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On certain nonlinear elliptic systems with indefinite terms *

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Abstract

We consider an elliptic quasi linear system with indefinite term on a bounded domain. Under suitable conditions, existence and positivity results for solutions are given.

1 Introduction

The purpose of this article is to find positive solutions to the system

$$\begin{gathered} -\Delta_p u = m(x)\frac{\partial H}{\partial u}(u,v) \quad \text{in } \Omega \\ -\Delta_q v = m(x)\frac{\partial H}{\partial v}(u,v) \quad \text{in } \Omega \\ u = v = 0 \quad \text{on } \partial\Omega \end{gathered} \tag{1.1}$$

where Ω is a bounded regular domain of $\mathbb{R}^N$, with a smooth boundary $\partial\Omega$, $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian with $1 < p < N$, m is a continuous function on $\overline{\Omega}$ which changes sign, and H is a potential function which will be specified later.

The case where the sign of m does not change has been studied by F. de Thélin and J. Vélin [9]. These authors treat the system (1.1) with a function H having the following properties

- There exists $C > 0$, for all $x \in \Omega$, for all $(u,v) \in D_3$ such that $0 \leq H(x,u,v) \leq C(|u|^{p'} + |v|^{q'})$
- There exists $C' > 0$, for all $x \in \Omega$, for all $(u,v) \in D_2$ such that $H(x,u,v) \leq C'$
- There exists a positive function a in $L^\infty(\Omega)$, such that for each $x \in \Omega$ and $(u,v) \in D_1 \cap \mathbb{R}^2_+$, $H(x,u,v) = a(x)u^{\alpha+1}v^{\beta+1}$,

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where

$$D_1 = \{(u,v) \in \mathbb{R}^2 : |u| \geq A \text{ or } |v| \geq A\},$$
$$D_2 = \{(u,v) \in \mathbb{R}^2 \backslash D_1 : |u| \geq \delta \text{ or } |v| \geq \delta\},$$

and $D_3 = \mathbb{R}^2 \backslash (D_1 \cup D_2)$ with $A > \delta > 0$, $1 < p' < p^* := \frac{Np}{N-p}$, and $1 < q' < q^*$.

They established the existence results under the conditions

$$\frac{\alpha+1}{p} + \frac{\beta+1}{q} > 1 \quad \text{and} \quad \frac{\alpha+1}{p^*} + \frac{\beta+1}{q^*} < 1$$

by using a suitable application of the variational method due to Ambrosetti-Rabinowitz [2]. M. Bouchekif [4] generalized the work of F. de Thélin and J.Velin [9] for the large class of functions of the form

$$H(u,v) = a|u|^\gamma + c|v|^\delta + b|u|^{\alpha+1}|v|^{\beta+1}$$

where α, $\beta \geq 0$; γ, $\delta > 1$ and a, b and c are real numbers. The case where the system (1.1) is governed by a single operator Δ_p has been studied by Baghli [3].

Our aim is to extend to the system (1.1) the results obtained in the scalar case (see [5]). Our existence results follow from modified quasilinear system in order to apply the Palais-Smale condition (P.S.) and then the Moser's Iterative Scheme as in T. Ôtani [6] or in F. de Thélin and J. Vélin [9]. We consider only weak solutions, and assume that H satisfies the following hypothesis.

(H1) $H \in C^1(\mathbb{R}^+ \times \mathbb{R}^+)$

(H2) $H(u,v) = o(u^p + v^q)$ as $(u,v) \to (0^+, 0^+)$

(H3) There exists $R_0 > 0$ and μ, $1 < \mu < \min(p^*/p, q^*/q)$, such that

$$\frac{u}{p}\frac{\partial H}{\partial u}(u,v) + \frac{v}{q}\frac{\partial H}{\partial v}(u,v) \geq \mu H(u,v) > 0 \ \forall (u,v) \in \mathbb{R}_+^* \times \mathbb{R}_+^*, \ u^p + v^q \geq R_0.$$

2 Preliminaries and existence results

The values of $H(u,v)$ are irrelevant for $u \leq 0$ or $v \leq 0$. We set

$$I(u,v) = \frac{1}{p}\int_\Omega |\nabla u|^p dx + \frac{1}{q}\int_\Omega |\nabla v|^q dx - \int_\Omega m(x) H(u,v) dx$$

defined on $E := W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega)$. The solutions of the system (1.1) are critical points of the functional I. Note that the functional I does not satisfy in general the Palais-Smale condition since

$$B_\mu H(u,v) := \frac{u}{p}\frac{\partial H}{\partial u}(u,v) + \frac{v}{q}\frac{\partial H}{\partial v}(u,v) - \mu H(u,v)$$

is not always bounded. In order to apply Ambrosetti-Rabinowitz Theorem [2], we modify H so that the corresponding $B_\mu H(u,v)$ becomes bounded. Let

$$A(R) = \max\Big\{\frac{H(u,v)}{(u^p+v^q)^\mu} : R \le u^p+v^q \le R+1\Big\}$$

and

$$\begin{aligned} C_R = \max\Big\{ &\sup_{u^p+v^q\le R+1} \big|\frac{\partial H}{\partial u}(u,v)\big| + 2p\mu A(R)(R+1)^{\mu+1-\frac{1}{p}} \sup_{R\le r\le R+1} |\eta'_R(r)|;\\ &\sup_{u^p+v^q\le R+1} \big|\frac{\partial H}{\partial v}(u,v)\big| + 2q\mu A(R)(R+1)^{\mu+1-\frac{1}{q}} \sup_{R\le r\le R+1} |\eta'_R(r)|\Big\} \end{aligned}$$

where $\eta_R \in C^1(\mathbb{R})$ is a cutting function defined by

$$\eta_R(r) \begin{cases} = 1 & \text{if } r \le R \\ < 0 & \text{if } R < r < R+1 \\ = 0 & \text{if } r \ge R+1. \end{cases}$$

Our main result is the following:

Theorem 2.1 *Assume that $(H_i)_{i=1,2,3}$ hold and $C_R = o(R^{\frac{p^*q^*}{(p^*-p)(q^*-q)}\mu})$ for R sufficiently large. Then the system (1.1) has at least one nontrivial solution (u,v) in $E \cap [L^\infty(\Omega)]^2$ with u and v positive.*

Before proving this theorem, we truncate the potential function H.

