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ON THE MINIMUM RANK OF NOT NECESSARILY SYMMETRIC MATRICES: A PRELIMINARY STUDY*

FRANCESCO BARIOLI†, SHAUN M. FALLAT‡, H. TRACY HALL§, DANIEL HERSHKOWITZ¶, LESLIE HOGBEN‖, HEIN VAN DER HOLST**, AND BRYAN SHADER††

Abstract. The minimum rank of a directed graph Γ is defined to be the smallest possible rank over all real matrices whose ijth entry is nonzero whenever (i, j) is an arc in Γ and is zero otherwise. The symmetric minimum rank of a simple graph G is defined to be the smallest possible rank over all symmetric real matrices whose ijth entry (for $i \neq j$) is nonzero whenever $\{i, j\}$ is an edge in G and is zero otherwise. Maximum nullity is equal to the difference between the order of the graph and minimum rank in either case. Definitions of various graph parameters used to bound symmetric maximum nullity, including path cover number and zero forcing number, are extended to digraphs, and additional parameters related to minimum rank are introduced. It is shown that for directed trees, maximum nullity, path cover number, and zero forcing number are equal, providing a method to compute minimum rank for directed trees. It is shown that the minimum rank problem for any given digraph or zero-nonzero pattern may be converted into a symmetric minimum rank problem.

Key words. Minimum rank, Maximum nullity, symmetric minimum rank, Asymmetric minimum rank, Path cover number, Zero forcing set, Zero forcing number, Edit distance, Triangle number, Minimum degree, Ditree, Directed tree, Inverse eigenvalue problem, Rank, Graph, Symmetric matrix.

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†Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga TN, 37403 (francesco-barioli@utc.edu).

‡Department of Mathematics and Statistics, University of Regina, Regina, SK, Canada (sfallat@math.uregina.ca). Research supported in part by an NSERC research grant.

§Department of Mathematics, Brigham Young University, Provo UT 84602 (H.Tracy@gmail.com).

¶Faculty of Mathematics, Technion, Haifa 32000, Israel (hershkow@technion.ac.il).

‖Department of Mathematics, Iowa State University, Ames, IA 50011 (lhogben@iastate.edu) and American Institute of Mathematics, 360 Portage Ave, Palo Alto, CA 94306 (hogben@aimath.org).

**Faculty of Mathematics and Computer Science, Eindhoven University of Technology 5600 MB Eindhoven, The Netherlands (H.v.d.Holst@tue.nl).

††Department of Mathematics, University of Wyoming, Laramie, WY 82071 (bshader@uwyo.edu).