GLOBAL EXISTENCE OF SOLUTIONS FOR REACTION-DIFFUSION SYSTEMS WITH A FULL MATRIX OF DIFFUSION COEFFICIENTS AND NONHOMOGENEOUS BOUNDARY CONDITIONS.

SAID KOUACHI

Abstract. In this article, we generalize the results obtained in [16] concerning uniform bounds and so global existence of solutions for reaction-diffusion systems with a full matrix of diffusion coefficients satisfying a balance law and with homogeneous Neumann boundary conditions. Our techniques are based on invariant regions and Lyapunov functional methods. We demonstrate that our results remain valid for nonhomogeneous boundary conditions and with out balance law's condition. The nonlinear reaction term has been supposed to be of polynomial growth.

The author thanks the superior education ministry and the national agency of development and scientific research (ANDRU) for their financial support.

1. **INTRODUCTION**

We consider the following reaction-diffusion system

$$\frac{\partial u}{\partial t}\text{-}a\Delta u\text{-}b\Delta v = f(u,v) \quad \text{in } \mathbb{R}^+ \times \Omega \tag{1.1}$$

$$\frac{\partial v}{\partial t}\text{-}c\Delta u\text{-}a\Delta v = g(u,v) \quad \text{in } \mathbb{R}^+ \times \Omega, \tag{1.2}$$

with the boundary conditions

$$\lambda u + (1-\lambda)\frac{\partial u}{\partial \eta} = \beta_1 \text{ and } \lambda v + (1-\lambda)\frac{\partial v}{\partial \eta} = \beta_2 \quad \text{on } \mathbb{R}^+ \times \partial\Omega, \tag{1.3}$$

and the initial data

$$u(0,x) = u_0(x), \qquad v(0,x) = v_0(x) \text{ in } \Omega, \tag{1.4}$$

where

$$(i)\ 0 < \lambda < 1 \text{ and } \beta_i \in \mathbb{R},\ i = 1, 2,$$

(Robin nonhomogeneous boundary conditions) or homogeneous Neumann boundary conditions

$$(ii)\ \lambda = \beta_i = 0,\ i = 1, 2,$$

1991 *Mathematics Subject Classification.* 35K57, 35K45.

Key words and phrases. Reaction diffusion systems, Invariant regions, Lyapunov functionals, Global existence.

or homogeneous Dirichlet boundary conditions

$$(iii)\ 1-\lambda=\beta_i=0,\ i=1,2.$$

Ω is an open bounded domain of class $\mathbb{C}^1$ in $\mathbb{R}^N$, with boundary $\partial\Omega$, $\frac{\partial}{\partial\eta}$ denotes the outward normal derivative on $\partial\Omega$, $\sigma,\ \rho,\ a$, b and c are positive constants satisfying the condition $2a>(b+c)$ which reflects the parabolicity of the system. The initial data are assumed to be in the following region

$$\Sigma=\begin{cases}\left\{(u_0,v_0)\in IR^2 \text{ such that } v_0\geq\mu\left|u_0\right|\right\} \text{ if } \beta_2\geq\mu\left|\beta_1\right|,\\ \left\{(u_0,v_0)\in IR^2 \text{ such that } \mu u_0\geq\left|v_0\right|\right\} \text{ if } \beta_1\geq\mu\left|\beta_2\right|,\end{cases}$$

where

$$\mu=\sqrt{\frac{c}{b}}.$$

One will treat the first case, the second will be discussed at the last section. We suppose that the reaction terms f and g are continuously differentiable, polynomially bounded on Σ, $(f(r,s),g(r,s))$ is in Σ for all (r,s) in $\partial\Sigma$ (we say that (f,g) points into Σ on $\partial\Sigma$); that is

$$(1.5)\qquad g(s,\mu s)\geq\mu f(s,\mu s) \text{ and } g(-s,\mu s)\geq-\mu f(-s,\mu s) \text{ for all } s\geq 0$$

and for positive constants $C<\mu$ sufficiently close to μ, we have

$$(1.6)\qquad Cf(u,v)+g(u,v)\leq C_1\left(u+v+1\right) \text{ for all } u \text{ and } v \text{ in } \Sigma$$

where $C_1$is a positive constant.