The modified problem

Let $R \ge R_0$ be fixed, and set

$$H_R(u,v) := \eta_R(u^p+v^q)H(u,v) + (1-\eta_R(u^p+v^q))A(R)(u^p+v^q)^\mu,$$

By construction H_R is C^1 and nonnegative. Let

$$\begin{aligned} M_R :=& (R+1)\max_{u^p+v^q\le R+1}\big[\eta'_R(u^p+v^q)(H(u,v)-A(R)(u^p+v^q)^\mu)\big]\\ &+ \max_{u^p+v^q\le R+1} B_\mu H(u,v), \end{aligned}$$

Lemma 2.2 *H_R satisfies (H1)-(H3) and the following estimates*

$$0 \le B_\mu H_R(u,v) \le M_R, \quad \forall (u,v) \in \mathbb{R}^+ \times \mathbb{R}^+, \tag{2.1}$$

$$\begin{gathered} \big|\frac{\partial H_R}{\partial u}(u,v)\big| \le C_R + \mu p A(R) u^{p-1}(u^p+v^q)^{\mu-1},\\ \big|\frac{\partial H_R}{\partial v}(u,v)\big| \le C_R + \mu q A(R) v^{q-1}(u^p+v^q)^{\mu-1}, \quad \forall (u,v) \in \mathbb{R}^2_+, \end{gathered} \tag{2.2}$$

$$H_R(u,v) \ge \frac{m_{R_0}}{R_0^\mu}(u^p+v^q)^\mu \quad \forall (u,v) \in \mathbb{R}^*_+ \times \mathbb{R}^*_+, \text{ such that } u^p+v^q \ge R_0, \tag{2.3}$$

with $m_{R_0} := \min\{H(u,v); u^p+v^q = R_0\}$.

Proof. (H1) and (H2) can be easly verified for H_R. We verify for (H3) as follows: For any $\nu > 1$,we have

$$
\begin{aligned}
&B_\nu H_R(u,v)\\
&= (u^p+v^q)\eta_R'(u^p+v^q)[H(u,v)-A(R)(u^p+v^q)^\mu]+\eta_R(u^p+v^q)B_\nu H(u,v),
\end{aligned}
$$

for $R_0 \le u^p+v^q \le R$;

$$
B_\nu H_R(u,v) = B_\nu H(u,v) \ge B_\mu H(u,v) \ge 0 \quad \text{for } 1<\nu\le\mu
$$

for $R \le u^p+v^q \le R+1$;

$$
B_\nu H_R(u,v) \ge \eta_R(u^p+v^q)B_\nu H(u,v) \ge \eta_R(u^p+v^q)B_\mu H(u,v) \ge 0 \text{ for } 1<\nu\le\mu;
$$

finally for $u^p+v^q \ge R+1$, $B_\nu H_R(u,v)=0$ for any $\nu>1$. Thus (H3) holds for H_R.

Conditions (2.1) and (2.2) result from straightforward computations. Using (H3), we have

$$
H_R(u,v) \ge \frac{m_{R_0}}{R_0^\mu}(u^p+v^q)^\mu \quad ,\forall (u,v)\in \mathbb{R}_+^*\times\mathbb{R}_+^* \text{ such that } u^p+v^q \ge R_0. \tag{2.4}
$$

In fact, put $f(t) := H_R(t^{1/p}u, t^{\frac{1}{q}}v)$ with $u^p+v^q \ge R_0$ then

$$
\begin{aligned}
f'(t) =& \frac{1}{t}\Big[\frac{t^{1/p}u}{p}\frac{\partial H_R}{\partial u}(t^{1/p}u, t^{\frac{1}{q}}v) + \frac{t^{\frac{1}{q}}v}{q}\frac{\partial H_R}{\partial v}(t^{1/p}u, t^{\frac{1}{q}}v)\Big]\\
\ge& \frac{\mu}{t}f(t) \quad \text{for all } t \ge t_0 := \frac{R_0}{u^p+v^q}(\le 1).
\end{aligned} \tag{2.5}
$$

Integrating (2.5) between t_0 and t, we obtain

$$
\frac{f(t)}{f(t_0)} \ge \frac{t^\mu}{t_0^\mu} \quad \text{for all} \quad t \ge t_0 \tag{2.6}
$$

and taking $t=1$ in (2.6), we have

$$
H_R(u,v) = f(1) \ge \frac{(u^p+v^q)^\mu}{R_0^\mu} f(t_0)
$$

and $f(t_0) = H_R(u_1,v_1) = H(u_1,v_1)$, where $u_1 = (\frac{R_0}{u^p+v^q})^{1/p}u$, and $v_1 = (\frac{R_0}{u^p+v^q})^{1/q}v$. Consequently,

$$
\min_{u^p+v^q\ge R_0} f(t_0(u,v)) = \min_{u^p+v^q=R_0} H(u,v),
$$

hence (2.4) follows. Now, consider the modified system

$$
\begin{gathered}
-\Delta_p u = m(x)\frac{\partial H_R}{\partial u}(u,v) \quad \text{in } \Omega\\
-\Delta_q v = m(x)\frac{\partial H_R}{\partial v}(u,v) \quad \text{in } \Omega\\
u=v=0 \quad \text{on } \partial\Omega
\end{gathered} \tag{2.7}
$$

which has an associated functional I_R defined on E as

$$I_R(u,v)=\frac{1}{p}\int_\Omega |\nabla u|^p dx+\frac{1}{q}\int_\Omega |\nabla v|^q dx-\int_\Omega m(x)H_R(u,v)dx.$$

Lemma 2.3 *Under the hypotheses (H1)-(H3), the functional I_R satisfies the Palais-Smale condition.*

