

THE q -NUMERICAL RANGE OF 3×3 TRIDIAGONAL MATRICES*

MAO-TING CHIEN[†] AND HIROSHI NAKAZATO[‡]

Abstract. For $0 \leq q \leq 1$, we examine the q -numerical ranges of 3×3 tridiagonal matrices $A(b)$ that interpolate between the circular range $W_0(A(b))$ and the elliptical range $W_1(A(b))$ as q varies from 0 to 1. We show that for $q \leq (1-b)^2/(2(1+b^2))$, $W_q(A(b))$ is a circular disc centered at the origin with radius $(1+b^2)^{1/2}$, but $W_{4/5}(A(2))$ is not even an elliptical disc.

Key words. Tridiagonal matrix, Davis-Wielandt shell, q -numerical range.

AMS subject classifications. 15A60.

*Received by the editors July 8, 2009. Accepted for publication July 16, 2010. Handling Editor: Bit-Shun Tam.

[†]Corresponding author. Department of Mathematics, Soochow University, Taipei, 11102, Taiwan (mtchien@scu.edu.tw). Supported in part by Taiwan National Science Council.

[‡]Department of Mathematical Sciences, Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan (nakahr@cc.hirosaki-u.ac.jp).