The system (1.1)-(1.2) may be regarded as a perturbation of the simple and trivial case where $b=c=0$; for which nonnegative solutions exist globally in time. Always in this case with homogeneous Neumann boundary conditions but when the coefficient of $-\Delta u$ in the first equation is different of the one of $-\Delta v$ in the second one (diagonal case), N. Alikakos [1] established global existence and L^∞-bounds of solutions for positive initial data in the case

$$(*)\qquad g(u,v)=-f(u,v)=-uv^\beta$$

where $1<\beta<\frac{(n+2)}{n}$. The reactions given by (*) satisfy in fact a condition analogous to (1.5) and form a special case; since (f,g) point into Σ on $\partial\Sigma$ by taking $\Sigma=\mathbb{R}^+\times\mathbb{R}^+$. K. Masuda [20] showed that solutions to this system exist globally for every $\beta>1$ and converge to a constant vector as $t\to+\infty$. A. Haraux and A. Youkana [6] have generalized the method of K. Masuda to handle nonlinearities $uF(v)$ that are from a particular case of our one; since they took also $\Sigma=\mathbb{R}^+\times\mathbb{R}^+$. Recently S. Kouachi and A. Youkana [18] have generalized the method of A. Haraux and A. Youkana by adding $-c\Delta v$ to the left-hand side of the diagonal case and by taking nonlinearities $f(u,v)$ of a weak exponential growth. J. I. Kanel and M. Kirane [10] have proved global existence , in the case $g(u,v)=-\ f(u,v)=-uv^n$

and n is an odd integer, under an embarrassing condition which can be written in our case as

$$|b - c| < C_p,$$

where C_p contains a constant from an estimate of Solonnikov. Recently they ameliorate their results in [11] to obtain global existence under the conditions

$$b < \left(\frac{a^2}{a^2 + c^2}\right) c$$

and

$$|F(v)| \leq C_F (1 + |v|^{1+\alpha},$$

where α and C_F are positive constants with $\alpha < 1$ sufficiently small and $g(u, v) = -f(u, v) = -uF(v)$. All techniques used by authors cited above showed their limitations because some are based on the embedding theorem of Sobolev as Alikakos [1], Hollis-Martin-Pierre [8], ... another as Kanel-Kirane [11] use a properties of the Neumann function for the heat equation for which one of it's restriction the coefficient of $-\Delta u$ in equation (1.1) must be larger than the one of $-\Delta v$ in equation (1.2) whereas it isn't the case of problem (1.1)-(1.4).

This article is a continuation of [16] where the reaction terms satisfied the condition $\sigma g + \rho f \equiv 0$ with σ and ρ was any positive constants, and the function $g(u, v)$ positive and polynomially bounded. In that article we have considered the homogeneous Neumann boundary conditions and established global existence of solutions with initial data in the invariant region Σ considered here.

The components $u(t, x)$ and $v(t, x)$ represent either chemical concentrations or biological population densities and system (1.1)-(1.2) is a mathematical model describing various chemical and biological phenomena (see E. L. Cussler [2], P. L. Garcia-Ybarra and P. Clavin [4], S. R. De Groot and P. Mazur [5], J. Jorne [9], J. S. Kirkaldy [14], A. I. Lee and J. M. Hill [19] and J. Savchik, B. Changs and H. Rabitz [22].