Proof. Let (u_n,v_n) be an element of E such that $I_R(u_n,v_n)$ is bounded and $I'_R(u_n,v_n)\to 0$ strongly in $W_0^{-1,p'}(\Omega)\times W_0^{-1,q'}(\Omega)$ (dual space of E).
Claim 1. (u_n,v_n) is bounded in E. In fact, for any M, we have

$$-M\le \frac{1}{p}\int_\Omega |\nabla u_n|^p dx+\frac{1}{q}\int_\Omega |\nabla v_n|^q dx-\int_\Omega m(x)H_R(u_n,v_n)dx\le M;$$

and for $\varepsilon\in(0,1)$, we have again

$$\begin{aligned}-\varepsilon \quad\le\quad & \frac{1}{p}\int_\Omega |\nabla u_n|^p dx+\frac{1}{q}\int_\Omega |\nabla v_n|^q dx\\ & -\int_\Omega m(x)\big[\frac{u_n}{p}\frac{\partial H_R}{\partial u}(u_n,v_n)+\frac{v_n}{q}\frac{\partial H_R}{\partial v}(u_n,v_n)\big]dx\le\varepsilon.\end{aligned}$$

Then we obtain

$$\begin{aligned}\frac{\mu-1}{p}\int_\Omega |\nabla u_n|^p dx+\frac{\mu-1}{q}\int_\Omega |\nabla v_n|^q dx &\le M\mu-\int_\Omega m(x)B_\mu H_R(u,v)dx\\ &\le M\mu+1+|m|_0M_R(\operatorname{meas}\Omega)\end{aligned}$$

where $|m|_0:=\max_{x\in\bar\Omega}(|m(x)|)$. Hence (u_n,v_n) is bounded in E.
Claim 2. (u_n,v_n) converges strongly in E. Since (u_n,v_n) is bounded in E, there exists a subsequence denoted again by (u_n,v_n) which converges weakly in E and strongly in the space $L^\zeta(\Omega)\times L^\eta(\Omega)$ for any ζ and η such that, $1<\zeta<p^*$ and $1<\eta<q^*$. ¿From the definition of I'_R, we write

$$\begin{aligned}&\int_\Omega (|\nabla u_n|^{p-2}\nabla u_n-|\nabla u_l|^{p-2}\nabla u_l)\nabla(u_n-u_l)dx\\ &=\langle I'_R(u_n,v_n)-I'_R(u_l,v_l),(u_n-u_l,0)\rangle\\ &\quad+\int_\Omega m(x)\Big[\frac{\partial H_R}{\partial u}(u_n,v_n)-\frac{\partial H_R}{\partial u}(u_l,v_l)\Big](u_n-u_l)dx.\end{aligned}$$

By assumptions on I'_R, $\langle I'_R(u_n,v_n)-I'_R(u_l,v_l),(u_n-u_l,0)\rangle$ converges to 0 as n and l tend to $+\infty$. In what follows, C denotes a generic positive constant. Now, we prove that

$$C_{n,l}:=\int_\Omega m(x)[\frac{\partial H_R}{\partial u}(u_n,v_n)-\frac{\partial H_R}{\partial u}(u_l,v_l)](u_n-u_l)dx$$

converges to 0 as n and l tend to $+\infty$. We have

$$|C_{n,l}| \leq |m|_0 \int_\Omega [|\frac{\partial H_R}{\partial u}(u_n, v_n)| + |\frac{\partial H_R}{\partial u}(u_l, v_l)|]|u_n - u_l|dx$$

and

$$\begin{aligned}
&\int_\Omega |\frac{\partial H_R}{\partial u}(u_n, v_n)||u_n - u_l|dx \\
&\leq \int_\Omega (C_R + \mu p A(R)|u_n|^{p-1}(|u_n|^p + |v_n|^q)^{\mu-1})|u_n - u_l|dx \\
&\leq 2^{\mu-1}C_R \int_\Omega (1 + |u_n|^{\mu p-1} + |u_n|^{p-1}|v_n|^{q\mu-q})|u_n - u_l|dx \\
&\leq 2^{\mu-1}C_R \Big[\int_\Omega |u_n - u_l|dx + \int_\Omega |u_n|^{\mu p-1}|u_n - u_l|dx \\
&\quad + \int_\Omega |u_n|^{p-1}|v_n|^{q\mu-q}|u_n - u_l|dx\Big].
\end{aligned}$$

Using Hölder's inequality and Sobolev's embeddings, we obtain

$$\begin{aligned}
&\int_\Omega \Big|\frac{\partial H_R}{\partial u}(u_n, v_n)\Big||u_n - u_l|dx \\
&\leq 2^{\mu-1}C_R(\operatorname{meas}\Omega)^{\frac{p-1}{p}}\Big[\int_\Omega |u_n - u_l|^p dx\Big]^{1/p} \\
&\quad + 2^{\mu-1}C_R\Big[\int_\Omega |u_n|^{\mu p}dx\Big]^{\frac{\mu p-1}{\mu p}}\Big[\int_\Omega |u_n - u_l|^{\mu p}dx\Big]^{\frac{1}{\mu p}} \\
&\quad + 2^{\mu-1}C_R\Big[\int_\Omega |u_n|^{\mu p}dx\Big]^{\frac{p-1}{\mu p}}\Big[\int_\Omega |v_n|^{\mu q}dx\Big]^{\frac{\mu-1}{\mu}}\Big[\int_\Omega |u_n - u_l|^{\mu p}dx\Big]^{\frac{1}{\mu p}},
\end{aligned}$$

(because $(u_n) \in W_0^{1,p}(\Omega)$ and $\mu p < p^*$, $(v_n) \in W_0^{1,q}(\Omega)$ and $\mu q < q^*$). Then

$$\int_\Omega \Big|\frac{\partial H_R}{\partial u}(u_n, v_n)\Big||u_n - u_l|dx \leq C\|u_n - u_l\|_{L^p(\Omega)} + C\|u_n - u_l\|_{L^{\mu p}(\Omega)}.$$

Similarly, we obtain

$$\int_\Omega |\frac{\partial H_R}{\partial u}(u_l, v_l)||u_n - u_l|dx \leq C\|u_n - u_l\|_{L^p(\Omega)} + C\|u_n - u_l\|_{L^{\mu p}(\Omega)},$$

and so $|C_{n,l}| \leq |m|_0(C\|u_n - u_l\|_{L^p} + C\|u_n - u_l\|_{L^{\mu p}})$. Hence $C_{n,l}$ converges to 0 as n and l tend to $+\infty$.

