



The finiteness conjecture for 3×3 binary matrices

Thomas Mejsstrik^a

Abstract

The invariant polytope algorithm was a breakthrough in the joint spectral radius computation, allowing to find the exact value of the joint spectral radius for most matrix families [7, 8]. This algorithm found many applications in problems of functional analysis, approximation theory, combinatorics, etc..

In this paper we propose a modification of the invariant polytope algorithm enlarging the class of problems to which it is applicable. Precisely, we introduce mixed numeric and symbolic computations. A further minor modification of augmenting the input set with additional matrices speeds up the algorithm in certain cases.

With this modifications we are able to automatically prove the finiteness conjecture for all pairs of binary 3×3 matrices and sign 2×2 matrices.

1 Introduction

In this paper we are concerned with the maximal asymptotic growth rate of products of matrices, the so called *joint spectral radius*. It has been defined in 1960 [18] and since found applications in many seemingly unconnected areas of mathematics and engineering, e.g. for computing the regularity of wavelets and of subdivision schemes [4], the capacity of codes [16], the stability of linear switched systems [6].

Definition 1.1. Given a finite set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$. The *joint spectral radius* (JSR) of \mathcal{A} is defined as

$$\text{JSR}(\mathcal{A}) := \lim_{n \rightarrow \infty} \max_{A_j \in \mathcal{A}} \|A_{j_n} \cdots A_{j_1}\|^{1/n}, \quad (1)$$

where $\|\cdot\|$ is any sub-multiplicative matrix norm.

An open question in the joint spectral radius theory is the so called *finiteness conjecture* [13]:

Given a matrix set, does there exist a finite product whose powers' spectral radii attain the growth rate equal to its joint spectral radius?

The finiteness conjecture has been proven false, in the sense that such a finite product does not always exist; although this case seems to be exceptional [1, 2, 12, 11, 9]. In this paper we prove the finiteness conjecture for pairs of binary matrices of dimension 3.

1.1 Overview and main results

In Section 2 we present the *invariant polytope algorithm* (ipa) for computing the JSR of a finite set of square matrices. In Section 2.1 we discuss how mixed numeric-symbolic computations can be used to widen the classes of matrices where the ipa is applicable. In Section 2.2 we show how we can augment our input set of matrices to obtain faster termination properties.

Finally, in Section 3 we discuss how the ipa, together with the discussed modifications, can automatically prove the finiteness conjecture for pairs of binary matrices of dimension 2 and 3, as well for pairs of sign matrices of dimension 2.

1.2 Notation

The set of all integers is denoted by \mathbb{N} , integers including zero by \mathbb{N}_0 , reals by \mathbb{R} , non-negative reals by \mathbb{R}_+ , complex numbers by \mathbb{C} . Given $X \subseteq \mathbb{C}^s$, where $s \in \mathbb{N}$ is the dimension, we denote the closure of X by $\text{cl}(X)$ and its interior by X° . Products of sets are understood element wise, e.g. $A \cdot B = \{a \cdot b : a \in A, b \in B\}$. Comparisons of matrices are understood element wise. For a matrix A we denote by A^T its transpose and for a square matrix by $\rho(A)$ its spectral radius.

We will make use of various convex hulls of sets throughout the paper.

Definition 1.2. • For $V \subseteq \mathbb{R}^s$, we define its *convex hull* $\text{co} V$ as the intersection of all convex sets containing V .

• For $V \subseteq \mathbb{R}_+^s$, we define the *cone hull* of V (in the first orthant) by

$$\text{co}_+ V = \{x \in \mathbb{R}_+^s : x = y - z, y \in \text{co}(V), z \in \mathbb{R}_+^s\} \subseteq \mathbb{R}_+^s. \quad (2)$$

• For $V \subseteq \mathbb{R}^s$, we define the *symmetric convex hull* of V by

$$\text{co}_s V = \text{co}\{V, -V\} \subseteq \mathbb{R}^s. \quad (3)$$

^aUniversity of Vienna, Austria, e-mail: thomas.mejsstrik@gmx.at; The author is sponsored by the Austrian Science Foundation (FWF) grant P 33352.

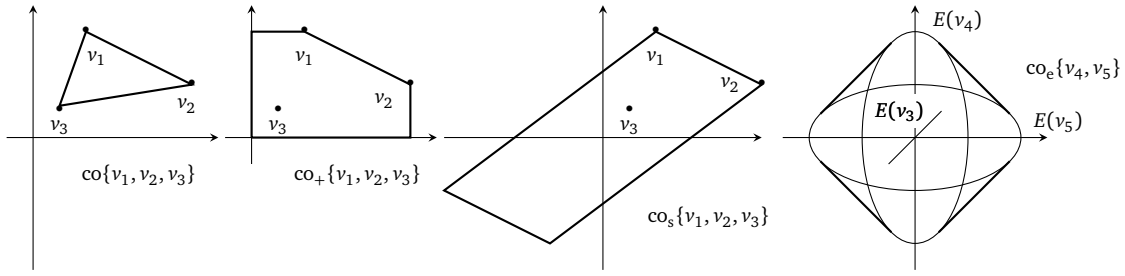


Figure 1: Various convex hulls. $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ i \end{bmatrix}$, $v_5 = \begin{bmatrix} i \\ 2 \end{bmatrix}$.

- For $v = a+ib \in \mathbb{C}^s$ we define its corresponding ellipse $E(v) = E(a, b) \subseteq \mathbb{R}^s$ as the two dimensional subset $\{a \cos t + b \sin t : t \in \mathbb{R}\} \subseteq \mathbb{R}^s$. For $V \subseteq \mathbb{C}^s$, we define the *elliptic convex hull* of V by

$$\text{co}_e V = \text{co} \{E(v) : v \in V\} \subseteq \mathbb{R}^s. \tag{4}$$

- For simplicity, we denote with $\text{co}_* V$ any of the convex hulls co_+ , co_s , co_e , depending on the context.

We will use the aforementioned convex hulls to define norms via their unit ball.

Definition 1.3. Let $P \subseteq \mathbb{R}^s$ be a compact, convex set with non-empty interior, and such that $rP \subseteq P$ for all $|r| \leq 1$. We define the *Minkowski norm* $\|\cdot\|_P : \mathbb{R}^s \rightarrow \mathbb{R}$ by

$$\|x\|_P = \min \{r > 0 : x \in rP\}. \tag{5}$$

2 The invariant polytope algorithm

The *invariant polytope algorithm (ipa)* for the computation of the JSR makes use of the inequality [4],

$$\max_{A_j \in \mathcal{A}} \rho(A_{j_k} \cdots A_{j_1})^{1/k} \leq \text{JSR}(\mathcal{A}) \leq \max_{A_j \in \mathcal{A}} \|A_{j_k} \cdots A_{j_1}\|^{1/k}, \tag{6}$$

which holds for any $k \in \mathbb{N}$ and any sub-multiplicative norm $\|\cdot\|$. Before we can describe how the ipa works, we need a few further definitions.

Definition 2.1. • For a product $A_{j_k} \cdots A_{j_1}$ we say the number $\rho(A_{j_k} \cdots A_{j_1})^{1/k}$ is its *averaged spectral radius*.

If there exists a product $\Pi = A_{j_n} \cdots A_{j_1}$, $A_j \in \mathcal{A}$, such that $\rho(\Pi)^{1/n} = \text{JSR}(\mathcal{A})$, i.e. its averaged spectral radius equals the joint spectral radius, we call the product a *spectral maximizing product (s.m.p.)*.

- Given a matrix $\Pi \in \mathbb{R}^{s \times s}$, we call the eigenvalues largest in modulus the *leading eigenvalues* and the corresponding eigenvectors the *leading eigenvectors*. If there exists only one largest eigenvalue in modulus (counted with algebraic multiplicity), we say the leading eigenvalue is *simple*.

- Given a bounded set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$ with $\lambda = \text{JSR}(\mathcal{A}) > 0$ and let $\tilde{\mathcal{A}}$ be the set of normalized matrices $\tilde{A}_j = A_j/\lambda : A_j \in \mathcal{A}$. \mathcal{A} is said to possess a spectral gap (at $\text{JSR}(\mathcal{A})$) if there exists $\gamma < 1$ and for every product $\tilde{\Pi} = \tilde{A}_{j_n} \cdots \tilde{A}_{j_1}$, $\tilde{A}_j \in \tilde{\mathcal{A}}$, which is not an s.m.p., it holds that $\rho(\tilde{\Pi}) < \gamma$.

We are now in the position to describe the ipa, which runs in two stages: Firstly, it guesses spectral maximizing products Π_n , $n = 1, \dots, N$; Secondly, it tries to construct the unit ball P of a vector norm, for whose induced matrix norm all normalized matrices $\tilde{A}_j = A_j/\rho(\Pi_1)^{1/\text{len}(\Pi_1)}$, $A_j \in \mathcal{A}$, have norm less than or equal to 1. If the second part succeeds, then, by Inequality (6), we obtain the exact value of the joint spectral radius.

The construction of the set P is done iteratively. Starting with (properly scaled leading eigenvectors) of the s.m.p.-candidates, in each step it is checked whether all images of all points not yet mapped into the interior (of the convex hull of all formerly computed points) are mapped into the interior (of the convex hull of all formerly computed points). Depending on the structure of the input set, different convex hulls need to be used; *Case (P)*: If all entries of the matrices A_j are non-negative, then we can take non-negative leading eigenvectors of the s.m.p.-candidates as starting vectors and use the cone hull co_+ . *Case (R)*: If the matrices A_j have positive and negative entries and all leading eigenvectors are real, then we use the symmetric convex hull co_s . *Case (C)*: In all other cases we need to use the elliptic convex hull co_e . If eventually all points are mapped into the interior, then an invariant polytope is found and the algorithm terminates.

A simplified pseudo code implementation is given in Algorithm 1. For a more thorough discussion of the algorithm see [7, 8, 14]; For a discussion about the containment problem see [7, 17].

A crucial point in Algorithm 1 is the line **if $r \notin \text{co}_* V^\circ$ then** : If we cannot prove that a vector r is not contained in the interior of $\text{co}_* V$, then we have to add it to the set V . Otherwise we would not get rigorous results using this algorithm. Furthermore, with this procedure sufficient and necessary conditions for the termination of the ipa are known.

Theorem 2.1 ([8]). Let $\mathcal{A} = \{A_j \in \mathbb{R}^{s \times s} : j = 1, \dots, J\}$ be a finite set of matrices. The ipa terminates if and only if the set \mathcal{A}

- has a spectral gap,
- has only finitely many s.m.p.s Π_n , $n = 1, \dots, N$ (up to powers and cyclic permutations), and
- each s.m.p. has only one simple leading eigenvector v_n (up to complex conjugates).