2. EXISTENCE.

In this section, we prove that if (f, g) points into Σ on $\partial\Sigma$, then Σ is an invariant region for problem (1.1)-(1.4), i.e. the solution remains in Σ for any initial data in Σ. At this stage and once the invariant regions are constructed, both problems of local and global existence become easier to be established: For the first problem we demonstrate that system (1.1)-(1.2) with boundary conditions (1.3) and initial data in Σ is equivalent to a problem for which local existence over the whole time interval $[0, T_{\max}[$ can be obtained by known procedure and for the second, since we use usual techniques based on Lyapunov functionals which are not directly applicable to problem (1.1)-(1.4) and need invariant regions (see M. Kirane and S. Kouachi [12], [13], S. Kouachi [15]and [16] and S. Kouachi and A. Youkana [18]).

2.1. **Invariant regions.** The main result of this subsection is the following

Proposition 1. *Suppose that (f, g) points into Σ on $\partial\Sigma$, then for any (u_0, v_0) in Σ the solution $(u(t, .), v(t, .))$ of the problem (1.1)-(1.4) remains in Σ for all t in $[0, T^*[$.*

Proof. The proof is similar to that in S. Kouachi [16].
Multiplying equation (1.1) through by $\sqrt{c}$ and equation (1.2) by $\sqrt{b}$, subtracting the resulting equations one time and adding them an other time we get

$$\frac{\partial w}{\partial t} - \lambda_1 \Delta w = F(w, z) \text{ in }]0, T^*[\times \Omega \tag{2.2}$$

$$\frac{\partial z}{\partial t} - \lambda_2 \Delta z = G(w, z) \text{ in }]0, T^*[\times \Omega, \tag{2.3}$$

with the boundary conditions

$$\lambda w + (1-\lambda)\frac{\partial w}{\partial \eta} = \rho_1 \text{ and } \lambda z + (1-\lambda)\frac{\partial z}{\partial \eta} = \rho_2 \text{ on }]0, T^*[\times \partial\Omega \ , \tag{2.4}$$

and the initial data

$$w(0, x) = w_0(x), \qquad z(0, x) = z_0(x) \qquad \text{in } \Omega, \tag{2.5}$$

where

$$w(t, x) = \sqrt{c}u(t, x) + \sqrt{b}v(t, x) \text{ and } z(t, x) = -\sqrt{c}u(t, x) + \sqrt{b}v(t, x), \tag{2.6}$$

for all (t, x) in $]0, T^*[\times \Omega$,

$$F(w, z) = \sqrt{b}g + \sqrt{c}f \text{ and } G(w, z) = \sqrt{b}g - \sqrt{c}f \quad \text{for all } (u, v) \text{ in } \Sigma, \tag{2.7}$$

$$\lambda_1 = a + \sqrt{bc} \text{ and } \lambda_2 = a - \sqrt{bc}$$

and

$$\rho_1 = \sqrt{c}\beta_1 + \sqrt{b}\beta_2 \text{ and } \rho_2 = -\sqrt{c}\beta_1 + \sqrt{b}\beta_2$$

First, let's notice that the condition of parabolicity of the system (1.1)-(1.2) implies the one of the (2.2)-(2.3) system; since $2a > (b + c) \Rightarrow a - \sqrt{bc} > 0$.
Now, it suffices to prove that the region

$$\{(w_0, z_0) \in IR^2 \text{ such that } w_0 \geq 0, z_0 \geq 0\} = IR^+ \times IR^+,$$

is invariant for system (2.2)-(2.3). Since, from (1.5) we have $F(0, z) = \sqrt{b}g(-v, \mu v) + \sqrt{c}f(-v, \mu v) \geq 0$ for all $z \geq 0$ and all $v \geq 0$, then $w(t, x) \geq 0$ for all $(t, x) \in]0, T^*[\times \Omega$, thanks to the invariant region's method (see Smoller [23]) and because $G(w, 0) = \sqrt{b}g(v, \mu v) - \sqrt{c}f(v, \mu v) \geq 0$ for all $w \geq 0$ and all $v \geq 0$, then $z(t, x) \geq 0$ for all $(t, x) \in]0, T^*[\times \Omega$, then Σ is an invariant region for the system (1.1)-(1.3). ∎