We have the following algebraic relation [8]

$$\begin{aligned}
&|\nabla u_n - \nabla u_l|^p \\
&\leq C\big[(|\nabla u_n|^{p-2}\nabla u_n - |\nabla u_l|^{p-2}\nabla u_l)\nabla(u_n - u_l)\big]^{s/2}\big(|\nabla u_n|^p + |\nabla u_l|^p\big)^{1-\frac{s}{2}},
\end{aligned} \tag{2.8}$$

where $s = \begin{cases} p & \text{for } 1 < p \leq 2 \\ 2 & \text{for } 2 < p \end{cases}$. Integrating (2.8) on Ω, and using Hölder's inequality in the right hand side, we obtain

$$\begin{aligned} &\|u_n - u_l\|_{1,p}^p \\ &\leq C\Big[\int_\Omega (|\nabla u_n|^{p-2}\nabla u_n - |\nabla u_l|^{p-2}\nabla u_l)\nabla(u_n - u_l)dx\Big]^{\frac{s}{2}}\big(\|u_n\|_{1,p}^p + \|u_l\|_{1,p}^p\big)^{1-\frac{s}{2}}. \end{aligned}$$

Now since

$$\int_\Omega (|\nabla u_n|^{p-2}\nabla u_n - |\nabla u_l|^{p-2}\nabla u_l)\nabla(u_n - u_l)dx \to 0$$

as n and l tend to $+\infty$, the sequence (u_n) converges strongly in $W_0^{1,p}(\Omega)$. Similarly we prove that the sequence (v_n) converges strongly in $W_0^{1,q}(\Omega)$.

The next lemma shows that I_R satisfies the geometric assumptions of the Mountain-Pass Theorem.

Proposition 2.4 *Under assumptions (H1)-(H3) we have*

1. *There exist two positive real numbers ρ, σ and a neighborhood V_ρ of the origin of E, such that for any element (u, v) on the boundary of V_ρ: $I_R(u, v) \geq \sigma > 0$.*
2. *There exist (ϕ, θ) in E such that $I_R(\phi, \theta) < 0$.*

Proof. From (H2) and taking into account that $H_R(u, v) = H(u, v)$ for $u^p + v^q \leq R$, we can write

$$\forall \varepsilon > 0, \exists \delta_\varepsilon > 0 : u^p + v^q \leq \delta_\varepsilon \Longrightarrow H_R(u, v) \leq \varepsilon(u^p + v^q),$$

and since $H_R(u, v)/(u^p + v^q)^\mu$ is uniformly bounded as $u^p + v^q$ tends to $+\infty$

$$\exists M(\varepsilon, R) > 0 : u^p + v^q \geq \delta_\varepsilon \Longrightarrow H_R(u, v) \leq M(u^p + v^q)^\mu.$$

Then for every (u, v) in $\mathbb{R}^+ \times \mathbb{R}^+$ we have

$$H_R(u, v) \leq \varepsilon(u^p + v^q) + M(u^p + v^q)^\mu.$$

Hence

$$\begin{aligned} &\int_\Omega m(x)H_R(u, v)dx \\ &\leq |m|_0\Big[\varepsilon\int_\Omega (u^p + v^q)dx + M\int_\Omega (u^p + v^q)^\mu dx\Big] \\ &\leq |m|_0\Big[\int_\Omega (\varepsilon u^p + 2^{\mu-1}Mu^{p\mu})dx + \int_\Omega (\varepsilon v^q + 2^{\mu-1}Mv^{q\mu})dx\Big] \\ &\leq C|m|_0\big[\varepsilon(\|u\|_{1,p}^p + \|v\|_{1,q}^q) + M(\|u\|_{1,p}^{\mu p} + \|v\|_{1,q}^{\mu q})\big]. \end{aligned}$$

For $I_R(u,v)$, we obtain

$$\begin{aligned} I_R(u,v) &\geq \|u\|_{1,p}^p\big[\frac{1}{p} - C|m|_0(\varepsilon + M\|u\|_{1,p}^{\mu p-p})\big] \\ &\quad + \|v\|_{1,q}^q\big[\frac{1}{q} - C|m|_0(\varepsilon + M\|v\|_{1,q}^{\mu q-q})\big] \geq \sigma > 0, \end{aligned}$$

for every (u,v) in the sphere $S(0,\rho)$ of E where ρ is such that $0 < \rho < \min(\rho_1,\rho_2)$ with

$$\rho_1 = \big[\frac{1}{pMC|m|_0} - \frac{\varepsilon}{M}\big]^{\frac{1}{\mu p-p}} \quad \text{and} \quad \rho_2 = \big[\frac{1}{qMC|m|_0} - \frac{\varepsilon}{M}\big]^{\frac{1}{\mu q-q}}$$

with ε sufficiently small.

2. Choose $(\phi,\theta) \in E$ such that: $\phi > 0$, $\theta > 0$,

$$\operatorname{supp}\phi \subset \Omega^+, \quad \operatorname{supp}\theta \subset \Omega^+,$$

where $\Omega^+ = \{x \in \Omega; m(x) > 0\}$. Hence, for t sufficiently large,

$$\begin{aligned} I_R(t^{1/p}\phi, t^{1/q}\theta) &= \frac{t}{p}\|\phi\|_{1,p}^p + \frac{t}{q}\|\theta\|_{1,q}^q - \int_\Omega m(x)H_R(t^{1/p}\phi, t^{1/q}\theta)dx \\ &\leq t\big[\frac{\|\phi\|_{1,p}^p}{p} + \frac{\|\theta\|_{1,q}^q}{q}\big] - t^\mu \frac{m_{R_0}}{R_0^\mu}\int_\Omega m(x)(\phi^p + \theta^q)^\mu dx \end{aligned}$$

and so $\lim_{t\to+\infty} I_R(t^{1/p}\phi, t^{1/q}\theta) = -\infty$, (because $\mu > 1$). By continuity of I_R on E, there exists (ϕ,θ) in $E \setminus B(0,\rho)$ such that $I_R(\phi,\theta) < 0$. By the usual Mountain-Pass Theorem, we know that there exists a critical point of I_R which we denote by (u_R, v_R), and corresponding to a critical value $c_R \geq \sigma$. Since (u_R^+, v_R^+), where $u_R^+ := \max(u_R, 0)$, is also solution for the system $(S_{p,q}^{H_R})$, we assume $u_R \geq 0$ and $v_R \geq 0$. Positivity of u_R and v_R follows from Harnack's inequality (see J. Serrin [7]). We prove now that, under some conditions, (u_R, v_R) is also solution of the system (2.7).

