

PATHS OF MATRICES WITH THE STRONG PERRON-FROBENIUS PROPERTY CONVERGING TO A GIVEN MATRIX WITH THE PERRON-FROBENIUS PROPERTY*

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Abstract. A matrix is said to have the Perron-Frobenius property (strong Perron-Frobenius property) if its spectral radius is an eigenvalue (a simple positive and strictly dominant eigenvalue) with a corresponding semipositive (positive) eigenvector. It is known that a matrix A with the Perron-Frobenius property can always be the limit of a sequence of matrices $A(\varepsilon)$ with the strong Perron-Frobenius property such that $\|A - A(\varepsilon)\| \leq \varepsilon$. In this note, the form that the parameterized matrices $A(\varepsilon)$ and their spectral characteristics can take are studied. It is shown to be possible to have $A(\varepsilon)$ cubic, its spectral radius quadratic and the corresponding positive eigenvector linear (all as functions of ε); further, if the spectral radius of A is simple, positive and strictly dominant, then $A(\varepsilon)$ can be taken to be quadratic and its spectral radius linear (in ε). Two other cases are discussed: when A is normal it is shown that the sequence of approximating matrices $A(\varepsilon)$ can be written as a quadratic polynomial in trigonometric functions, and when A has semipositive left and right Perron-Frobenius eigenvectors and $\rho(A)$ is simple, the sequence $A(\varepsilon)$ can be represented as a polynomial in trigonometric functions of degree at most six.

Key words. Perron-Frobenius property, Generalization of nonnegative matrices, Eventually nonnegative matrices, Eventually positive matrices, Perturbation.

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