Then system (1.1)-(1.2) with boundary conditions (1.3) and initial data in Σ is equivalent to system (2.2)-(2.3) with boundary conditions (2.4) and positive initial data (2.5). As it has been mentioned at the beginning of this section and since ρ_1 and ρ_2 are positive, for any initial data in $\mathbb{C}(\overline{\Omega})$ or $\mathbb{L}^p(\Omega)$, $p \in (1, +\infty)$; local existence and uniqueness of solutions to the initial value problem (2.2)-(2.5) and consequently those of problem (1.1)-(1.4) follow from the basic existence theory for abstract semilinear differential equations (see A. Friedman [3], D. Henry [7] and Pazy [21]). The solutions are classical on $]0, T^*[$, where T^* denotes the eventual blowing-up time in $\mathbb{L}^{\infty}(\Omega)$. The local solution is continued globally by a priori estimates.

Once invariant regions are constructed, one can apply Lyapunov technique and establish global existence of unique solutions for (1.1)-(1.4).

2.2. **Global existence.** As the determinant of the linear algebraic system (2.6), with regard to variables u and v, is different from zero, then to prove global existence of solutions of problem (1.1)-(1.4) comes back in even to prove it for problem (2.2)-(2.5). To this subject, it is well known that (see Henry [7]) it suffices to derive an uniform estimate of $\|F(w,z)\|_p$ and $\|G(w,z)\|_p$ on $[0,T^*[$ for some $p > N/2$, where $\|.\|_p$denotes the usual norms in spaces $\mathbb{L}^p(\Omega)$ defined by

$$\|u\|_p^p = \frac{1}{|\Omega|}\int_\Omega |u(x)|^p\, dx,\ 1 \leq p < \infty \text{ and } \|u\|_\infty = \underset{x\in\Omega}{ess\,\sup}\,|u(x)|\,.$$

Let us define, for all positive integer n, the finite sequence

$$\theta_i = \theta^{i^2},\ i = 0, 1, ..., n \tag{2.8}$$

where θ is a positive constant sufficiently large such that

$$\theta > \frac{a}{\sqrt{a^2 - bc}}. \tag{2.9}$$

The main result and in some sense the heart of the paper is:

Theorem 2. *Let $(w(t,.), z(t,.))$ be any positive solution of the problem (2.2)-(2.5) and let the functional*

$$t \longrightarrow L(t) = \int_\Omega H_n\left(w(t,x), z(t,x)\right) dx, \tag{2.10}$$

where

$$H_n(w,z) = \sum_{i=0}^{n} C_n^i \theta_i w^i z^{n-i}. \tag{2.11}$$

Then the functional L is uniformly bounded on the interval $[0,T^*]$.

Proof. Differentiating L with respect to t yields

$$\begin{aligned} L'(t) &= \int_\Omega \left[\frac{\partial H_n}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial H_n}{\partial z}\frac{\partial z}{\partial t}\right] dx \\ &= \int_\Omega \left(\lambda_1 \frac{\partial H_n}{\partial w}\Delta w + \lambda_2 \frac{\partial H_n}{\partial z}\Delta z\right) dx + \int_\Omega \left(f\frac{\partial H_n}{\partial w} + \frac{\partial H_n}{\partial z} g\right) dx \\ &= I + J. \end{aligned}$$

By simple use of Green's formula we have

$$I = I_1 + I_2,$$

where

$$I_1 = \int_{\partial\Omega} \left(\lambda_1 \frac{\partial H_n}{\partial w} \frac{\partial w}{\partial \eta} + \lambda_2 \frac{\partial H_n}{\partial z} \frac{\partial z}{\partial \eta} \right) dx \tag{2.12}$$

and

$$I_2 = -\int_{\Omega} \left(\lambda_1 \frac{\partial^2 H_n}{\partial w^2} |\nabla w|^2 + 2a \frac{\partial^2 H_n}{\partial w \partial z} \nabla w \nabla z + \lambda_2 \frac{\partial^2 H_n}{\partial z^2} |\nabla z|^2 \right) dx. \tag{2.13}$$