3 Existence results

We adapt the Moser iteration used in [6, 9] to construct two strictly unbounded sequences $(\lambda_k)_{k\in\mathbb{N}}$ and $(\mu_k)_{k\in\mathbb{N}}$ such that (u_R, v_R) satisfies

$$\text{if } \left\{ \begin{array}{l} u_R \in L^{\lambda_k}(\Omega) \\ v_R \in L^{\mu_k}(\Omega) \end{array} \right\} \quad \text{then} \quad \left\{ \begin{array}{l} u_R \in L^{\lambda_{k+1}}(\Omega) \\ v_R \in L^{\mu_{k+1}}(\Omega). \end{array} \right\}$$

Bootstrap argument

Proposition 3.1 *Under the assumptions of Theorem 2.1, there exist two sequences $(\lambda_k)_k$ and $(\mu_k)_k$ such that*

1. *For each k, u_R and v_R belong to $L^{\lambda_k}(\Omega)$ and $L^{\mu_k}(\Omega)$ respectively*

2. *There exist two positive constants C_p and C_q such that*
$$\|u_R\|_\infty \le \limsup_{k\to+\infty}\|u_R\|_{L^{\lambda_k}} \le C_p, \quad \textit{and} \quad \|v_R\|_\infty \le \limsup_{k\to+\infty}\|v_R\|_{L^{\mu_k}} \le C_q.$$

Lemma 3.2 *Let $(a_k)_{k\in\mathbb{N}}$ and $(b_k)_{k\in\mathbb{N}}$ be two positive sequences satisfying, for each integer k, the relations*
$$\frac{p+a_k}{\lambda_k} + \frac{q(\mu-1)}{\mu_k} = 1, \quad \textit{and} \quad \frac{q+b_k}{\mu_k} + \frac{p(\mu-1)}{\lambda_k} = 1. \tag{3.1}$$
If u_R and v_R are in $L^{\lambda_k}(\Omega)$ and $L^{\mu_k}(\Omega)$ respectively, $\lambda_{k+1} \le (1+\frac{a_k}{p})\pi_p$, $\mu_{k+1} \le (1+\frac{b_k}{q})\pi_q$ with $1<\pi_p<p^*$ and $1<\pi_q<q^*$, then we have:
$$\|u_R\|_{\lambda_{k+1}}^{\lambda_{k+1}} \le K_p\Big\{\theta_p\big[1+\frac{a_k}{p}\big]\big[C_R|m|_0(\|u_R\|_{\lambda_k}^{\lambda_k}+\|v_R\|_{\mu_k}^{\mu_k})\big]^{1/p}\Big\}^{\frac{\lambda_{k+1}}{1+\frac{a_k}{p}}}, \tag{3.2}$$
$$\|v_R\|_{\mu_{k+1}}^{\mu_{k+1}} \le K_q\Big\{\theta_q\big[1+\frac{b_k}{q}\big]\big[C_R|m|_0(\|u_R\|_{\lambda_k}^{\lambda_k}+\|v_R\|_{\mu_k}^{\mu_k})\big]^{\frac{1}{q}}\Big\}^{\frac{\mu_{k+1}}{1+\frac{b_k}{q}}} \tag{3.3}$$
where $\|z\|_\beta$ is $\|z\|_{L^\beta(\Omega)}$ and K_p, K_q, θ_p, and θ_q are positive constants.

Proof. Remark that if, for an infinite number of integers k, $\|u_R\|_{\lambda_k} \le 1$ then $\|u_R\|_\infty \le 1$ and proposition 1 is proved. So we suppose that $\|u_R\|_{\lambda_k} \ge 1$ for all $k\in\mathbb{N}$. Let ζ_n, $n\in\mathbb{N}$, be C^1 functions such that
$$\begin{aligned} &\zeta_n(s) = s && \text{if } s\le n\\ &\zeta_n(s) = n+1 && \text{if } s \ge n+2\\ &0<\zeta_n'(s)<1 && \text{if } s\in\mathbb{R}^+. \end{aligned}$$
Put $u_n := \zeta_n(u_R)$, then $u_n^{1+a_k} \in W_0^{1,p}(\Omega)\cap L^\infty(\Omega)$ and u_R satisfies the first equation of the system (2.7). Multiply this equation by $u_n^{1+a_k}$ and integrate over Ω to get
$$\begin{aligned} \int_\Omega -\Delta_p u_R.u_n^{1+a_k}dx &= \int_\Omega m(x)\frac{\partial H_R}{\partial u}(u_R,v_R)u_n^{1+a_k}dx\\ &\le 2^{\mu-1}C_R|m|_0\int_\Omega(1+u_R^{p\mu-1}+u_R^{p-1}v_R^{q\mu-q})u_n^{1+a_k}dx. \end{aligned}$$
Since $u_n\le u_R$, we have
$$\begin{aligned} &\int_\Omega -\Delta_p u_R.u_n^{1+a_k}dx\\ &\le 2^{\mu-1}C_R|m|_0\Big\{\int_\Omega u_R^{1+a_k}dx + \int_\Omega u_R^{p\mu+a_k}dx + \int_\Omega u_R^{p+a_k}v_R^{q\mu-q}dx\Big\}. \end{aligned}$$
Using Hölder's inequality, we obtain
$$\begin{aligned} \int_\Omega -\Delta_p u_R.u_n^{1+a_k}dx \le\ & 2^{\mu-1}C_R|m|_0\Big\{(\operatorname{meas}\Omega)^{1-\frac{1+a_k}{\lambda_k}}\|u_R\|_{\lambda_k}^{1+a_k} + \|u_R\|_{p\mu+a_k}^{p\mu+a_k}\\ &+\|u_R\|_{\lambda_k}^{p+a_k}\|v_R\|_{\mu_k}^{q\mu-q}\Big\}. \end{aligned}$$

We shall show below that $p\mu + a_k = \lambda_k$. Since $\|u_R\|_{\lambda_k} \geq 1$, we get

$$\begin{aligned}
&\int_\Omega -\Delta_p u_R . u_n^{1+a_k}\,dx\\
&\quad \leq 2^{\mu-1} C_R |m|_0 \max(1,\,\mathrm{meas}\,\Omega)\big[2\|u_R\|_{\lambda_k}^{\lambda_k} + \|u_R\|_{\lambda_k}^{p+a_k}\|v_R\|_{\mu_k}^{q\mu-q}\big].
\end{aligned}$$

