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CERTAIN MATRICES RELATED TO THE FIBONACCI SEQUENCE HAVING RECURSIVE ENTRIES*

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Abstract. Let $\phi = (\phi_i)_{i\geq 1}$ and $\psi = (\psi_i)_{i\geq 1}$ be two arbitrary sequences with $\phi_1 = \psi_1$. Let $A_{\phi,\psi}(n)$ denote the matrix of order n with entries $a_{i,j}$, $1 \leq i,j \leq n$, where $a_{1,j} = \phi_j$ and $a_{i,1} = \psi_i$ for $1 \leq i \leq n$, and where $a_{i,j} = a_{i-1,j-1} + a_{i-1,j}$ for $2 \leq i,j \leq n$. It is of interest to evaluate the determinant of $A_{\phi,\psi}(n)$, where one of the sequences ϕ or ψ is the Fibonacci sequence (i.e., $1, 1, 2, 3, 5, 8, \ldots$) and the other is one of the following sequences:

$$\alpha^{(k)} = (\overbrace{1, 1, \ldots, 1}^{k-\text{times}}, 0, 0, 0, \ldots)\ ,$$
$$\chi^{(k)} = (1^k, 2^k, 3^k, \ldots, i^k, \ldots),$$
$$\xi^{(k)} = (1, k, k^2, \ldots, k^{i-1}, \ldots) \quad \text{(a geometric sequence)},$$
$$\gamma^{(k)} = (1, 1+k, 1+2k, \ldots, 1+(i-1)k, \ldots) \quad \text{(an arithmetic sequence)}.$$

For some sequences of the above type the inverse of $A_{\phi,\psi}(n)$ is found. In the final part of this paper, the determinant of a generalized Pascal triangle associated to the Fibonacci sequence is found.

Key words. Inverse matrix, Determinant, LU-factorization, Fibonacci sequence, Generalized Pascal triangle, Recursive relation.

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