Algorithm 1: Invariant polytope algorithm

Data: irreducible, finite set of matrices $\mathcal{A} = \{A_j \in \mathbb{R}^{s \times s} : j = 1, \dots, J\}$

Result upon Termination: $\lambda = \text{JSR}(\mathcal{A})$, invariant polytope $\text{co}_* V$

Search for s.m.p.s Π_1, \dots, Π_N , set $\lambda := \rho(\Pi_1)^{1/\text{len}\Pi_1}$

Scale matrices $\tilde{\mathcal{A}} := \{\lambda^{-1}A_j : j = 1, \dots, J\}$

Select leading eigenvectors $V := \{v_0, \dots, v_N\}$

Set $R_{\text{new}} := V$

while $R_{\text{new}} \neq \emptyset$ **do**

 Set $R := R_{\text{new}}$

 Set $R_{\text{new}} := \emptyset$

for $r \in \tilde{\mathcal{A}}R$ **do**

if $r \notin \text{co}_* V^\circ$ **then**

 Set $V := V \cup r$

 Set $R_{\text{new}} := R_{\text{new}} \cup r$

Return $\lambda, \text{co}_* V$

2.1 Mixed numeric/symbolic computations

The conditions on the matrix set \mathcal{A} in Theorem 2.1 sound rather restricting, but it turns out that most matrix families from applications fulfil them. Notable exceptions are when the scaled set $\tilde{\mathcal{A}}$ has a matrix product which is the identity matrix, or when vertices are mapped onto the boundary of the current polytope. In both cases the ipa cannot terminate, since the algorithm always checks whether images of vertices are mapped into the interior of the current polytope.

To overcome these problems one can revert to a symbolic computation of the norm. Unfortunately, a purely symbolic computation is computationally not feasible, because too expensive. Thus, we resort to a mixed numerical and symbolic algorithm to replace the aforementioned line in the algorithm with **if** $s \notin \text{co}_* V$ **then** whenever possible. We distinguish between two cases.

Case 1: Whenever a new vertex point is near to an existing vertex point, we compare their exact coordinates symbolically. This can be done efficiently and just needs some matrix-vector multiplications.

Case 2: Slightly more complicated but still feasible; Whenever the norm of a new vertex is near 1, we compute an exact upper bound of its norm symbolically. This is efficiently possible whenever the leading eigenvectors are all real, i.e. in cases (R) and (P). Indeed, the problem of determining whether a point is inside or outside of a polytope can be stated as an LP problem [7], which does not only answer the containment problem, but also reports the vertices of a face of the polytope through which a ray through the point in question passes. This face can be used to symbolically compute an *upper* bound of the norm. For the case when one leading eigenvector is complex (case (C)) we yet do not have devised an efficient algorithm for the second problem.

Example 2.1 shows how mixed numeric/symbolic computation can be used to solve examples where vertices of the polytope are mapped onto other vertices.

Example 2.1. Let

$$A_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix}.$$

The only s.m.p. of this set is given by A_2 , see below for the proof. The ipa cannot compute the joint spectral radius of this set exactly due to two reasons: (1) The matrix A_2 has multiple leading eigenvalues ± 1 , and furthermore (2), $A_2 = A_2^3$ and thus vertices of any polytope are mapped onto itself after three iterations.

With mixed symbolic and numeric computation we obtain that, with leading eigenvector $v_0 = [\sqrt{2} + 2 \quad \sqrt{2} \quad -2]^T$, the polytope $P = \text{co}_s \{v_0, A_1 v_0, A_1 A_1 v_0, A_2 A_1 v_0, A_2 A_2 A_1 v_0\}$ is invariant under both matrices A_1, A_2 . See Figure 2 for the tree generated by the ipa.

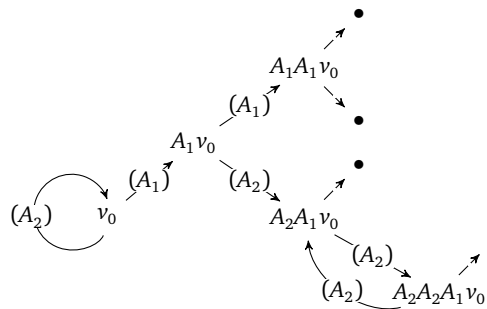


Figure 2: Tree generated by the ipa with mixed numeric/symbolic computations for Example 2.1. The starting vector v_0 is the leading eigenvector of A_2 . Arrows depict how vertices are mapped under the given matrix product. Vertices plotted as \bullet (instead written as text), are mapped to the interior of the polytope $P = \text{co}_s \{v_0, A_1 v_0, A_1 A_1 v_0, A_2 A_1 v_0, A_2 A_2 A_1 v_0\}$.

Proof. We prove that A_2 is the only s.m.p. of the set $\{A_1, A_2\}$. First note that $\rho(A_2) = 1$. The claim follows by Gripenberg's algorithm [5]: Since the norm $\|A_1\|_2 = \sqrt{2}/2 < \rho(A_2)$, each product which is a candidate for an s.m.p. has to start with either $\cdots A_1 A_2$ or $\cdots A_2 A_2$, where

$$A_1 A_2^1 = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \text{and} \quad A_2 A_2^1 = \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Again, $\|A_1 A_2^1\|_2 = \sqrt{\frac{\sqrt{5}+3}{8}} \simeq 0.80902 < 1$, and thus each product which is a candidate for an s.m.p. has to start with either $\cdots A_1 A_2^2$ or $\cdots A_2 A_2^2$, where

$$A_1 A_2^2 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad A_2 A_2^2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix}.$$

And again, the norm $\|A_1 A_2^2\|_2 = \sqrt{\frac{\sqrt{5}+3}{8}} \simeq 0.80902 < 1$, and thus each product which is a candidate for an s.m.p. has to start with either $\cdots A_1 A_2^3$ or $\cdots A_2 A_2^3$. Since $A_1 A_2^3 = A_1 A_2^2$ and $A_2 A_2^3 = A_2 A_2^2$ we conclude that all s.m.p. candidates are of the form $A_1 A_2^n$ and $A_2 A_2^n$. The former are no s.m.p.s, since their spectral radii is $1/2$, thus A_2 is the only s.m.p. \square

2.2 Limit matrices

In some cases it speeds up the computation when one adds matrices to the input set \mathcal{A} in question. In particular, given a set of matrices \mathcal{A} , its joint spectral radius $\text{JSR}(\mathcal{A})$ does not change when elements of the closure $\text{cl}\mathcal{A}$ or its convex hull $\text{co}\mathcal{A}$ (to be understood in the Hausdorff distance using a matrix norm) are added to \mathcal{A} [10, Proposition 1.8],

$$\text{JSR}(\mathcal{A}) = \text{JSR}(\text{cl}\mathcal{A}) = \text{JSR}(\text{co}\mathcal{A}). \quad (7)$$

Lemma 2.2 shows that we may also add limit matrices to the set in question.

Lemma 2.2. *Given matrices $\tilde{\mathcal{A}} \subseteq \mathbb{R}^{s \times s}$ with $\text{JSR}(\tilde{\mathcal{A}}) = 1$; If the ipa terminates for the set $\tilde{\mathcal{A}}$, then the ipa terminates for the set $\mathcal{L} \cup \tilde{\mathcal{A}}$, where \mathcal{L} is the set of all matrices of the limit set of all possible products of matrices of $\tilde{\mathcal{A}}$, i.e. of the set $\{\prod_{k=1}^n \tilde{A}_k : n \in \mathbb{N}, \tilde{A}_j \in \tilde{\mathcal{A}}\}$.*

Proof. If the ipa terminates, then there exists $K \in \mathbb{N}$ such that $\tilde{A}_j v_n \in \text{co}_* V$ for all $\tilde{A}_j \in \tilde{\mathcal{A}}$ with $V = \bigcup_{k=0}^K \tilde{\mathcal{A}}^k \{v_1, \dots, v_N\}$, where $v_n, n = 1, \dots, N$, are the starting vectors for the ipa. Thus, for each $v_n \in V$, there exists $M \in \mathbb{N}$ such that $\tilde{\Pi}_j^m v \in \text{co}_* V$ for all $m \geq M$ and all s.m.p.s $\tilde{\Pi}_j$. In particular, $\tilde{\Pi}_l v \in \text{co}_* V$ for all $\tilde{\Pi}_l \in \mathcal{L}$. \square

Currently we use a heuristic to decide whether to add matrices and which of them. The according rules have not stabilized yet, and thus, we do not report them.

3 The finiteness conjecture

Recalling the definition of a spectral maximizing product (s.m.p.) in Section 2, we make the following definition:

Definition 3.1. A bounded set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$ is said to possess the *finiteness property* if there exists a finite product $\Pi = A_{j_n} \cdots A_{j_1}, A_{j_i} \in \mathcal{A}$ such that $\rho(\Pi)^{1/n} = \text{JSR}(\mathcal{A})$.

As already mentioned in the beginning, it has been shown that there exist sets of matrices such that the normalized spectral radius of every finite product is strictly less than the JSR. In other words, not all sets of matrices possess an s.m.p.. It is an open question whether pairs of *binary matrices* with entries $\{0, 1\}$ or *sign matrices* with entries $\{-1, 0, 1\}$ always possess an s.m.p. [11, 10]. Using the ipa we can check special cases of this question.

Theorem 3.1. *The finiteness conjecture holds for all pairs of*

- (a) *binary matrices of dimension 2 (i.e. with entries $\{0, 1\}$),*
- (b) *sign matrices of dimension 2 (i.e. with entries $\{-1, 0, 1\}$), and*
- (c) *binary matrices of dimension 3 (i.e. with entries $\{0, 1\}$).*

Remark 1. Point 3.1 (a) is already proven in [10, Chapter 4]; Point 3.1 (b) is already proven in [3].

Proof. With our proposed algorithm a proof of (a) and (b) takes some minutes. The proof of 3.1 (c) takes two days (CPU: AMD Ryzen 3600, 6 cores, 64 GB RAM). The used scripts to proof the results can be found at github.com/tommsch/dolomites (and/or tommsch.com/science.php) and are named `fc_2.m`, `fc_2s.m`, `fc_3.m`. The results in condensed form are tabulated in the Appendix; in more detail they can be found online. \square

Remark 2. If the algorithm would be implemented in a performant language (like C), this approach of checking the finiteness conjecture could also be used for pairs of sign matrices of dimension 3, of which there are approximately 20 million cases to be checked. For larger matrices, this approach is not feasible any more.

3.1 Diminishing the number of cases

To proof 3.1 (c) we have to consider $2^{18} = 262144$ cases (To proof (b) we have to consider $3^8 = 6561$ cases, for (a) $2^8 = 256$ cases). This number can be reduced significantly: For some sets of matrices a concrete s.m.p. is known, other sets share certain symmetries, so that in total we check 15908 cases (For (b) we check 166 cases, for (a) we check 6 cases). We could exploit even more symmetries, but since those are computational hard to check, the total time needed to proof the statement most likely would increase.