Moreover, using the relation (3.1), we obtain

$$\|u_R\|_{\lambda_k}^{p+a_k}\|v_R\|_{\mu_k}^{q\mu-q} \leq \|u_R\|_{\lambda_k}^{\lambda_k} + \|v_R\|_{\mu_k}^{\mu_k},$$

so, with $c_0 := 3\max(1, \mathrm{meas}\,\Omega)$,

$$\int_\Omega -\Delta_p u_R . u_n^{1+a_k}\,dx \leq 2^{\mu-1} c_0 C_R |m|_0 \big[\|u_R\|_{\lambda_k}^{\lambda_k} + \|v_R\|_{\mu_k}^{\mu_k}\big]. \tag{3.4}$$

On the other hand we have

$$\begin{aligned}
\int_\Omega -\Delta_p u_R . u_n^{1+a_k}\,dx &= (1+a_k)\int_\Omega |\nabla u_R|^p \zeta_n'(u_R) u_n^{a_k}\,dx\\
&\geq (1+a_k)\int_\Omega |\nabla u_R|^p (\zeta_n'(u_R))^p u_n^{a_k}\,dx\\
&= (1+a_k)\int_\Omega |\nabla u_n|^p u_n^{a_k}\,dx
\end{aligned}$$

and thus

$$\int_\Omega -\Delta_p u_R . u_n^{1+a_k}\,dx \geq \int_\Omega |\nabla u_n|^p u_n^{a_k}\,dx\,. \tag{3.5}$$

Since $u_n^{1+\frac{a_k}{p}} \in W_0^{1,p}(\Omega)$, the continuous imbedding of $W_0^{1,p}(\Omega)$ in $L^{\pi_p}(\Omega)$ implies the existence of a positive constant c such that

$$\begin{aligned}
\Big(\int_\Omega |\,u_n^{1+\frac{a_k}{p}\pi_p}|dx\Big)^{\frac{1}{\pi_p}} &\leq c\Big(\int_\Omega |\nabla u_n^{1+\frac{a_k}{p}}|^p dx\Big)^{1/p}\\
&= c\big[1+\frac{a_k}{p}\big]\Big(\int_\Omega u_n^{a_k}|\nabla u_n|^p dx\Big)^{1/p}.
\end{aligned} \tag{3.6}$$

By assumption, we have $\lambda_{k+1} \leq j_k := \big[1+\frac{a_k}{p}\big]\pi_p$. Then

$$\|u_n\|_{\lambda_{k+1}} \leq (\mathrm{meas}\,\Omega)^{m_k}\|u_n\|_{j_k}, \quad \text{where } m_k := \frac{1}{\lambda_{k+1}} - \frac{1}{(1+\frac{a_k}{p})\pi_p}$$

and thus

$$\|u_n\|_{\lambda_{k+1}}^{\lambda_{k+1}} \leq K_p \|u_n\|_{j_k}^{\lambda_{k+1}}$$

where K_p is a positive constant greater than $(\mathrm{meas}\,\Omega)^{m_k\lambda_{k+1}}$ independently of the integer k. By the relation (3.6),

$$\|u_n\|_{j_k}^{\lambda_{k+1}} \leq \Big[c[1+\frac{a_k}{p}]\Big(\int_\Omega u_n^{a_k}|\nabla u_n|^p dx\Big)^{1/p}\Big]^{\frac{\lambda_{k+1}}{1+\frac{a_k}{p}}}. \tag{3.7}$$

Combining the inequalities (3.4)-(3.7), we deduce

$$\|u_n\|_{\lambda_{k+1}}^{\lambda_{k+1}} \le K_p \Big\{\theta_p\big[1+\frac{a_k}{p}\big]\big\{C_R|m|_0(\|u_R\|_{\lambda_k}^{\lambda_k}+\|v_R\|_{\mu_k}^{\mu_k})\big\}^{1/p}\Big\}^{\frac{\lambda_{k+1}}{1+\frac{a_k}{p}}},$$

with $\theta_p = 2^{\frac{\mu-1}{p}} c_0^{1/p} c$. Hence, by letting $n \to +\infty$, we obtain (3.2). Similarly we show (3.3).

Construction and definition of $(\lambda_k)_k$ and $(\mu_k)_k$.

Here we construct the sequences $(\lambda_k)_k$ and $(\mu_k)_k$ using tools similar as those in [O] or [TV]. The first terms of each sequence cannot be determined directly by using the Rellich-Kondrachov continuous imbedding result. So, we first construct two other sequences $(\widehat{\lambda}_k)_k$ and $(\widehat{\mu}_k)_k$, such that for each k, u_R and v_R belong to $L^{\widehat{\lambda}_k}(\Omega)$ and $L^{\widehat{\mu}_k}(\Omega)$ respectively. By a suitable choice of k_0, $\widehat{\lambda}_{k_0}$ and $\widehat{\mu}_{k_0}$ determine the first terms of $(\lambda_k)_k$ and $(\mu_k)_k$

Construction of $(\widehat{\lambda}_k)_k$ and $(\widehat{\mu}_k)_k$.

Suppose $p \le q$, and fix a number s, such that $cp/p^* < s < 1/\mu$. Put

$$\widehat{C} := \frac{1}{2} + \frac{s}{2}\frac{p^*}{p}.$$

Remark that $\widehat{C} > 1$, $1 < \mu p\widehat{C} < p^*$ and $1 < \mu q\widehat{C} < q^*$. Now, we take $\widehat{\lambda}_k = \mu p\widehat{C}^k$ and $\widehat{\mu}_k = \mu q\widehat{C}^k$. By definition of (a_k), we have

$$\frac{p+a_k}{\widehat{\lambda}_k} + \frac{\mu-1}{\widehat{\mu}_k}q = 1$$

then $a_k = \widehat{\lambda}_k - p\mu$. Similarly, we find $b_k = \widehat{\mu}_k - q\mu$.