First, let's calculate the first and second partial derivatives of H_n with respect to w and z: We have

$$\frac{\partial H_n}{\partial w} = \sum_{i=1}^{n} \left(iC_n^i \theta_i w^{i-1} z^{n-i} \right) \text{ and } \frac{\partial H_n}{\partial z} = \sum_{i=0}^{n-1} \left((n-i) C_n^i \theta_i w^i z^{n-i-1} \right).$$

Using the formula

$$iC_n^i = nC_{n-1}^{i-1}, \text{ for all } \ i = 1, ..., n \tag{2.14}$$

and changing the index i by $i-1$, we get

$$\frac{\partial H_n}{\partial w} = (n-1) \sum_{i=0}^{n-1} \left(C_{n-1}^i \theta_{i+1} w^i z^{n-1-i} \right). \tag{2.15}$$

For $\dfrac{\partial H_n}{\partial z}$, using (2.14) and the fact that

$$C_n^i = C_n^{n-i}, \text{ for all } \ i = 0, ..., n, \tag{2.16}$$

we get

$$\frac{\partial H_n}{\partial z} = (n-1) \sum_{i=0}^{n-1} \left(C_{n-1}^i \theta_i w^i z^{n-1-i} \right). \tag{2.17}$$

While applying formulas (2.15) and (2.17) to $\dfrac{\partial H_n}{\partial w}$ and (2.17) to $\dfrac{\partial H_n}{\partial z}$, we deduce by analogy

$$\frac{\partial^2 H_n}{\partial w^2} = (n-1)(n-2) \sum_{i=0}^{n-2} \left(C_{n-2}^i \theta_{i+2} w^i z^{n-2-i} \right), \tag{2.18}$$

$$\frac{\partial^2 H_n}{\partial w \partial z} = (n-1)(n-2) \sum_{i=0}^{n-2} \left(C_{n-2}^i \theta_{i+1} w^i z^{n-2-i} \right) \tag{2.19}$$

and

$$\frac{\partial^2 H_n}{\partial z^2} = (n-1)(n-2) \sum_{i=0}^{n-2} \left(C_{n-2}^i \theta_i w^i z^{n-2-i} \right). \tag{2.20}$$

Now we claim that there exists a positive constant C_2 independent of $t \in [0, T_{\max}[$ such that

$$I_1 \le C_2 \text{ for all } t \in [0, T_{\max}[. \tag{2.21}$$

To see this, we follow the same reasoning as in S. Kouachi [18]:
(i) If $0<\lambda<1$, using the boundary conditions (1.4) we get

$$I_1=\int\limits_{\partial\Omega}\left(\lambda_1\frac{\partial H_n}{\partial w}(\gamma_1-\alpha w)+\lambda_2\frac{\partial H_n}{\partial z}(\gamma_2-\alpha z)\right)dx,$$

where $\alpha=\dfrac{\lambda}{1-\lambda}$ and $\gamma_i=\dfrac{\rho_i}{1-\lambda},\ i=1,\ 2.$ Since

$$\begin{aligned} H(w,z) &= \lambda_1\frac{\partial H_n}{\partial w}(\gamma_1-\alpha w)+\lambda_2\frac{\partial H_n}{\partial z}(\gamma_2-\alpha z) \\ &= P_{n-1}(w,z)-Q_n(w,z), \end{aligned}$$

where P_{n-1} and Q_n are polynomials with positive coefficients and respective degrees $n-1$ and n and since the solution is positive, then

$$(**)\qquad \limsup_{(|w|+|z|)\to+\infty} H(w,z)=-\infty,$$

which prove that H is uniformly bounded on $\mathbb{R}_+^2$ and consequently (2.21).
(ii) If $\lambda=0$, then $I_1=0$ on $[0,T_{\max}[$.
(iii) The case of homogeneous Dirichlet conditions is trivial; since in this case the positivity of the solution on $[0,T_{\max}[\times\Omega$ implies $\dfrac{\partial w}{\partial\eta}\leq 0$ and $\dfrac{\partial z}{\partial\eta}\leq 0$ on $[0,T_{\max}[\times\partial\Omega$. Consequently one gets again (2.21) with $C_2=0$.