Lemma 3.2. *Given $A_1, A_2 \in \mathbb{R}^{s \times s}$; The following pairs have the same joint spectral radius and the finiteness property holds for all or none of them:*

- $\{A_1, A_2\}$,
- $\{\pm A_1, \pm A_2\}$,
- $\{A_2, A_1\}$,
- $\{P^T A_1 P, P^T A_2 P\}$ where P is a permutation matrix, and
- $\{A_1^T, A_2^T\}$,
- $\{S^{-1} A_1 S, S^{-1} A_2 S\}$ where S is an invertible matrix.

Proof. For the proof we use the definition of the joint spectral radius 1.1 with the 2-norm. The statements then follow from the facts that $\|A\|_2 = \|A^T\|_2 = \|-A\|_2$ and $PP^T = SS^{-1} = I$. \square

Lemma 3.3. *Given $A_1, A_2, A_0 \in \mathbb{N}_0^{s \times s}$; If $A_2 \leq A_1$, then $\text{JSR}(\{A_2, A_0\}) \leq \text{JSR}(\{A_1, A_0\})$.*

Proof. For the proof we use the definition of the joint spectral radius 1.1 with the Frobenius norm $\|\cdot\|_F$, and let $X = A_{j_n} \cdots A_{j_1}$, $j_i \in \{2, 0\}$, be a given product. We first construct a new product $\tilde{X} = A_{\tilde{j}_n} \cdots A_{\tilde{j}_1}$, $\tilde{j}_i \in \{1, 0\}$, from X , by replacing all occurrences of A_2 by A_1 . It follows that $\|X\|_F^{1/n} \leq \|\tilde{X}\|_F^{1/n}$, and thus $\text{JSR}(\{A_2, A_0\}) \leq \text{JSR}(\{A_1, A_0\})$. \square

Lemma 3.4. *Given $A_1, A_2 \in \mathbb{N}_0^{s \times s}$; The finiteness property holds whenever $\text{JSR}(\{A_1, A_2\}) \leq 1$.*

Proof. Since the norm of a non-zero integer matrix is always greater equal than one, it is not possible that the joint spectral radius of a set of integer matrices is strictly between 0 and 1. If $\text{JSR}(\{A_1, A_2\}) = 0$, then clearly both A_1 and A_2 are s.m.p.s. The second case is non trivial and its proof is given in [10, Chapter 3.4]. \square

Corollary 3.5. *Given $A_1, A_2 \in \mathbb{N}_0^{s \times s}$; The finiteness property holds whenever*

- (a) $A_2 \leq A_1$
- (b) $A_1 A_2 \leq A_1^2$
- (c) $A_2 \leq I$
- (d) $A_2 A_1 \leq A_1 A_2$

Proof. Again, we use the definition of the joint spectral radius 1.1 with the Frobenius norm $\|\cdot\|_F$, and let $A_{j_n} \cdots A_{j_1}$, $j \in \{1, 2\}$, be a given product.

- (a) and (b) It follows that $\|A_{j_n} \cdots A_{j_1}\|_F^{1/n} \leq \|A_1\|_F^{1/n}$, and thus $\text{JSR}(\{A_1, A_2\}) = \rho(A_1)$
- (c) It follows that $\|A_{j_n} \cdots A_{j_1}\|_F^{1/n} \leq \|A_1\|_F^{1/\tilde{n}}$ with $\tilde{n} \leq n$, and thus, $\text{JSR}(\{A_1, A_2\}) \leq \rho(A_1)$ which implies $\text{JSR}(\{A_1, A_2\}) = \rho(A_1)$.
- (d) It follows that $\|A_{j_n} \cdots A_{j_1}\|_F^{1/n} \leq \|A_1^{n_1} A_2^{n_2}\|_F^{1/n}$ for some $n_1 + n_2 = n$, and thus, $\text{JSR}(\{A_1, A_2\}) = \max\{\rho(A_1), \rho(A_2)\}$. \square

Lemma 3.6. *If there exists a norm $\|\cdot\|$ such that $\max\{\rho(A_1), \rho(A_2)\} = \max\{\|A_1\|, \|A_2\|\}$, then the finiteness property holds. In particular, the finiteness property holds for sets of normal matrices, and thus, symmetric matrices. A matrix A is normal, iff $A^T A = A A^T$.*

Proof. The first part follows from Inequality (6), which reads for products of length 1 as $\max_{A_j \in \mathcal{A}} \rho(A_j) \leq \text{JSR}(\mathcal{A}) \leq \max_{A_j \in \mathcal{A}} \|A_j\|$. By the assumptions we have equality here, and thus $\text{JSR}(\{A_1, A_2\}) = \max_{A_j \in \mathcal{A}} \rho(A_j)$.

The second parts about normal matrices follows now by using the 2-norm, which equals the matrix' the largest singular value. For normal matrices the largest singular value equals the largest eigenvalue in magnitude, and thus $\max_{A_j \in \mathcal{A}} \rho(A_j) = \max_{A_j \in \mathcal{A}} \|A_j\|_2$. \square

Definition 3.2. *Given a finite set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$; If there exists $V \in \mathbb{R}^{s \times s}$ such that $VA_j V^{-1} = \begin{bmatrix} B_j & C_j \\ 0 & D_j \end{bmatrix}$ for all $A_j \in \mathcal{A}$, then \mathcal{A} is reducible.*

Theorem 3.7. *In the notation from Definition 3.2; If \mathcal{A} is reducible, then $\text{JSR}(\mathcal{A}) = \max\{\text{JSR}(\mathcal{B}), \text{JSR}(\mathcal{D})\}$, $\mathcal{B} = \{B_j : j = 1, \dots, J\}$, $\mathcal{D} = \{D_j : j = 1, \dots, J\}$.*

The first rigorous proof known to the author can be found in [10, Proposition 1.5]. Although the proof is straight forward, it is also rather technical and we abstain from giving it here.

Corollary 3.8. *Given $A_1, A_2 \in Z^{s \times s}$, $Z \subseteq \mathbb{Z}$ and using the notation from Definition 3.2; The finiteness conjecture holds whenever there exists $S \in \mathbb{C}^{s \times s}$ such that the matrices $S^{-1} A_j S$, have joint block diagonal form with blocks $B_j \in Z^{s_B \times s_B}$, $D_j \in Z^{s_D \times s_D}$, $j = 1, 2$, $s_B < l$, and the finites property holds for all pairs of matrices in $Z^{s_B \times s_B}$.*

Proof. This follows from Lemma 3.2 and Theorem 3.7. \square

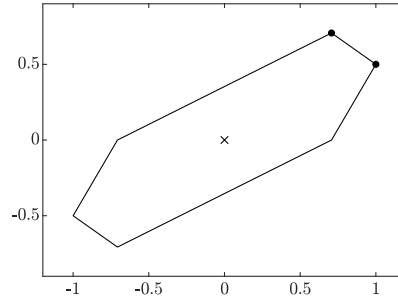


Figure 3: Invariant polytope for the matrices $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$ of Example 4.1. The cross \times denotes the origin, the dots \bullet the leading eigenvectors of the matrices (and s.m.p.s) A_1 and A_2 .

4 Implementation notes

Our Matlab implementation of the algorithm can be found on Gitlab [15], and is extensively documented. The file `manual.pdf` gives an overview of the toolbox, in depth documentation can be found directly in the source files, and can be viewed by typing `help functionname` or `edit functionname`, e.g. `help tjcsr` or `edit tjcsr`, in Matlab.

The main function for the JSR computation is `tjcsr`, short for *invariant polytope algorithm*. Depending on the input, our implementation chooses its parameters automatically and usually there is no need for the user to specify options by hand. For example, of the 15910 cases checked for Theorem 3.1 (c), manual intervention was only necessary for 2 cases.

Example 4.1 presents how to use the `tjcsr` algorithm.

Example 4.1. Given the matrices $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$. To compute their joint spectral radius, the matrices must be passed as a cell array to the algorithm, e.g. by typing:

```
tjcsr( {[0 1;0 1],[1 0;1 -1]} )
```

The algorithm (version 1.2022.05.25) produces the following output:

```
Input: 2 matrices of dimension 2

A lot of candidates found. Nearly all orderings are smp's.
Set <'epseigenplane',inf, 'epsspectralradius',inf, 'maxsmpdepth',5, 'balancing',-1, 'ncdelta',1>.
JSR (of block) 1: 1.0000 1.6180
Duplicate leading eigenvectors occurred. Enable symbolic computation. Set <'epssym',5e-12>.
JSR (of block) 1: 1.0000 1.6180
Case (R).
Selected candidates: | 1 | 2 |
Number of vertices: 2 ( candidates )
Balance 2 Trees. Balancing vector found: [ 1, 341/305]
JSR = [ 1, 1.61803398875 ], norm= Inf, #test: 1/1, #V:2/2 | 0
JSR = [ 1, 1.61803398875 ], norm= 2.41421508763, _
Number of vertices of polytope: 3
Products which give lower bounds of JSR: | 1 |
Algorithm terminated correctly. Exact value found.
JSR = 1
```

One can see that the algorithm restarts two times. The first time because too many s.m.p. candidates are found and appropriate options are set: `<'epseigenplane',inf, 'epsspectralradius',inf, 'maxsmpdepth',5, 'balancing',-1, 'ncdelta',1>`. The second time because the s.m.p. candidates do not have a unique leading eigenvector and mixed symbolic and numeric computation is enabled `<'epssym',5e-12>`.

The third time the algorithm terminates after two iterations. It reports two s.m.p.s A_1 and A_2 , and a joint spectral radius of 1. The constructed polytope with 3 vertices is given in Figure 3. The figure is produced by calling `tjcsr` with the option `'plot': tjcsr([0 1;0 1],[1 0;1 -1], 'plot','polytope')`. A complete list of all options can be found in the file `tjcsr_option`, and viewed by typing `tjcsr help` or `edit tjcsr_option`.

Remark 3. Since our implementation of the algorithm is in Matlab, which has very restricted capabilities for symbolic computations, mixed symbolic computation only works when the leading eigenvectors are expressible in a “simple” closed form, e.g. for integer matrices of dimension less than or equal to 3.

A List of cases

A.1 2×2 binary matrices

The following list reports \mathcal{A} : the set of matrices and *s.m.p.*: the shortest s.m.p. found for the set \mathcal{A} . All unreported cases can be reduced to a simpler one, or an s.m.p. is known due to the structure of the set \mathcal{A} , by the Lemmata presented in Section 3.1.