Lemma 3.3 *For each integer k, $u_R \in L^{\widehat{\lambda}_k}(\Omega)$ and $v_R \in L^{\widehat{\mu}_k}(\Omega)$.*

Proof. By induction. For $k = 0$, $\widehat{\lambda}_0 = \mu p < p^*$, $\widehat{\mu}_0 = \mu q < q^*$, and since $(u_R, v_R) \in E$, by the Sobolev imbedding theorem, we have $u_R \in L^{\widehat{\lambda}_0}(\Omega)$ and $v_R \in L^{\widehat{\mu}_0}(\Omega)$.

Suppose that the proposition is true for all integers k' such that $0 \le k' \le k$. Take

$$\pi_p = \mu p\widehat{C} \quad \text{and} \quad \pi_q = \mu q\widehat{C}.$$

Since $u_R \in L^{\widehat{\lambda}_k}(\Omega)$ and

$$[1+\frac{a_k}{p}]\pi_p = \big[1+\frac{\widehat{\lambda}_k-\mu p}{p}\big]\mu p\widehat{C} = \mu p\widehat{C} + \mu^2 p\widehat{C}^{k+1} - \mu^2 p\widehat{C} \ge \mu p\widehat{C}^{k+1}$$

i.e. $[1+\frac{a_k}{p}]\pi_p \ge \widehat{\lambda}_{k+1}$, Lemma 3 allows us to write $u_R \in L^{\widehat{\lambda}_{k+1}}(\Omega)$ and $v_R \in L^{\widehat{\mu}_{k+1}}(\Omega)$.

Construction of $(\lambda_k)_k$ and $(\mu_k)_k$ Put

$$C = \frac{N}{N-p}, \quad \text{and} \quad \delta = \big[\frac{p}{N}\mu \widehat{C}^{k_0} - (\mu - 1)\big]C,$$

where the integer k_0 is chosen so as to have $\delta > 0$. The sequences $(\lambda_k)_k$ and $(\mu_k)_k$ are defined by $\lambda_k = pf_k$ and $\mu_k = qf_k$, where

$$f_k = \frac{C}{C-1}[\delta C^{k-1} + (\mu - 1)].$$

We remark that the three last sequences are strictly increasing and unbounded. Furthermore (f_k) satisfies the relation $f_{k+1} = C[f_k - (\mu - 1)]$.

Proof of Proposition 2. 1. We show by induction that for all integer k, $u_R \in L^{\lambda_k}(\Omega)$ and $v_R \in L^{\mu_k}(\Omega)$. For $k = 0$,

$$\lambda_0 = pf_0 = \frac{pC}{C-1}\big[\frac{\delta}{C} + (\mu - 1)\big] = p\frac{N}{p}\big[\frac{p}{N}\mu \widehat{C}^{k_0}\big] = \widehat{\lambda}_{k_0},$$

and similarly, $\mu_0 = \widehat{\mu}_{k_0}$.

By Lemma 4, $u_R \in L^{\lambda_0}(\Omega)$ and $v_R \in L^{\mu_0}(\Omega)$. Suppose that $(u_R, v_R) \in L^{\lambda_k}(\Omega) \times L^{\mu_k}(\Omega)$. First we establish that $\lambda_k = a_k + p\mu$. By condition (3.1),

$$1 = \frac{p + a_k}{\lambda_k} + q\frac{\mu - 1}{\mu_k} = \frac{p}{\lambda_k} - \frac{q}{\mu_k} + \frac{a_k}{\lambda_k} + \mu\frac{q}{\mu_k},$$

thus

$$\frac{a_k}{pf_k} + \frac{\mu}{f_k} = 1$$

which implies $a_k = p(f_k - \mu) = \lambda_k - p\mu$, and similarly $\mu_k = b_k + q\mu = q(f_k - \mu)$. Now when we take $\pi_p = Cp$ and $\pi_q = Cq$, we then have

$$\big[1 + \frac{a_k}{p}\big]\pi_p = (1 + f_k - \mu)Cp = pf_{k+1} = \lambda_{k+1}.$$

and similarly $[1 + \frac{b_k}{q}]\pi_q = \mu_{k+1}$. Since $(u_R, v_R) \in L^{\lambda_k}(\Omega) \times L^{\mu_k}(\Omega)$, we conclude, according to Lemma 3, that

$$(u_R, v_R) \in L^{\lambda_{k+1}}(\Omega) \times L^{\mu_{k+1}}(\Omega).$$

So $u_R \in L^{\lambda_k}(\Omega)$, and $v_R \in L^{\mu_k}(\Omega)$, for all integer k.
2. Now we prove that u_R and v_R are bounded. By Lemma 3, we have

$$\|u_R\|_{\lambda_{k+1}}^{\lambda_{k+1}} \le K_p\Big\{\theta_p\big[1 + \frac{a_k}{p}\big]\big\{C_R|m|_0(\|u_R\|_{\lambda_k}^{\lambda_k} + \|v_R\|_{\mu_k}^{\mu_k})\big\}^{1/p}\Big\}^{\frac{\lambda_{k+1}}{1+\frac{a_k}{p}}},$$

$$\|v_R\|_{\mu_{k+1}}^{\mu_{k+1}} \le K_q\Big\{\theta_q\big[1 + \frac{b_k}{q}\big]\big\{C_R|m|_0(\|u_R\|_{\lambda_k}^{\lambda_k} + \|v_R\|_{\mu_k}^{\mu_k})\big\}^{\frac{1}{q}}\Big\}^{\frac{\mu_{k+1}}{1+\frac{b_k}{q}}}.$$