$$I_2=-n(n-1)\sum_{i=0}^{n-2}C_{n-2}^i w^i z^{n-2-i}\int\limits_{\Omega}\left(\lambda_1\theta_{i+2}\left|\nabla w\right|^2+2a\theta_{i+1}\nabla w\nabla z+\lambda_2\theta_i\left|\nabla z\right|^2\right)dx.$$

Using (2.8) and (2.9) we deduce that the quadratic forms (with respect to ∇w and ∇z) are positive since

$$(a\theta_{i+1})^2-\lambda_1\lambda_2\theta_i\theta_{i+2}=\theta^{2(i+1)^2}\left(a^2-(a^2-bc)\theta^2\right)<0,\ i=0,...,n-2. \tag{2.22}$$

Then

$$I_2\leq 0. \tag{2.23}$$

Using (2.15) and (2.17) we get

$$J=n\sum_{i=0}^{n-1}C_{n-1}^i w^i z^{n-1-i}\int\limits_{\Omega}\left[(\theta_{i+1}F(w,z)+\theta_i G(w,z))\right]dx.$$

Using the expressions (2.7), we obtain

$$J=n\sum_{i=0}^{n-1}C_{n-1}^i w^i z^{n-1-i}\int\limits_{\Omega}\left[\left(\sqrt{c}(\theta_{i+1}-\theta_i)f+\sqrt{b}(\theta_{i+1}+\theta_i)g\right)\right]dx,$$

which can be written

$$J=n\sum_{i=0}^{n-1}C_{n-1}^i(\theta_{i+1}+\theta_i)w^i z^{n-1-i}\int\limits_{\Omega}\left[\sqrt{c}\left(\frac{\left(\frac{\theta_{i+1}}{\theta_i}\right)-1}{\left(\frac{\theta_{i+1}}{\theta_i}\right)+1}\right)f+\sqrt{b}g\right]dx.$$

Since the function $x \to \frac{x-1}{x+1}$ is increasing with $\lim_{x\to+\infty}\left(\frac{x-1}{x+1}\right) = 1$ and since the sequence $\left\{\frac{\theta_{i+1}}{\theta_i}\right\}_{0\le i\le n}$ is increasing by choosing θ sufficiently large with $\lim_{i\to+\infty}\left(\frac{\theta_{i+1}}{\theta_i}\right) = +\infty$, we get by using condition (1.6) and relation (2.6) successively

$$J \le C_3 \int_\Omega \left[\sum_{i=1}^{p} (w+z+1)\, C_{p-1}^{i-1} w^{i-1} z^{p-i}\right] dx.$$

Following the same reasoning as in S. Kouachi [15] we get

$$J \le C_4 L(t) \text{ on } [0, T^*].$$

Then we have

$$\overset{\bullet}{L}(t) \le C_5 L(t) + C_6 L^{(p-1)/p}(t) \text{ on } [0, T^*].$$

While putting

$$Z = L^{1/p},$$

one gets

$$p\overset{\bullet}{Z} \le C_5 Z + C_6.$$

The resolution of this linear differential inequality gives the uniform boundedness of the functional L on the interval $[0, T^*]$, what finishes at the same time our reasoning and the proof of the theorem.

