \beta x_2^*)\tau}}{1 - (1 - 2D)e^{-(a_1 - \beta x_2^*)\tau}}, \\ re^{-w\tau_1} &> d + \frac{D\beta v_1^* e^{(a_1\tau - w\tau_1)}}{(1 - D)e^{a_1\tau} - (1 - 2D)e^{(a_1 - (a_2 + \beta x_2^*))\tau}}, \end{aligned} \quad (4.30)$$

*hold, system (2.3) is permanent. Here*

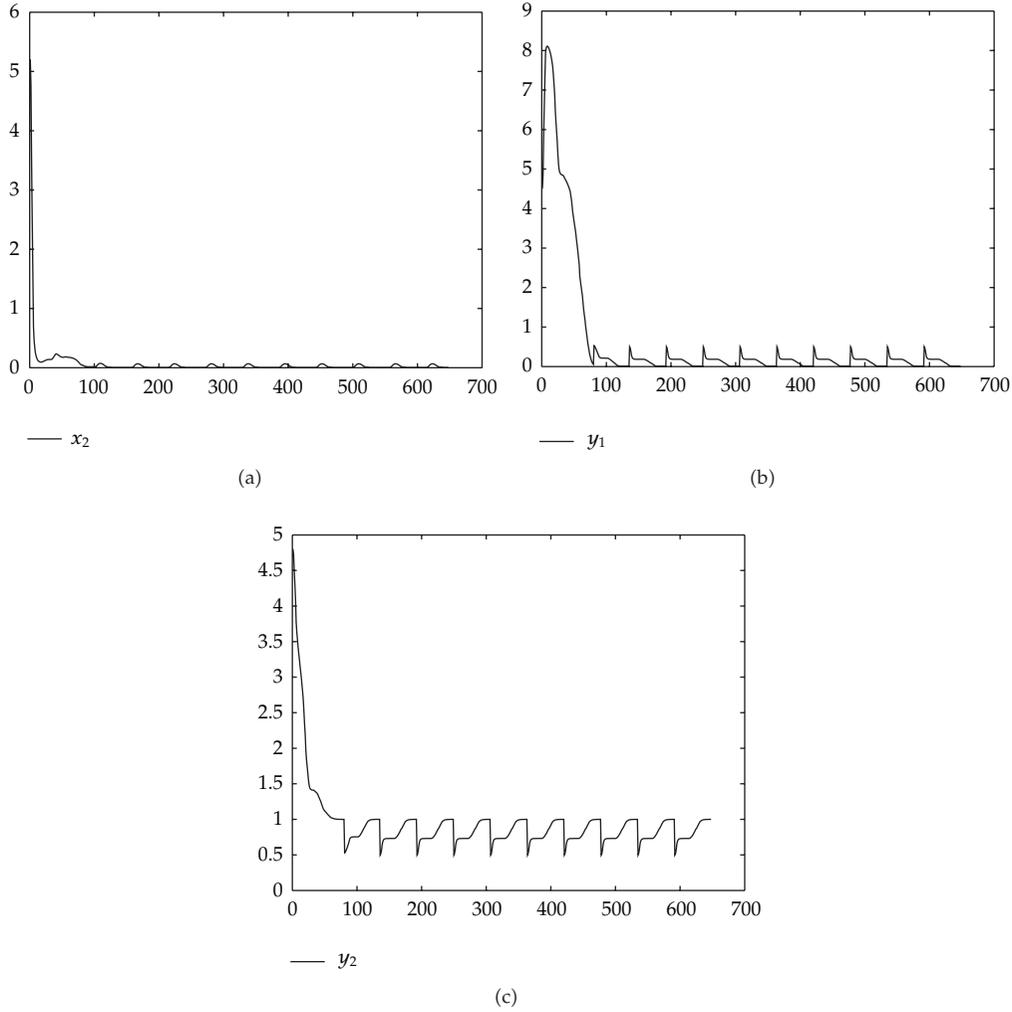
$$v_1^* = \frac{a_2((1 - D)(e^{a_2\tau} + e^{-(a_1 + \beta x_2^*)\tau}) - (1 - 2D)e^{(a_2 - (a_1 + \beta x_2^*))\tau} - 1)}{b_2(e^{a_2\tau} - 1)(1 - (1 - D)e^{(a_2 - (a_1 + \beta x_2^*))\tau})}. \quad (4.31)$$



**Figure 1:** Dynamical behavior of system (2.5) on prey-extinction periodic solution with  $x_2(0) = 5.2, y_1(0) = 5.0, y_2(0) = 5.6, r = 1, \tau_1 = 3, d = 3, a_1 = 1, a_2 = 1, b_2 = 1, \beta = 4, k = 0.9, D = 0.5, w = 1,$  and  $\tau = 1$ . (a) Time-series of  $x_2(t)$ , (b) time-series of  $y_1(t)$ , (c) time-series of  $y_2(t)$ .