\mathcal{A}	<i>s.m.p.</i>	\mathcal{A}	<i>s.m.p.</i>	\mathcal{A}	<i>s.m.p.</i>
$\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$	$A_1 A_2^4$	$\left\{ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$	$A_1 A_2^3$	$\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$	$A_1 A_2$
$\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right\}$	$A_1^2 A_2$	$\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$	$A_1 A_2^2$	$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$	$A_1 A_2$

A.2 2 x 2 sign matrices

To save space, we use the following abbreviations: + for +1, - for -1, o for 0. In addition to A and s.m.p., as reported in Section A.1, we mark all cases where at least one leading eigenvalues is complex (case (C)). If the case is not explicitly mentioned, then it is case (R), meaning that all leading eigenvalues are real.

Table with 12 columns and multiple rows. Each column contains a 2x2 sign matrix (A), its s.m.p. (s.m.p.), and its case (case). The matrices are composed of +, -, and o symbols. The s.m.p. values are integers, and the cases are labeled (C) for complex or (R) for real eigenvalues.

A.3 3 x 3 binary matrices

Due to the large number of cases and the need to save space, we give the matrices as a pair of two numbers, where the binary representation of the numbers corresponds to the entries in the matrices. The rightmost digit in the binary representation corresponds to the entry in the second matrix, third row, third column. For example, the pair 3/477, written in the binary system as 000'000'0112/111'011'1012 corresponds to the matrix pair 3/477 ≐ 000'000'0112/111'011'1012 ≐ { [0 0 0], [0 0 1], [0 0 1] }, [1 0 1], [1 1 1] }. Matrix pairs with the same s.m.p. are subsumed under one entry. For example, the s.m.p. A2 of the matrix pair 3/477 can be found in the section with A1 = 3 and the cell with the entry 476-78, A2.

Large table with 12 columns and many rows. Each row contains a pair of numbers (s.m.p.) and a corresponding entry (matrix pair). The columns are grouped by the value of A1 (e.g., A1 = 2, A1 = 3, A1 = 6, A1 = 7, A1 = 10, A1 = 11). Each cell contains a pair of numbers representing the matrix pair and a label for the second matrix (A2).

172-74	A ₁	178-82	A ₁	183	A ₁ A ₂ ⁴	188-90	A ₂	230	A ₁	231	A ₂	246-47	A ₂	326	A ₁	327	A ₂	342	A ₁	343	A ₂	352-54	A ₁	
355	A ₂	356	A ₁	357-59	A ₂	360-62	A ₂	364	A ₁	365-66	A ₂	368-69	A ₁	370	A ₂	372	A ₁	373-75	A ₂	376	A ₁	377	A ₁ A ₂ ³	
378	A ₂	380	A ₁	381-82	A ₂	418-23	A ₁	428-30	A ₂	434-39	A ₂	444-46	A ₂	486-87	A ₂	502-3	A ₂	A ₁ = 12		67	A ₁ A ₂ ²	71	A ₂	
75	A ₂	83	A ₁ A ₂ ³	86	A ₁ A ₂ ⁴	87	A ₂	90-91	A ₂	99	A ₂	102-3	A ₂	106-7	A ₂	114-15	A ₂	118-19	A ₂	122-23	A ₂	131	A ₁ A ₂	
134-135	A ₁ A ₂	139	A ₂	147	A ₁ A ₂ ³	149	A ₁ A ₂ ⁵	150	A ₁ A ₂ ²	151	A ₁ A ₂ ²	184-87	A ₂	193	A ₁ A ₂	194	A ₁	203	A ₂	208	A ₂	209	A ₁ A ₂	
166	A ₂	167	A ₁ A ₂ ³	169	A ₁ A ₂ ⁶	170-71	A ₂	176-82	A ₂	183	A ₁ A ₂	200-201	A ₁ A ₂	202	A ₁ A ₂ A ₂ ²	203	A ₂	208	A ₂	209	A ₁ A ₂	226-32	A ₂	
197	A ₂	198	A ₁ A ₂ A ₂ ²	211	A ₁ A ₂ ²	212	A ₁ A ₂ ⁴	213	A ₂	214	A ₁ A ₂ ²	215	A ₁ A ₂ ²	216	A ₂	217	A ₁ A ₂ ³	218-19	A ₂	219	A ₁ A ₂	225	A ₁ A ₂	
210	A ₂	211	A ₁ A ₂ ²	240-51	A ₂	323	A ₁ A ₂ ³	326-27	A ₂	330	A ₁ A ₂ ²	331	A ₂	338	A ₁ A ₂ ⁶	339	A ₁ A ₂ ²	342-43	A ₂	346-47	A ₂	354-55	A ₂	
233	A ₁ A ₂ ³	234-35	A ₂	370-71	A ₂	374-75	A ₂	378-79	A ₂	386	A ₂	387	A ₁ A ₂	388	A ₂	389	A ₁ A ₂ ²	390	A ₂	391	A ₁ A ₂ ³	393	A ₁ A ₂ ²	
358-59	A ₂	362-63	A ₂	400-402	A ₂	403	A ₁ A ₂ ²	404	A ₂	404	A ₂	405	A ₁ A ₂ ²	406-407	A ₁ A ₂ ²	407	A ₁ A ₂ ²	408	A ₁ A ₂ ⁴	409	A ₁ A ₂ ⁵	410-11	A ₂	
394	A ₁ A ₂ ⁴	395	A ₂	449	A ₁ A ₂ ³	450	A ₂	451	A ₁ A ₂ ²	452-55	A ₂	456	A ₁ A ₂ ²	457	A ₁ A ₂ ³	458	A ₁ A ₂ ²	459	A ₁ A ₂ ²	464	A ₁ A ₂ ⁴	464	A ₁ A ₂ ⁴	
416-27	A ₂	432-43	A ₂	466	A ₂	467	A ₁ A ₂ ²	472-473	A ₁ A ₂ ²	474-75	A ₂	478	A ₂	480-88	A ₂	489	A ₁ A ₂ ²	490-91	A ₂	496-7	A ₂	A ₁ = 13		
