

SPECTRA OF WEIGHTED COMPOUND GRAPHS OF GENERALIZED BETHE TREES*

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Abstract. A generalized Bethe tree is a rooted tree in which vertices at the same distance from the root have the same degree. Let \mathcal{G}_m be a connected weighted graph on m vertices. Let $\{\mathcal{B}_i : 1 \leq i \leq m\}$ be a set of trees such that, for $i = 1, 2, \dots, m$,

(i) \mathcal{B}_i is a generalized Bethe tree of k_i levels,

(ii) the vertices of \mathcal{B}_i at the level j have degree d_{i,k_i-j+1} for $j = 1, 2, \dots, k_i$, and

(iii) the edges of \mathcal{B}_i joining the vertices at the level j with the vertices at the level $(j+1)$ have weight w_{i,k_i-j} for $j = 1, 2, \dots, k_i - 1$.

Let $\mathcal{G}_m \{\mathcal{B}_i : 1 \leq i \leq m\}$ be the graph obtained from \mathcal{G}_m and the trees $\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_m$ by identifying the root vertex of \mathcal{B}_i with the i th vertex of \mathcal{G}_m . A complete characterization is given of the eigenvalues of the Laplacian and adjacency matrices of $\mathcal{G}_m \{\mathcal{B}_i : 1 \leq i \leq m\}$ together with results about their multiplicities. Finally, these results are applied to the particular case $\mathcal{B}_1 = \mathcal{B}_2 = \dots = \mathcal{B}_m$.

Key words. Weighted graph, Generalized Bethe tree, Laplacian matrix, Adjacency matrix, Spectral radius, Algebraic connectivity.

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