*Proof.* Let  $(x_1(t), x_2(t), y_1(t), y_2(t))$  be any solution of system (2.3). From system (2.3) and Theorem 4.3, we have

$$\begin{aligned} \frac{dx_1(t)}{dt} &= x_1(t)(a_1 - b_1x_1(t)), & t \neq n\tau, \\ \frac{dx_2(t)}{dt} &\geq -a_2x_2(t), \\ \Delta x_1(t) &= D(x_2(t) - x_1(t)), & t = n\tau, n = 1, 2, \dots \\ \Delta x_2(t) &= D(x_1(t) - x_2(t)), \end{aligned} \tag{4.32}$$



**Figure 2:** Dynamical behavior of the permanence of system (2.5) with  $x_2(0) = 5.2$ ,  $y_1(0) = 5.0$ ,  $y_2(0) = 5.6$ ,  $\tau_1 = 1$ ,  $d = 3$ ,  $a_1 = 1$ ,  $a_2 = 1$ ,  $b_2 = 1$ ,  $\beta = 4$ ,  $k = 0.9$ ,  $D = 0.5$ ,  $w = 1$ , and  $\tau = 1$ . (a) Time-series of  $x_2(t)$ . (b) Time-series of  $y_1(t)$ . (c) Time-series of  $y_2(t)$ .

By the same argument as those in the proof of Theorem 4.1, we have that  $x_1(t) \geq x_1^{l*} - \varepsilon = p^1$ , and  $x_2(t) \geq x_1^{l*} e^{-a_2 t} - \varepsilon = p^2$ , and  $\varepsilon > 0$  is small enough. Here

$$x_1^{l*} = \frac{a_2((1-D)(e^{a_2 \tau} + e^{-a_1 \tau}) - (1-2D)e^{(a_2-a_1)\tau} - 1)}{b_2(e^{a_2 \tau} - 1)(1 - (1-D)e^{(a_2-a_1)\tau})}, \quad (4.33)$$

$$x_2^{l*} = \frac{D e^{a_2 \tau} x_1^{l*}}{(1-D)e^{a_2 \tau} - (1-2D)e^{(a_2-a_1)\tau}}. \quad (4.34)$$

In view of Theorem 4.1, the third equation of system (2.3) becomes

$$\frac{dy_1(t)}{dt} \geq r(p_2 g - M e^{-w \tau_1}) - w y_1(t). \quad (4.35)$$

It is easy to obtain  $y_1(t) \geq \delta - \varepsilon$ , and  $\varepsilon > 0$  is small enough, where  $\delta = r(p_2g - Me^{-w\tau_1})/w - \varepsilon$ . By Theorem 4.3 and the above discussion, system (2.3) is permanent. The proof of Theorem 4.4 is complete.  $\square$

## 5. Discussion

In this paper, we investigate a delayed predator-prey model with impulsive diffusion on predator and stage structure on prey. We analyze that the prey-extinction periodic solution of system (2.3) is globally attractive, and we also obtain the permanent condition of system (2.3). It is assumed that  $x_2(0) = 5.2$ ,  $y_1(0) = 5.0$ ,  $y_2(0) = 5.6$ ,  $r = 1$ ,  $\tau_1 = 3$ ,  $d = 3$ ,  $a_1 = 1$ ,  $a_2 = 1$ ,  $b_2 = 1$ ,  $\beta = 4$ ,  $k = 0.9$ ,  $D = 0.5$ ,  $w = 1$ , and  $\tau = 1$ . Then the prey-extinction periodic solution is a global attractivity (see Figure 1). We also assume that  $x_2(0) = 5.2$ ,  $y_1(0) = 5.0$ ,  $y_2(0) = 5.6$ ,  $r = 1$ ,  $\tau_1 = 1$ ,  $d = 3$ ,  $a_1 = 1$ ,  $b_1 = 1$ ,  $a_2 = 1$ ,  $b_2 = 1$ ,  $\beta = 4$ ,  $k = 0.9$ ,  $D = 0.5$ ,  $w = 1$ ,  $\tau = 1$ , then, system (2.5) is permanent (see Figure 2). From Theorems 4.1 and 4.4, we can easily guess that there must exist a threshold  $\tau_1^*$ . If  $\tau_1 > \tau_1^*$ , the prey-extinction periodic solution  $(\widetilde{x_1(t)}, \widetilde{x_2(t)}, 0, 0)$  of (2.3) is globally attractive. If  $\tau_1 < \tau_1^*$ , system (2.3) is permanent. The results show that the diffusion and the mature time of the prey play important roles for the permanence of system (2.3) and provide tactical basis for the biological resource protection.

## Acknowledgments

This paper is supported by National Natural Science Foundation of China (10961008), the Nomarch Foundation of Guizhou Province (2008035), and the Science Technology Foundation of Guizhou (2010J2130).

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