465	A ₁ A ₂ ²	466	A ₂	470	A ₁ A ₂ ²	71	A ₂	74	A ₁ A ₂ ²	75	A ₂	78	A ₂	82	A ₁ A ₂ ²	83	A ₁ A ₂ ²	86	A ₁ A ₂ ²	87	A ₁ A ₂ ³	90-91	A ₂	
66	A ₁ A ₂ ²	67	A ₁ A ₂ ²	69	A ₁ A ₂ ²	102-3	A ₂	106-7	A ₂	110	A ₂	114-15	A ₂	118-19	A ₂	122-23	A ₂	126	A ₂	128	A ₁	129	A ₁ A ₂	
94	A ₂	98	A ₂	99	A ₁ A ₂	133	A ₁ A ₂	134	A ₁ A ₂ ²	135	A ₁ A ₂	136	A ₁	137	A ₁ A ₂	138	A ₁ A ₂ ²	139	A ₂	140	A ₂	142	A ₁ A ₂ ²	
130	A ₁ A ₂ ²	131	A ₁ A ₂	132	A ₁	147	A ₁ A ₂	148	A ₁ A ₂ ²	149	A ₁ A ₂ ²	150	A ₂	151	A ₁ A ₂ ²	152	A ₁ A ₂ ²	153	A ₁ A ₂ ²	154-56	A ₂	158	A ₂	
144	A ₁	145	A ₁ A ₂	146	A ₂	163	A ₁ A ₂	164	A ₁	165	A ₁ A ₂	166	A ₁ A ₂	167	A ₁ A ₂	168	A ₁	169	A ₁ A ₂	170-72	A ₂	174	A ₂	
160	A ₁	161	A ₁ A ₂ ²	162	A ₂	190	A ₂	192	A ₁ A ₂ ²	193	A ₁ A ₂	194	A ₂	195	A ₁ A ₂	196	A ₂	197	A ₂	198	A ₂	199	A ₁ A ₂	
176-82	A ₂	183	A ₁ A ₂	184-88	A ₂	202	A ₁ A ₂ ²	203	A ₁ A ₂ ²	204	A ₁ A ₂ ²	206	A ₂	208	A ₂	209	A ₁ A ₂ ²	210	A ₂	211	A ₂	212	A ₁ A ₂ ²	
200	A ₁ A ₂ ²	201	A ₁ A ₂	216	A ₂	217	A ₁ A ₂	218-20	A ₂	222	A ₂	224	A ₁ A ₂ ²	225	A ₁ A ₂	226	A ₁ A ₂ ²	227	A ₁ A ₂ ²	228-31	A ₂	232	A ₁ A ₂ ²	
233	A ₁ A ₂	234-36	A ₂	238	A ₂	240-52	A ₂	254	A ₂	322	A ₁ A ₂ ²	323	A ₁ A ₂	326-27	A ₂	330	A ₁ A ₂ ²	331	A ₁ A ₂ ²	334	A ₂	338	A ₁ A ₂ ³	
339	A ₁ A ₂ A ₂ ²	342-43	A ₂	346-47	A ₂	350	A ₂	354-55	A ₂	358-59	A ₂	362-63	A ₂	366	A ₂	370-71	A ₂	374-75	A ₂	378-79	A ₂	384	A ₂	
382	A ₂	384	A ₁	385	A ₁ A ₂	386	A ₁ A ₂ ²	387	A ₁ A ₂	388	A ₁ A ₂ ²	389	A ₁ A ₂ ³	390	A ₁ A ₂ ²	391	A ₁ A ₂ ²	392	A ₂	393	A ₁ A ₂ ³	394	A ₁ A ₂ ⁴	
395	A ₁ A ₂ ⁵	396	A ₂	398	A ₂	400	A ₁ A ₂ ⁴	401-402	A ₁ A ₂ ²	403	A ₁ A ₂	404	A ₁ A ₂ ⁴	405	A ₁ A ₂ ⁵	406	A ₁ A ₂ ²	407	A ₁ A ₂ ²	408	A ₁ A ₂ ³	409	A ₁ A ₂ ⁴	
409	A ₁ A ₂ ⁵	410-12	A ₂	414	A ₂	416-28	A ₂	430	A ₂	432-44	A ₂	446	A ₂	448	A ₁ A ₂	449	A ₁ A ₂	450	A ₁ A ₂ ²	451	A ₁ A ₂ ²	452-55	A ₂	
456	A ₁ A ₂ ²	457-459	A ₂	460	A ₂	462	A ₂	464	A ₁ A ₂ ²	466	A ₁ A ₂ ²	465	A ₁ A ₂ ²	466	A ₁ A ₂ ²	467	A ₁ A ₂	468-69	A ₂	470	A ₁ A ₂ ⁵	471	A ₂	
472	A ₁ A ₂ ²	473	A ₁ A ₂ ³	474	A ₁ A ₂ ⁵	475-76	A ₂	478	A ₂	480-88	A ₂	489	A ₁ A ₂ ⁵	490-92	A ₂	494	A ₂	496-8	A ₂	510	A ₂	A ₁ = 14		
65	A ₁	67	A ₁ A ₂ ²	69	A ₂	71	A ₂	72-74	A ₁ A ₂	75	A ₂	76	A ₁ A ₂	77	A ₂	78	A ₁ A ₂ ²	81	A ₁ A ₂	82	A ₁ A ₂ ²	82	A ₁ A ₂ ²	
83	A ₁ A ₂ ²	85	A ₂	86	A ₁ A ₂ ²	87	A ₂	88-89	A ₁ A ₂	90-91	A ₂	92	A ₁ A ₂ ²	93	A ₂	97	A ₁ A ₂ ²	99	A ₂	100	A ₁	100	A ₁	
101-3	A ₂	104	A ₁ A ₂	105	A ₁ A ₂ ²	106	A ₁ A ₂	107	A ₂	108	A ₁ A ₂ ²	109	A ₂	113	A ₁ A ₂ ²	114-15	A ₂	116	A ₁ A ₂ ²	117-19	A ₂	120	A ₁ A ₂	
121	A ₁ A ₂ ²	122-23	A ₂	124	A ₁ A ₂	125	A ₁ A ₂ ²	130	A ₁	131	A ₁ A ₂	134	A ₁	135	A ₁ A ₂	137	A ₁ A ₂	138	A ₂	A ₂ A ₁ A ₂	139	A ₂	139	A ₂
141	A ₂	144-146	A ₁ A ₂ ²	147	A ₁ A ₂	148	A ₁ A ₂ ²	149	A ₁ A ₂ ²	149	A ₁ A ₂ ²	150	A ₁ A ₂ ²	151	A ₁ A ₂ ²	152	A ₁ A ₂ ²	153	A ₁ A ₂ ²	154-57	A ₂	162	A ₁	
163	A ₁ A ₂	164	A ₁	165	A ₁ A ₂ ⁴	167	A ₁ A ₂ ²	169	A ₁ A ₂ ³	169	A ₁ A ₂ ³	170-73	A ₂	176-82	A ₂	183	A ₁ A ₂ ⁶	184	A ₂	185	A ₁ A ₂ ⁶	186-89	A ₂	
193	A ₁ A ₂	194	A ₁	A ₁ A ₂ A ₂ ²	195	A ₁ A ₂	197	A ₂	198	A ₂	A ₂ A ₁ A ₂	199	A ₂	199	A ₂	200-202	A ₁ A ₂	203	A ₂	203	A ₂	204	A ₁ A ₂	
205	A ₂	208	A ₂	209	A ₁ A ₂ ²	210	A ₂	211	A ₁ A ₂ ²	212	A ₂	212	A ₂	213	A ₁ A ₂ ²	214	A ₂	214	A ₁ A ₂ ²	215	A ₁ A ₂ ²	216-217	A ₁ A ₂	
218-21	A ₂	225	A ₁ A ₂ ³	226-31	A ₂	232	A ₁ A ₂ ²	233	A ₁ A ₂ ²	234-37	A ₂	240-48	A ₂	249	A ₁ A ₂ ³	250-53	A ₂	321	A ₁ A ₂ ⁴	323	A ₁ A ₂ ³	324-27	A ₂	
328	A ₁ A ₂ ²	329	A ₁ A ₂ ²	330	A ₁ A ₂	331	A ₁ A ₂ ²	332-33	A ₂	337	A ₁ A ₂ ²	338	A ₂ A ₂ ⁴	339	A ₁ A ₂ ²	340-43	A ₂	344	A ₁ A ₂ ²	345	A ₁ A ₂ ²	346-49	A ₂	
352	A ₁ A ₂ ²	353	A ₁ A ₂ ²	354-59	A ₂	360	A ₁ A ₂ ²	361	A ₁ A ₂ ²	362-65	A ₂	368-69	A ₂	A ₁ A ₂ ³	370-75	A ₂	376	A ₁ A ₂ ²	377	A ₁ A ₂ ²	378-81	A ₂		
386	A ₁	387	A ₁ A ₂	388	A ₁	389	A ₁ A ₂ ⁵	390	A ₁	391	A ₁ A ₂ ³	393	A ₁ A ₂ ³	394	A ₁ A ₂ ²	395-97	A ₂	400-402	A ₂	403	A ₁ A ₂ ²	403	A ₁ A ₂ ²	
404	A ₁ A ₂ ³	405	A ₁ A ₂ ⁵																					