Corollary 3. *Suppose that the functions $f(r,s)$ and $g(r,s)$ are continuously differentiable on Σ, point into Σ on $\partial\Sigma$ and satisfy condition (1.5). Then all solutions of (1.1)-(1.4) with initial data in Σ and uniformly bounded on Ω are in $\mathbb{L}^\infty(0, T^*; \mathbb{L}^p(\Omega))$ for all $p \ge 1$.*

Proof. The proof is an immediate consequence of theorem 2, expressions (2.6) and the trivial inequalities

$$\int_\Omega (w(t,x) + z(t,x))^p \, dx \le L(t) \text{ on } [0, T^*[.$$

Proposition 4. *Under hypothesis of corollary 3 if the reactions $f(r,s)$ and $g(r,s)$ are polynomially bounded, then all solutions of (1.1)-(1.3) with initial data in Σ and uniformly bounded on Ω are global in time.*

Proof. As it has been mentioned above; it suffices to derive an uniform estimate of $\|F(w,z)\|_p$ and $\|G(w,z)\|_p$ on $[0, T^*[$ for some $p > n/2$. Since the functions $f(u,v)$ and $g(u,v)$ are polynomially bounded on Σ, then using relations (2.6) and (2.7) we get that $F(w,z)$ and $G(w,z)$ are too and the proof becomes an immediate consequence of corollary 2.3.

3. Final remarks

If $\beta_1 \geq \mu |\beta_2|$, all previous results remain valid in the region

$$\Sigma = \left\{ (u_0, v_0) \in IR^2 \text{ such that } u_0 \geq \sqrt{\frac{b}{c}} |v_0| \right\}.$$

In this case, system (2.2)-(2.3) becomes

$$(2.2)' \qquad \frac{\partial w}{\partial t} - \lambda_2 \Delta w = F(w,z) \ \text{ in } \ ]0,T^*[\times \Omega,$$

$$(2.3)' \qquad \frac{\partial z}{\partial t} - \lambda_1 \Delta z = G(w,z) \ \text{ in } \ ]0,T^*[\times \Omega,$$

where

$$(2.6)' \qquad w = \sqrt{c}u - \sqrt{b}v \text{ and } \ z = \sqrt{c}u + \sqrt{b}v \text{ on } \ ]0,T^*[\times \Omega$$

and

$$(2.7)' \qquad F(w,z) = (\sqrt{c}f - \sqrt{b}g)(u,v) \text{ and } G(w,z) = (\sqrt{c}f(u,v) + \sqrt{b}g)(u,v),$$

with the boundary conditions

$$(2.4)' \qquad \lambda w + (1-\lambda)\frac{\partial w}{\partial \eta} = \rho_1 \text{ and } \lambda z + (1-\lambda)\frac{\partial z}{\partial \eta} = \rho_2 \text{ on } \ ]0,T^*[\times \partial\Omega \ ,$$

where

$$\rho_1 = \sqrt{c}\beta_1 - \sqrt{b}\beta_2 \geq 0 \text{ and } \rho_2 = \sqrt{c}\beta_1 + \sqrt{b}\beta_2 \geq 0.$$

The conditions (1.5) and (1.6) become respectively

$$(1.5)' \qquad \mu f(s,\mu s) \geq g(s,\mu s) \text{ and } \mu f(s,-\mu s) \geq -g(s,-\mu s) \ \text{ for all } s \geq 0$$

and

$$(1.6)' \qquad Cf(u,v) + g(u,v) \leq C_1' (u+v+1) \quad \text{for all } (u,v) \text{ in } \Sigma,$$

for positive constants $C > \mu$ sufficiently close to μ where C_1' is a positive constant.

Remark 1. *The case where boundary condition imposed to u are of different type to those imposed to v remains open for some technical difficulties.*

References

[1] N. Alikakos, L^p-Bounds of Solutions of Reaction-Diffusion Equations.Comm. P. D. E. 4 (1979). 827-828.

[2] E. L. Cussler, Multicomponent Diffusion, Chemical Engineering Monographs 3. Elsevier Scientific Publishing Company, Amsterdam, 1976.

[3] A.Friedman, Partial Differential Equations of Parabolic Type. Prentice Hall Englewood Chiffs. N. J. 1964.