We remark that
$$\frac{\lambda_{k+1}}{1+\frac{a_k}{p}} = pC \quad \text{and} \quad \frac{\mu_{k+1}}{1+\frac{b_k}{q}} = qC.$$
Consequently,
$$\|u_R\|_{\lambda_{k+1}}^{\lambda_{k+1}} \le 2^C K_p \theta_p^{pC}\big[1+\frac{a_k}{p}\big]_\infty^{pC}(|m|_0 C_R)^C \max\big(\|u_R\|_{\lambda_k}^{\lambda_k C}, \|v_R\|_{\mu_k}^{\mu_k C}\big),$$
$$\|v_R\|_{\mu_{k+1}}^{\mu_{k+1}} \le 2^C K_q \theta_q^{qC}\big[1+\frac{b_k}{q}\big]^{qC}(|m|_0 C_R)^C \max\big(\|u_R\|_{\lambda_k}^{\lambda_k C}, \|v_R\|_{\mu_k}^{\mu_k C}\big).$$
We have
$$1+\frac{a_k}{p} = 1+\frac{b_k}{q} = 1+f_k-\mu < \frac{C}{C-1}\big[\frac{\delta}{C}+\mu-1\big]C^k.$$
Take
$$A := \frac{C}{C-1}\big[\frac{\delta}{C}+\mu-1\big][K_p+K_q]$$
and $\theta := 2|m|_0 \max(\theta_p^p, \theta_q^q)$, then we can write
$$\max\Big(\|u_R\|_{\lambda_{k+1}}^{\lambda_{k+1}}, \|v_R\|_{\mu_{k+1}}^{\mu_{k+1}}\Big) \le (A^q\theta)^C C^{kqC} C_R^C \max\Big(\|u_R\|_{\lambda_k}^{C\lambda_k}, \|v_R\|_{\mu_k}^{C\mu_k}\Big).$$
We construct an iterative relation
$$E_{k+1} \le r_k + CE_k$$
where $E_k = \ln\max(\|u_R\|_{\lambda_k}^{\lambda_k}, \|v_R\|_{\mu_k}^{\mu_k})$, and $r_k = ak+b$, with $a = \ln C^{qC}$ and $b = \ln[A^q\theta C_R]^C$. Proceeding step by step, we find
$$\begin{aligned} E_{k+1} &\le r_k + Cr_{k-1} + C^2 r_{k-2} + \cdots + C^k r_0 + C^{k+1}E_{0,} \\ E_{k+1} &\le C^{k+1}E_0 + \sum_{i=0}^{k} C^i r_{k-i}. \end{aligned}$$
Let us evaluate
$$\sigma_k := \sum_{i=0}^{k} C^i r_{k-i}.$$
We have $r_{k-i} = a(k-i)+b = ak+b-ai$, then
$$\begin{aligned} \sigma_k &= (ak+b)\sum_{i=0}^{k} C^i - a\sum_{i=0}^{k} iC^i \\ &= \frac{bC^{k+2}+(a-b)C^{k+1}+(1-C)ak-[C(a+b)-b]}{(C-1)^2}. \end{aligned}$$
Since $C>1$, and a, b are positive, we have
$$\sigma_k \le \frac{bC^{k+2}+(a-b)C^{k+1}}{(C-1)^2}$$

then

$$E_{k+1} \le \frac{bC^{k+2}}{(C-1)^2} + C^{k+1}\big[\frac{a-b}{(C-1)^2} + E_0\big].$$

By an appropriate choice for the constants K_p and K_q, we ensure that

$$\frac{b-a}{(C-1)^2} \ge E_0.$$

Recall that

$$b - a = C \ln \frac{A^q \theta C_R}{C^q} \quad \text{with} \quad A = \frac{C}{C-1}\big[\frac{\delta}{C} + \mu - 1\big][K_p + K_q];$$

hence $E_{k+1} \le bC^{k+2}/(C-1)^2$. By the definition of E_{k+1} and the last inequality, we obtain

$$\lambda_{k+1} \ln \|u_R\|_{\lambda_{k+1}} \le E_{k+1} \le \frac{bC^{k+2}}{(C-1)^2},$$

thus

$$\ln \|u_R\|_{\lambda_{k+1}} \le \frac{bC^{k+2}}{\lambda_{k+1}(C-1)^2}.$$

Letting $k \to +\infty$, we find

$$\ln \|u_R\|_\infty \le \frac{bC}{p\delta(C-1)}, \quad \text{or} \quad \ln \|u_R\|_\infty \le \frac{N}{\delta p^2} b.$$

Similarly

$$\ln \|v_R\|_\infty \le \frac{N}{\delta q^2} b.$$

We deduce the existence of constants C_p and C_q such that:

$$\|u_R\|_\infty \le C_p \quad \text{and} \quad \|v_R\|_\infty \le C_q.$$

Take

$$C_p = \exp \frac{N}{\delta p^2} b, \quad \text{and} \quad C_q = \exp \frac{N}{\delta q^2} b.$$

Then C_p and C_q, are greater than 1, which is compatible with the remark noted at the beginning of the proof of Lemma 3. This completes the proof of proposition 1.

Proof of Theorem 2.1. If $\|u_R\|_\infty^p + \|v_R\|_\infty^q < R$, then (u_R, v_R) furnishes a solution of the system (1.1). We have

$$\|u_R\|_\infty^p + \|v_R\|_\infty^q \le C_p^p + C_q^q \le 2 \exp \frac{N}{\delta p} b;$$

so it is sufficient to have $2 \exp \frac{N}{\delta p} b < R$ for R large enough, to get (u_R, v_R) solution of the initial system (1.1). Replacing b by its expression, we obtain $(A^q \theta C_R)^{\frac{CN}{\delta p}} < \frac{R}{2}$ i.e.

$$C_R < \frac{R^{\frac{\delta p}{CN}}}{2^{\frac{\delta p}{CN}} \theta A^q}.$$

But δ can be chosen such that

$$\frac{\delta p}{CN} > \frac{N^2}{pq}\mu = \frac{p^*q^*}{(p^*-p)(q^*-q)}\ \mu$$

and we can take $C_R < \frac{R^{\frac{p^*q^*}{(p^*-p)(q^*-q)}\ \mu}}{2^{\frac{\delta p}{CN}}\theta A^q}$. Then (u_R, v_R) is solution of system (1.1) if

$$C_R = o\Big(R^{\frac{p^*q^*}{(p^*-p)(q^*-q)}\mu}\Big)$$

for R sufficiently large.

Examples

Now, we present functions satisfying the hypotheses in our main result.

For $1 < \gamma < \min(\frac{p^*}{p}, \frac{q^*}{q})$, let

$$H(u,v) = (u^p + v^q)^\gamma$$

be defined on $\mathbb{R}^2_+$. Then H satisfies the hypotheses of Theorem 2.1.

For $\alpha, \beta \geq 0$, $\frac{\alpha+1}{p} + \frac{\beta+1}{q} > 1$ and $\frac{\alpha+1}{p^*} + \frac{\beta+1}{q^*} < 1$, let

$$H(u,v) = u^{\alpha+1}v^{\beta+1}\,.$$

be defined on $\mathbb{R}^2_+$. Then H satisfies the hypotheses of Theorem 2.1.

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