489	$A_1A_2^3$	490-1	A_2	504	A_2	505	$A_1A_2^4$	506-9	A_2	$A_1 = 23$	72	$A_1^2A_2$	73-75	A_1A_2	76	$A_2^2A_3$	77-78	A_1A_2								
79	A_2	88-90	A_1A_2	91	A_2	92-93	A_1A_2	94	A_2	94	A_2	96	$A_1^2A_2$	97	$A_1A_2^2$	98	$A_1^2A_2$	99	$A_1^2A_2$	100	$A_1^3A_2^2$					
101	A_2	102	$A_1^2A_2^2$	103	A_2	104	$A_2^2A_2$	105-106	A_1A_2	107	A_2	108	$A_1^2A_2$	109	A_1A_2	110-11	A_2	112	$A_1^2A_2$	113	$A_1^2A_2^2$					
114-15	A_2	116	$A_1^2A_2^2$	117	$A_1^2A_2^2$	118	A_2	120-121	A_1A_2	122-23	A_2	124	A_1A_2	125	$A_1^2A_2^2$	126	A_2	136	$A_1^2A_2$							
137-138	A_1A_2	139	$A_1A_2^2$	140	$A_1^2A_2$	141-142	A_1A_2	143	A_2	152-153	A_1A_2	154	$A_1A_2^2$	155	A_2	156	A_1A_2			156	A_1A_2					
157-58	A_2	168	$A_1^2A_2$	169	A_1A_2	170-71	A_2	172	$A_1^2A_2$	173-75	A_2	184	A_2	185	$A_1A_2^2$	186-90	A_2	200-204	A_1A_2	205	A_2					
206	A_1A_2	207	A_2	216-218	A_1A_2	219	A_2	220	A_1A_2	221-22	A_2	224	$A_1^2A_2$	225	$A_1^2A_2$	226	$A_2^2A_2$	227	A_2	228	$A_1A_2^2$					
229-31	A_2	232-234	A_1A_2	235-40	A_2	241	$A_1A_2^2$	242-46	A_2	248	A_1A_2	249	$A_1A_2^2$	250-54	A_2	328	$A_1^2A_2$	329-331	A_1A_2							
332-35	A_2	344-346	A_1A_2	347-50	A_2	352	$A_1^2A_2$	353	$A_1A_2^2$	354-59	A_2	360	$A_1^2A_2$	361	A_1A_2	362-67	A_2	368	$A_2^2A_3$	369	$A_1A_2^3$					
370-74	A_2	376	A_1A_2	377	$A_1A_2^2$	378-79	A_2	380	$A_1A_2^2A_1A_2$	400-41	A_2	381-82	A_2	392	$A_1^2A_2$	393-394	A_1A_2			395	$A_1A_2^2$					
396-99	A_2	408	A_1A_2	409	$A_1A_2^2$	410	$A_1A_2^3$	411-14	A_2	424-31	A_2	440-46	A_2	456-458	A_1A_2	459	$A_1A_2A_1A_2^2$	460-63	A_2	464	$A_2^2A_3$					
472-474	A_2	475	$A_1A_2^2$	476-78	A_2	480	A_2	481	$A_1A_2^2$	482-88	A_2	489	$A_1A_2^2$	490-2	A_2	504	A_2	505	$A_1A_2^3$	506-10	A_2					
$A_1 = 27$	70-71	A_1	86-87	A_1	96-97	A_1	99-10	A_1	111	A_2	112-22	A_1	124-26	A_1	230-31	A_1	247	A_2	326-27	A_1	342-43	A_1				
352-58	A_2	359	A_2	360-66	A_1	367	A_2	368-70	A_1	372-74	A_1	375	A_2	376-78	A_1	380-81	A_1	382	A_2	486-87	A_2	503	A_2			
$A_1 = 28$	67	A_1A_2	71	A_2	74	$A_1^2A_2A_1A_2^2$	75	A_2	78-79	A_2	82	A_1A_2	83	$A_1A_2^2$	86	A_1A_2	87	A_2	90-91	A_2						
99	A_2	102-3	A_2	106-7	A_2	110-11	A_2	114-15	A_2	118-19	A_2	122-23	A_2	130	$A_1^2A_2$	131	A_1A_2	134	$A_2^2A_2$	135	A_1A_2	149	$A_1A_2^3$			
137	A_2	122-23	A_2	126	A_2	139	A_2	141-43	A_2	144	$A_2^2A_2$	145	A_1A_2	146	$A_1^2A_2$	147	A_1A_2	148	$A_2^2A_2$	149	A_1A_2	150	$A_2^2A_2$			
151	$A_1A_2^2$	152-153	A_1A_2	154-55	A_2	162	$A_1^2A_2$	163	A_1A_2	164-165	$A_1A_2^2$	166	$A_2^2A_2$	167	$A_1^2A_2$	169	$A_1A_2^3$	169	$A_1A_2^2$	170-78	A_2					
179	$A_1A_2^2$	180-82	A_2	183	$A_1A_2^4$	184-87	A_2	193	A_1A_2	194	$A_2^2A_2$	195	A_1A_2	197	A_2	198	$A_1^2A_2$	199	A_2	200-202	A_1A_2					
203-7	A_2	208	$A_1^2A_2$	209	A_1A_2	210	$A_1A_2^2$	211	A_1A_2	212	A_1A_2	213	A_2	214	$A_1A_2^2$	215	A_1A_2	216	$A_1A_2^2$	217	A_2	218-217	A_1A_2			
218-19	A_2	225	$A_1A_2A_1A_2^3$	226-31	A_2	232	A_1A_2	233	$A_1A_2A_1A_2^2$	234-48	A_2	249	$A_1A_2^2$	250-51	A_2	249	$A_1A_2^2$	250-51	A_2	323	$A_1A_2^3$	326-27	A_2			
330	$A_1A_2^3$	331	$A_1A_2^6$	334-35	A_2	338	A_1A_2	339	$A_1A_2^2$	342-43	A_2	346-47	A_2	354-55	A_2	358-59	A_2	362-63	A_2	366-67	A_2	370-71	A_2			
374-75	A_2	378-79	A_2	386	$A_2^2A_2$	387	A_1A_2	388	$A_2^2A_2$	389	$A_2^2A_2$	390	$A_1A_2^2$	393	$A_1A_2^2$	394	A_1A_2	395-99	A_2							
400-401	$A_1A_2^3$	402	$A_2^2A_2$	403	A_1A_2	404	$A_2^2A_2$	405	$A_1A_2^2$	406	$A_2^2A_2$	407-409	$A_1A_2^2$			410	$A_1A_2^2$	411	A_2	416-43	A_2					
449	$A_1A_2^3$	450	$A_2^2A_2$	451	$A_1A_2^2$	452-55	A_2	456	A_1A_2	457	$A_2^2A_2$	459	$A_1A_2^2$	460-63	A_2	464	$A_2^2A_2$	465	$A_1A_2^2$	466	$A_2^2A_2$	466	$A_2^2A_2$	466	$A_2^2A_2$	
467	$A_1A_2A_1A_2$	468-69	A_2	470	$A_2^2A_2$	471	A_2	472	$A_1A_2^2$	473-474	A_2	475	A_2	476	A_2	477	A_2	480-88	A_2							
$A_1 = 29$	66	$A_1^2A_2$	67	A_1A_2	70	$A_1^2A_2$	71	A_2	74	$A_2^2A_2$	75	A_1A_2	78	$A_2^2A_2$	79	A_2	82	$A_2^2A_2$	83	A_1A_2	86	$A_2^2A_2$				
87	$A_1A_2^2$	90-91	A_2	94	A_2	98	$A_1^2A_2$	99	A_1A_2	102	$A_1^2A_2$	103	A_2	106	$A_1^2A_2$	107	A_2	110-11	A_2	114	$A_2^2A_2$	115	A_2	116	$A_2^2A_2$	
118-19	A_2	122-23	A_2	126	A_2	128	$A_2^2A_2$	129-130	$A_2^2A_2$	131	$A_2^2A_2$	132	$A_2^2A_2$	133-134	$A_2^2A_2$	134	$A_2^2A_2$	135	$A_2^2A_2$	136	$A_2^2A_2$	137	$A_2^2A_2$	138	$A_2^2A_2$	
137	A_1A_2	138	$A_2^2A_2$	139	A_1A_2	140	$A_2^2A_2$	141	A_2	142	$A_2^2A_2$	143	A_2	144	$A_1^2A_2$	145-146	$A_1^2A_2$	147	A_1A_2	148	A_1A_2	149	A_1A_2	150	A_1A_2	
149	$A_1A_2^2$	150	$A_2^2A_2$	151	A_1A_2	152	A_1A_2	153	A_1A_2	154	A_1A_2	155	A_2	156	$A_2^2A_2$	158	A_2	160	$A_2^2A_2$	161-162	A_1A_2	163	$A_2^2A_2$	164	$A_2^2A_2$	
163	A_1A_2	164	$A_2^2A_2$	165	$A_2^2A_2$	166	$A_2^2A_2$	167	$A_2^2A_2$	168	$A_2^2A_2$	169	A_1A_2	170	$A_2^2A_2$	171	A_2	172	$A_2^2A_2$	173-78	A_2	174	$A_2^2A_2$	175	$A_2^2A_2$	
180-81	A_2	182	$A_2^2A_2$	183	$A_1A_2^2$	184-88	A_2	190	A_2	192	$A_2^2A_2$	193	A_1A_2	194	$A_2^2A_2A_1A_2^2$	195	A_1A_2	196	$A_2^2A_2$	197	A_2	198	$A_2^2A_2$	199	$A_2^2A_2$	
198	$A_2^2A_2A_1A_2$	199	A_1A_2	200	$A_2^2A_2$	201	A_1A_2	202	$A_2^2A_2$	203	A_1A_2	204	$A_2^2A_2$	205	A_2	206	$A_1A_2^2$	207	A_2	208	$A_2^2A_2$					
209	A_1A_2	210	$A_2^2A_2$	211	A_1A_2	212	$A_1A_2^2$	213	$A_2^2A_2A_1A_2^2$	214	$A_1A_2^2$	215	$A_1A_2^2$	216	$A_1A_2^2$	217	$A_2^2A_2$	218	$A_2^2A_2$	219	$A_2^2A_2$	220	$A_2^2A_2$	221	$A_2^2A_2$	
219-20	A_2	222	A_2	224	A_1A_2	225	A_1A_2	226	A_1A_2	227	A_1A_2	228	$A_1A_2^2$	229-31	A_2	232	$A_2^2A_2$	233	A_1A_2	234	$A_2^2A_2$					
249	A_1A_2	250-52	A_2	254	A_2	322	A_1A_2	323	A_1A_2	326-27	A_2	330	$A_2^2A_2$	331	A_1A_2	334-35	A_2	338	$A_2^2A_2$	339	A_1A_2	342-43	A_2			
346	$A_1A_2^2A_1A_2^3A_1A_2^3$	347	A_2	350	A_2	354-55	A_2	358-59	A_2	362-63	A_2	366-67	A_2	370-71	A_2	374-75	A_2	378-79	A_2							
382	A_2	384	$A_2^2A_2$	385-386	A_1A_2	387	$A_1^2A_2$	388	$A_2^2A_2$	389	$A_2^2A_2$	390	$A_2^2A_2$	391	$A_2^2A_2$	392	$A_2^2A_2$	393	$A_2^2A_2$	394	$A_2^2A_2$	395	$A_2^2A_2$	396	$A_2^2A_2$	
395	A_1A_2	396-99	A_2	400	$A_1A_2^3$	401	$A_2^2A_2A_2^2$	402	$A_2^2A_2$	403	A_1A_2	404	$A_2^2A_2$	405	$A_2^2A_2$	406-407	$A_1A_2^2$	408	$A_1A_2^2$	409	$A_2^2A_2$	410	$A_2^2A_2$	411	$A_2^2A_2$	
409-410	$A_1A_2^2$	411-12	A_2	414	A_2	416-22	A_2	423	$A_1A_2^2$	424-44	A_2	446	A_2	448	A_1A_2	449	A_1A_2	450	$A_1A_2^2$	451	$A_2^2A_2$	452	$A_2^2A_2$	453	$A_2^2A_2$	
451	A_1A_2	452-55	A_2	456	$A_2^2A_2$	457	A_1A_2	458	$A_2^2A_2$	459	$A_1A_2^2$	460-63	A_2	464	$A_2^2A_2$	465	A_1A_2	466	$A_2^2A_2$	467	A_1A_2	468	$A_2^2A_2$	469	$A_2^2A_2$	
469	A_1A_2	470	$A_2^2A_2$	471	A_2	472	$A_2^2A_2$	473-474	A_2	475	$A_2^2A_2$	476	A_2	477	A_2	478	A_2	480-88	A_2							
510	A_2	$A_1 = 30$	64-74	A_1	75	$A_1A_2^2$	76-78	A_1	79	A_2	80-88	A_1	89-90	A_1A_2	91	A_2	92	A_2	93	A_1A_2						
96-2	A_1	103	A_2	104-6	A_1	107	A_2	108-10	A_1	111	A_2	112-14	A_1	115	A_2	116-18	A_1	119	A_2	120	A_1	121	A_1A_2			
122-23	A_2																									