[4] P. L. Garcia-Ybarra and P. Clavin, Cross transport effects in premixed flames. Progress in Astronautics and Aeronautics, Vol. 76, The American Institute of Aeronautics and Astronautics, New York, 463-481 (1981).

[5] S. R. De Groot and P. Mazur, Non-Equilibrium Thermodynamics. Dover Publications Inc., New York (1984).

[6] A. Haraux and A.Youkana, On a Result of K. Masuda Concerning Reaction-Diffusion Equations. Tôhoku. Math. J.40 (1988), 159-163.

[7] D. Henry, Geometric Theory of Semilinear Parabolic Equations. Lecture Notes in Mathematics 840, Springer-Verlag,New-York, 1984.

[8] S. L. Hollis, R. H. Martin $_{JR}$ and M. Pierre, Global Existence and bounds in Reaction-Diffusion Systems. Siam. J. Math. Anal, Vol 18, n^{0}3, May 1987.

[9] J. Jorne, The diffusion Lotka-Volterra oscillating system. J. Theor. Biol., 65, 133-139 (1977).

[10] J. I. Kanel and M. Kirane, Pointwise a priori bounds for a strongly coupled system of Reaction-Diffusion Equations with a balance law, Math. Methods in the applied Sciences, 171 (1999), 227-230.

[11] J. I. Kanel and M. Kirane, Pointwise a priori bounds for a strongly coupled system of Reaction-Diffusion Equations, International Journal of Diff. Eq. and Appl., 1, No. 1 (2000), 77-97.

[12] M. Kirane and S. Kouachi, Global Solutions to a System of Strongly Coupled Reaction-Diffusion Equations. Nonlinear Analysis Theory, Methods and Applications, Vol 126, n^{0}8, (1996). USA.

[13] M. Kirane and S. Kouachi, Asymptotic Behavior for a System Describing Epidemics with Migration and Spatial Spread of Infection. Dynamical Systems and Applications, Vol 12; number 1, (1993), 121-130.

[14] J. S. Kirkaldy, Diffusion in multicomponent metallic systems. Canadian J. of Physics, 35, 435-440 (1957).

[15] S. Kouachi, Global existence of solutions to reaction-diffusion systems via a Lyapunov functional. Electron. J. Diff. Eqns Vol. 2001(2001), No. 68, pp. 1-10.

[16] S. Kouachi, Uniform boundedness and global existence of solutions for reaction-diffusion systems with a balance law and a full matrix of diffusion coefficients. E. J. Qualitative Theory of Diff. Equ., No. 7. (2001), pp. 1-9.

[17] S. Kouachi, Functionals giving global existence of solutions to reaction-diffusion systems with nonhomogeneous boundary conditions. Submitted to Electron. J. Diff. Eqns.

[18] S. Kouachi and A. Youkana, Global existence for a class of reaction-diffusion systems. Bulletin of the Polish Academy of Science, Vol 49, n°3, (2001).

[19] A. I. Lee and J. M. Hill, On the general linear coupled system for diffusion in media with two diffusivities. J. Math. Anal. Appl., 89, 530-557 (1982).

[20] K. Masuda, On the Global Existence and Asymptotic Behavior of Solutions of Reaction-Diffusion Equations. Hokkaido. Math. J. 12 (1983), 360-370.

[21] A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations. Applied Math. Sciences 44, Springer-Verlag, New York (1983).

[22] J. Savchik, B. Changs and H. Rabitz, Application of moments to the general linear multicomponent reaction-diffusion equations. J. Phys. Chem. 37, 1990-1997 (1983).

[23] J. A. Smoller, Shock waves and reaction-diffusion equations. Springer-Verlag, New York (1983).

Laboratoire de mathémathiques, université d'Annaba, B. P. 12, Sidi Ammar, 23200, Annaba.

E-mail address: kouachi.said@caramail.com

Current address: Département de Mathématiques, Centre univ de Tébessa, B. P. 24, Tébessa, 12002, Algérie.