

## IDEMPOTENCE-PRESERVING MAPS BETWEEN MATRIX SPACES OVER FIELDS OF CHARACTERISTIC 2\*

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**Abstract.** Let  $M_n(\mathbb{F})$  be the space of all  $n \times n$  matrices over a field  $\mathbb{F}$  of characteristic 2 other than  $\mathbb{F}_2 = \{0, 1\}$ , and let  $P_n(\mathbb{F})$  be the subset of  $M_n(\mathbb{F})$  consisting of all  $n \times n$  idempotent matrices. Let  $m$  and  $n$  be integers with  $n \geq m$  and  $n \geq 3$ . We denote by  $\Phi_{n,m}(\mathbb{F})$  the set of all maps from  $M_n(\mathbb{F})$  to  $M_m(\mathbb{F})$  satisfying that  $A - \lambda B \in P_n(\mathbb{F})$  implies  $\phi(A) - \lambda\phi(B) \in P_m(\mathbb{F})$  for all  $A, B \in M_n(\mathbb{F})$  and  $\lambda \in \mathbb{F}$ . In this paper, we give a complete characterization of  $\Phi_{n,m}(\mathbb{F})$ .

**Key words.** Field; Characteristic; Idempotence; Preserving; Homogeneous

**AMS subject classifications.** 15A04.

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\*Received by the editors on June 10, 2009. Accepted for publication on July 31, 2010. Handling Editors: Roger A. Horn and Fuzhen Zhang.

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