329 $A_1A_2^2$	330 A_1A_2	331 $A_1A_2^5$	332-35 A_2	336 A_1A_2	337 $A_1A_2^2$	338 $A_1A_2^3$	339 $A_1A_2^4$	340-43 A_2	344 A_1A_2	345 $A_1A_2^2$	346-51 A_2
352 $A_1A_2^2$	353 $A_1A_2^3$	354-57 A_2	360 A_1A_2	361 $A_1A_2^2$	362-65 A_2	368 $A_1A_2^3$	369 $A_1A_2^4$	370-73 A_2	376 A_1A_2	377 $A_1A_2^2$	378-81 A_2
392 $A_1A_2^2$	393-394 $A_1A_2^2$	432-55 A_2	456 A_1A_2	457-458 $A_1A_2^2$	474 $A_1A_2^3$	475 $A_1A_2^4$	476-80 A_2	481 $A_1A_2^5$	482-85 A_2	488 A_2	489 $A_1A_2^2$
449-451 $A_1A_2^2$	471 A_2	472 A_1A_2	506-9 A_2	$A_1 = 39$	64-65 $A_1A_2^2$	66-67 A_1A_2	68 $A_1A_2^2$	69 A_1A_2	70 A_1A_2	71 A_1A_2	72 $A_1A_2^2$
496-1 A_2	504 A_2	505 $A_1A_2^4$	73-75 A_1A_2	91 A_2	92-93 A_1A_2	94-95 A_2	96-97 $A_1A_2^2$	98 A_1A_2	99 $A_1A_2^2$	100 A_1A_2	101-3 A_2
71 A_2	72 $A_1A_2^2$	73-75 A_1A_2	104 A_1A_2	105-106 A_1A_2	107 A_2	108 $A_1A_2^2$	109 A_1A_2	110 A_2	111-113 A_1A_2	114-15 A_2	116 A_1A_2
87 $A_1A_2^3$	88-90 $A_1A_2^2$	101-2 A_2	118 A_2	120-121 A_1A_2	122-23 A_2	124 A_1A_2	125 $A_1A_2^2$	126 A_2	127 $A_1A_2^2$	128 A_2	129-131 $A_1A_2^2$
100 $A_1A_2^2$	101-2 A_2	118 A_2	138-139 $A_1A_2^2$	140 $A_1A_2^2$	141-142 $A_1A_2^2$	143 A_2	144-145 A_2	146 A_1A_2	147 $A_1A_2^2$	148 A_2	149-150 $A_1A_2^2$
116 A_1A_2	117 $A_1A_2^2$	138-139 $A_1A_2^2$	168 $A_1A_2^2$	169 $A_1A_2^2$	170-74 A_2	184 A_2	185 $A_1A_2^4$	186-90 A_2	192-196 A_1A_2	197 A_2	198-206 $A_1A_2^2$
137 $A_1A_2A_1A_2^2$	157-59 A_2	168 $A_1A_2^2$	207 A_2	208-218 A_1A_2	219 A_2	220 A_1A_2	221-23 A_2	224-227 A_1A_2	228-30 A_2	229 $A_1A_2^2$	230-234 A_1A_2
156 $A_1A_2^2$	157-59 A_2	168 $A_1A_2^2$	235-38 A_2	240 A_2	241 A_1A_2	242-46 A_2	248-249 A_1A_2	250-54 A_2	320 $A_1A_2^2$	321 $A_1A_2^2$	322 A_1A_2
198-206 $A_1A_2^2$	232-234 A_1A_2	235-38 A_2	328 $A_1A_2^2$	329-330 A_1A_2	331 $A_1A_2^2$	332-35 A_2	336-338 A_1A_2	339 $A_1A_2^2$	340-43 A_2	341-44 A_2	342-46 $A_1A_2^2$
344-345 A_1A_2	376 A_1A_2	377 $A_1A_2^2$	346 $A_1A_2^3$	347-51 A_2	352 $A_1A_2^2$	353 $A_1A_2^2$	354-58 A_2	360 $A_1A_2^2$	362-66 A_2	368-369 $A_1A_2^3$	370-74 A_2
370-74 A_2	376 A_1A_2	377 $A_1A_2^2$	347 $A_1A_2^2$	378-79 A_2	380 $A_1A_2^2$	381-82 A_2	392-394 $A_1A_2^2$	395 $A_1A_2^2$	396-99 A_2	408 $A_1A_2^2$	409 $A_1A_2^2$
409 $A_1A_2^2$	410 $A_1A_2^4$	411-15 A_2	447 $A_1A_2^2$	424-30 A_2	440-46 A_2	448-451 A_1A_2	452-55 A_2	456-458 A_1A_2	473-474 $A_1A_2^2$	475 $A_1A_2^3$	476-80 A_2
464-466 A_2	482-86 A_2	488 A_2	489 $A_1A_2^5$	490-94 A_2	496-2 A_2	504 A_2	505 $A_1A_2^3$	506-10 A_2	$A_1 = 45$	66-67 A_1A_2	
481 $A_1A_2^3$	482-86 A_2	488 A_2	489 $A_1A_2^5$	490-94 A_2	496-2 A_2	504 A_2	505 $A_1A_2^3$	506-10 A_2	$A_1 = 45$	66-67 A_1A_2	
70-71 A_1A_2	72 A_1A_2	73-75 A_1A_2	102 A_1A_2	103 A_2	106 $A_1A_2^2$	107 A_2	110 A_2	114-15 A_2	86 $A_1A_2^5$	87 $A_1A_2^2$	90-91 A_2
98-99 A_1A_2	102 A_1A_2	104 A_1A_2	132 A_1	133-135 A_1A_2	148-150 $A_1A_2^2$	151 A_1A_2	152-153 $A_1A_2^2$	154-59 A_2	160 A_1	161-163 A_1A_2	126 A_2
129-131 A_1A_2	145-147 A_1A_2	165-167 A_1A_2	168 A_1	169 A_1A_2	170-72 A_2	174 A_2	176-78 A_2	179 A_1A_2	180-82 A_2	183 A_1A_2	128 A_1
144 A_1	145-147 A_1A_2	165-167 A_1A_2	168 A_1	169 A_1A_2	170-72 A_2	174 A_2	176-78 A_2	179 A_1A_2	180-82 A_2	183 A_1A_2	133 A_2
164 A_1	165-167 A_1A_2	192-203 A_1A_2	204 $A_1A_2^2$	205 A_2	206 A_1A_2	207 A_2	208-211 A_1A_2	212 $A_1A_2^2$	213-218 A_1A_2	219-23 A_2	143 A_2
190 A_2	192-203 A_1A_2	224-227 A_1A_2	228-29 A_2	230 A_1A_2	231 A_2	232-234 A_1A_2	235-36 A_2	238 A_2	240 A_2	241 A_1A_2	144 A_2
219-23 A_2	224-227 A_1A_2	250-52 A_2	254 A_2	322-323 A_1A_2	326-27 A_2	330 $A_1A_2^2$	331 $A_1A_2^2$	334-35 A_2	338-339 $A_1A_2^2$	374-75 A_2	145 A_2
242-48 A_2	249 A_1A_2	343 A_2	346 $A_1A_2^6$	347 A_2	350-51 A_2	354-55 A_2	358-59 A_2	362-63 A_2	366 A_2	370-71 A_2	146 A_2
342 $A_1A_2^6$	343 A_2	346 $A_1A_2^6$	385-387 A_1A_2	404-405 $A_1A_2^2$	388-389 $A_1A_2^2$	390-391 $A_1A_2^2$	392-394 $A_1A_2^2$	399-400 $A_1A_2^2$	408-409 $A_1A_2^2$	452-53 A_2	147 A_2
382 A_2	384 A_1	385-387 A_1A_2	403 A_1A_2	430 A_2	432-44 A_2	446 A_2	448-451 A_1A_2	469 A_2	470 $A_1A_2^2$	496-8 A_2	148 A_2
400 $A_1A_2^4$	401-402 $A_1A_2^2$	423 A_1A_2	424-28 A_2	464 $A_1A_2^2$	465-467 A_1A_2	489 $A_1A_2^2$	490-92 A_2	494 A_2	496-8 A_2	510 A_2	149 A_2
419 A_1A_2	420-22 A_2	423 A_1A_2	424-28 A_2	464 $A_1A_2^2$	465-467 A_1A_2	489 $A_1A_2^2$	490-92 A_2	494 A_2	496-8 A_2	510 A_2	150 A_2
456 $A_1A_2^2$	457-459 A_1A_2	475 $A_1A_2^3$	476-80 A_2	481 A_1A_2	482-88 A_2	489 $A_1A_2^2$	490-92 A_2	494 A_2	496-8 A_2	510 A_2	$A_1 = 46$
472-474 $A_1A_2^2$	475 $A_1A_2^3$	476-80 A_2	481 A_1A_2	482-88 A_2	489 $A_1A_2^2$	490-92 A_2	494 A_2	496-8 A_2	510 A_2	510 A_2	$A_1 = 46$
64 A_1	65 $A_1A_2^2$	66 $A_1A_2^2$	67 $A_1A_2^2$	68 A_1	69 A_2	70 A_2	71 A_2	72 A_1A_2	73-74 A_1A_2	74 A_2	75 A_1A_2
75 $A_1A_2^2$	76 A_1A_2	77-79 A_2	80-82 A_2	83 $A_1A_2^2$	84 A_1A_2	85 A_2	86 A_1A_2	87 $A_1A_2^4$	88-89 A_1A_2	90-91 A_2	91-95 A_2
90-91 A_2	92-93 A_1A_2	94-95 A_2	96 A_1	97 $A_1A_2^2$	98 A_2	99 $A_1A_2^2$	100 A_1	101-3 A_2	104 A_1A_2	105 $A_1A_2^2$	106 $A_1A_2^2$
105 $A_1A_2^2$	106 $A_1A_2^2$	107 A_2	108 A_1A_2	109 A_2	110 A_2	111-113 A_1A_2	114-15 A_2	116 A_1A_2	117-19 A_2	120-121 A_1A_2	121-23 A_2
120-121 A_1A_2	122-23 A_2	124 A_1A_2	125 $A_1A_2^3$	126 $A_1A_2^2$	127 $A_1A_2^2$	128-133 A_1A_2	129-131 $A_1A_2^2$	132 A_1	133-135 A_1A_2	136 A_1	137-139 $A_1A_2^2$
205 A_2	206 A_1A_2	207 A_2	208-218 A_1A_2	219 A_2	220 A_1A_2	221-23 A_2	224-227 A_1A_2	228-30 A_2	229 $A_1A_2^2$	230-234 A_1A_2	235-38 A_2
247 A_2	320 A_1	321 $A_1A_2^3$	322 $A_1A_2^2$	323 $A_1A_2^2$	324 $A_1A_2^2$	325 $A_1A_2^2$	326 A_1	327 A_2	328 A_1	329-330 A_1A_2	331 $A_1A_2^2$
332-35 A_2	336 A_1A_2	337-338 $A_1A_2^2$	339 $A_1A_2^3$	340-43 A_2	344 A_1A_2	345 $A_1A_2^2$	346 $A_1A_2^2$	347-51 A_2	352 $A_1A_2^2$	353 $A_1A_2^2$	354-59 A_2
354-59 A_2	360 A_1A_2	361 $A_1A_2^2$	362-65 A_2	368-369 $A_1A_2^3$	370-75 A_2	376 A_1A_2	377 $A_1A_2^2$	378-81 A_2	448-451 A_1A_2	475 $A_1A_2^3$	476-80 A_2
452-55 A_2	457-458 A_1A_2	459 $A_1A_2^2$	460-63 A_2	465 A_1A_2	467 $A_1A_2^2$	468 $A_1A_2^4$	469 A_2	470 $A_1A_2^3$	471 $A_1A_2^4$	472 A_1	473-474 $A_1A_2^2$
476-79 A_2	484-87 A_2	493 A_2	501 A_2	503 A_2	504 A_2	505 $A_1A_2^3$	506-10 A_2	507 A_1A_2	508 $A_1A_2^2$	509 $A_1A_2^2$	510 A_2
74 A_1	75 A_1A_2	76 A_1	77 A_1A_2	78 A_1	79 A_2	80 A_1	81 A_1A_2	82 A_1	83 A_1A_2	84 A_1	85 A_1A_2
86 A_1	87 A_1A_2	88 A_1	89-91 A_1A_2	92 A_1	93 A_1A_2	94-95 A_2	96-98 A_1	99 A_1A_2	100 A_1	101 A_1A_2	102 A_1
102 A_1	103 A_2	104 A_1	105 A_1A_2	106 A_1	107 A_1A_2	108 A_1	109 A_1A_2	110 A_1	111 A_1	112 A_1	113 A_1A_2
115 A_1A_2	116 A_1	117 $A_1A_2^2$	118 A_1	119 A_2	120 A_1	121 A_1A_2	122-23 A_2	124 A_1	125 A_1A_2	126 A_2	127 A_1A_2
131 A_1A_2	132-34 A_1	135 A_1A_2	136-38 A_1	139 A_1A_2	140-42 A_1	143 A_1A_2	144-46 A_1	147 A_1A_2	148-50 A_1	151 A_1A_2	152 A_1
153 A_1A_2	154 $A_1A_2^2$	155 A_2	156 A_1	157 $A_1A_2^2$	158-59 A_2	160-62 A_1	163 A_1A_2	164-66 A_1	167 A_1A_2	168-70 A_1	171 A_1A_2
172-74 A_1	176-78 A_1	179 A_1A_2	180-82 A_1	183 A_1A_2	184 A_1	185 $A_1A_2^2$	186-90 A_2	192 A_1	193-195 A_1A_2	196 A_1	197 A_1A_2
197-199 A_1A_2	200 A_1	201-203 A_1A_2	204 A_1	205-206 A_1A_2	207 A_2	208 A_1	209 A_2	210 A_1	211 A_2	212 A_1	213-221 A_1A_2
212 A_1	213-221 A_1A_2	222 $A_1A_2^2$	223 A_2	224 A_1	225-227 A_1A_2	228 A_1	229 A_2	230 A_1A_2	231 A_2	232 A_1	233-235 A_1A_2
232 A_1	233-235 A_1A_2	236 A_1	237-38 A_2	240 A_1	241 A_1A_2	242 A_2	243 A_1A_2	244-47 A_2	248-249 A_1A_2	249 A_1A_2	250-54 A_2
250-54 A_2	320-22 A_1	323 A_1A_2	324 A_1	325 A_2	326 A_1	327 A_2	328 A_1	329 A_1A_2	330 A_1	331 $A_1A_2^2$	332 A_1
333-35 A_2	336 A_1	337 A_1A_2	338 A_1	339 A_1A_2	340 A_1	341 A_2	342 $A_1A_2^4$	343 A_2	344 A_1	345 A_1A_2	346 $A_1A_2^2$
346-347 $A_1A_2^2$	348 A_1	349-51 A_2	352 A_1	353 $A_1A_2^2$	354 A_1	355 $A_1A_2^2$	356 A_1	357-59 A_2	360 A_1	361 A_1A_2	362 A_1
362 A_1	363 A_2	364 A_1	365-66 A_2	368 A_1							

