

## THE MINIMUM RANK PROBLEM OVER FINITE FIELDS\*

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**Abstract.** The structure of all graphs having minimum rank at most  $k$  over a finite field with  $q$  elements is characterized for any possible  $k$  and  $q$ . A strong connection between this characterization and polarities of projective geometries is explained. Using this connection, a few results in the minimum rank problem are derived by applying some known results from projective geometry.

**Key words.** Minimum rank, Symmetric matrix, Finite field, Projective geometry, Polarity graph, Bilinear symmetric form

**AMS subject classifications.** 05C50, 05C75, 15A03, 05B25, 51E20.

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