359	A ₂	360-62	A ₁	363	A ₁ A ₂ ²	364-66	A ₁	367	A ₂	368-74	A ₁	375	A ₂	376-78	A ₁	379	A ₂	380-81	A ₁	382	A ₂	448-55	A ₁		
457-58	A ₁	459	A ₁ A ₂	460	A ₁	461	A ₂	462	A ₁	463	A ₂	465	A ₁	467	A ₁ A ₂	468-70	A ₁	471	A ₁ A ₂	475	A ₁ A ₂	476	A ₁		
477	A ₂	478	A ₁ A ₂ ²	479	A ₂	484	A ₁	485-87	A ₂	493-95	A ₂	501	A ₂	503	A ₂	A ₁ = 78				86	A ₁	87	A ₁ A ₂ ³	96-97	A ₁
99	A ₂	100	A ₁	101	A ₂	102	A ₁	103	A ₂	104	A ₁	105	A ₁ A ₂	106	A ₁	107	A ₂	108	A ₁	109	A ₂	112-13	A ₁		
114-15	A ₂	116	A ₁	117-19	A ₂	120	A ₁	121	A ₁ A ₂	122-23	A ₂	124	A ₂	125	A ₁ A ₂ ²	126-63	A ₁	127	A ₂	128	A ₁	129	A ₂	130	A ₁
172	A ₁	173-75	A ₂	178	A ₂	179	A ₁ A ₂ ²	180-82	A ₂	183	A ₁ A ₂ ³	188-91	A ₂	246-47	A ₂	274-75	A ₁	278	A ₁	279	A ₁ A ₂	284	A ₁		
285	A ₁ A ₂ A ₁ A ₂ ²			286-87	A ₂	304-306	A ₁ A ₂		307	A ₁ A ₂ ²	308	A ₂ A ₂	309	A ₁ A ₂	310	A ₁ A ₂ ²	311	A ₁ A ₂ ²	312	A ₂ A ₂ ²	313	A ₁ A ₂			
314-15	A ₂	316	A ₁ A ₂	317	A ₁ A ₂	318	A ₁ A ₂ ⁴	319	A ₂	368	A ₁ A ₂ ²	369	A ₁ A ₂ ³	370	A ₂	372-75	A ₂	376	A ₁ A ₂ ²	377	A ₁ A ₂ ²	378-79	A ₂		
380	A ₁ A ₂ ⁴	381	A ₂	434-35	A ₂	438-39	A ₂	444-47	A ₂	A ₁ = 79				86-87	A ₁	96-97	A ₁	99-10	A ₁	112-22	A ₁	123	A ₂	124-26	A ₁
162-63	A ₁	166-67	A ₁	172-75	A ₁	178-83	A ₁	188-89	A ₁	190-91	A ₂	246	A ₁	247	A ₂	274-75	A ₁	278-79	A ₁	284-87	A ₁	304-18	A ₁		
319	A ₁ A ₂ ²	368-70	A ₁	372-74	A ₁	375	A ₂	376-78	A ₁	379	A ₂	380	A ₁	381	A ₁ A ₂ ²	382	A ₂	434-35	A ₁	438-39	A ₁	444-47	A ₂		
A ₁ = 84			99	A ₂	102-3	A ₂	106-7	A ₂	110-11	A ₂	114-15	A ₂	122-23	A ₂	163	A ₁ A ₂ ³	165	A ₁ A ₂ ³	166	A ₁ A ₂	167	A ₁ A ₂ ³	173-75	A ₂	
179	A ₂	181-82	A ₂	183	A ₁ A ₂ ⁵	189-91	A ₂	231	A ₂	271	A ₂	272	A ₂	287	A ₂	291	A ₁ A ₂ ⁴	295	A ₁ A ₂ ²	299	A ₂	303	A ₁ A ₂ ⁵	307	A ₁ A ₂ ⁴
311	A ₁ A ₂ ³	315	A ₂	319	A ₂	355	A ₂	359	A ₂	363	A ₂	367	A ₂	379	A ₂	423	A ₁ A ₂ ⁵	431	A ₂	439	A ₂	447	A ₂		
A ₁ = 85			99	A ₁	102	A ₁	103	A ₂	106	A ₁	107	A ₂	110	A ₁	111	A ₂	114	A ₁	115	A ₂	118	A ₁	122-23	A ₂	
126	A ₂	162-67	A ₁	172-74	A ₁	175	A ₂	178-82	A ₁	183	A ₁ A ₂ A ₁ A ₂ ³		188-91	A ₂	230	A ₁	231	A ₂	246	A ₂	270	A ₁			
271	A ₁ A ₂ ³	286	A ₁	287	A ₂	290-91	A ₁	294	A ₁	295	A ₁ A ₂	298	A ₁	299	A ₁ A ₂ ⁶	302	A ₁	303	A ₁ A ₂ ⁵	306-7	A ₁	310	A ₁		
311	A ₁ A ₂ ²	314	A ₁ A ₂ ²	315	A ₂	318	A ₁ A ₂ ²	319	A ₂	354	A ₁	355	A ₂	358-59	A ₂	362	A ₁	363	A ₂	366-67	A ₂	370	A ₂		
374	A ₂	378-79	A ₂	382	A ₂	418-22	A ₁	423	A ₁ A ₂ ³	428-31	A ₂	434-39	A ₂	444-47	A ₂	486-87	A ₂	A ₁ = 86				502	A ₂	503	A ₂
88-89	A ₁ A ₂			90-91	A ₂	92	A ₁ A ₂	93	A ₁ A ₂ ²	96	A ₁ A ₂ ²	97	A ₁ A ₂	98	A ₁ A ₂ ²	99	A ₂	100	A ₁ A ₂ ²	101-3	A ₂	117	A ₂		
104-106	A ₁ A ₂			107	A ₂	108	A ₂ A ₂	109-11	A ₂	112	A ₂ A ₂	113	A ₁ A ₂ ²	114-15	A ₂	116	A ₂ A ₂ A ₂ A ₂ A ₂ ²		A ₁ A ₂	120-110	A ₁ A ₂	153	A ₁ A ₂ ²		
120-121	A ₁ A ₂			122-23	A ₂	124	A ₁ A ₂	125	A ₁ A ₂ ²	136-138	A ₁ A ₂		139	A ₂	140	A ₁ A ₂	141-43	A ₂	152	A ₁ A ₂	153	A ₁ A ₂ ²			
154-55	A ₂	156	A ₁ A ₂	157-59	A ₂	160	A ₁ A ₂	161	A ₁ A ₂ ²	162	A ₁ A ₂ ²	163	A ₁ A ₂	164	A ₁ A ₂ ²	165-167	A ₁ A ₂		168	A ₂ A ₂	169	A ₁ A ₂ ²			
170-71	A ₂	172	A ₁ A ₂ ²	173-82	A ₂	183	A ₁ A ₂ ²	184	A ₂	185	A ₁ A ₂ ²	186-91	A ₂	200-202	A ₁ A ₂		203	A ₂	204	A ₁ A ₂	205-7	A ₂			
216-218	A ₁ A ₂			219	A ₂	220	A ₁ A ₂	221	A ₂	224	A ₁ A ₂ A ₁ A ₂		225-226	A ₁ A ₂		227-31	A ₂	232-233	A ₁ A ₂		234	A ₁ A ₂			
234-40	A ₂	241	A ₁ A ₂ ⁵	242-45	A ₂	248	A ₁ A ₂	249	A ₁ A ₂ ²	250-53	A ₂	264	A ₁ A ₂ ²	265	A ₁ A ₂ A ₁ A ₂		266	A ₁ A ₂ ²	267	A ₂	268	A ₁ A ₂ ³			
269	A ₁ A ₂ ²	270	A ₁ A ₂	271	A ₁ A ₂ ²	280-281	A ₁ A ₂		282-83	A ₂	284	A ₁ A ₂		293-294	A ₁ A ₂		295	A ₁ A ₂ ²	296-298	A ₁ A ₂		299	A ₂		
286-87	A ₂	288-290	A ₁ A ₂		291	A ₁ A ₂ ²	292	A ₂ A ₂	307	A ₁ A ₂ ³	308	A ₂ A ₂	309	A ₁ A ₂	310-311	A ₁ A ₂		312	A ₂ A ₂	313	A ₁ A ₂				
301-302	A ₁ A ₂			303	A ₁ A ₂ ³	304-306	A ₂ A ₂		317	A ₁ A ₂	318	A ₁ A ₂ A ₁ A ₂ ³		319	A ₂	320-330	A ₁ A ₂		331	A ₁ A ₂ ²	332-35	A ₂			
314	A ₁ A ₂ ⁴	315	A ₂	316	A ₁ A ₂ A ₂ A ₂ ²		317	A ₁ A ₂	318	A ₁ A ₂ A ₁ A ₂ ³		319	A ₂	361	A ₁ A ₂	362-67	A ₂	368	A ₂ A ₂ ²	369	A ₁ A ₂ ²	370-71	A ₂		
344-346	A ₁ A ₂			347-49	A ₂	352	A ₂ A ₂	353	A ₁ A ₂ ²	354-59	A ₂	360	A ₂ A ₂	361	A ₁ A ₂	362-67	A ₂	368	A ₂ A ₂ ²	369	A ₁ A ₂ ²	370-71	A ₂		
372	A ₁ A ₂ ²	373	A ₂	376	A ₁ A ₂	377	A ₁ A ₂ ²	378-79	A ₂	380	A ₂ A ₂ ³	381	A ₂	392	A ₁ A ₂ A ₁ A ₂		393-394	A ₁ A ₂		395-99	A ₂	473	A ₁ A ₂ ²		
408	A ₁ A ₂	409	A ₁ A ₂ ²	410	A ₁ A ₂ ²	411-22	A ₂	423	A ₁ A ₂ ²	424-47	A ₂	456-458	A ₁ A ₂		459	A ₁ A ₂ ³	460-63	A ₂	472	A ₁ A ₂	473	A ₁ A ₂ ²			
474	A ₁ A ₂	475-77	A ₂	480	A ₂	481	A ₁ A ₂ ⁴	482-88	A ₂	489	A ₁ A ₂ ²	490-1	A ₂	504	A ₂	505	A ₁ A ₂ ²	506-9	A ₂	A ₁ = 87					
88	A ₁ A ₂ A ₁ A ₂			89-90	A ₁ A ₂		91	A ₂	92	A ₂ A ₂	93	A ₁ A ₂	94	A ₂	96	A ₁	97	A ₁ A ₂ ²	98	A ₁	99	A ₁ A ₂ ⁴			
100	A ₁	101	A ₁ A ₂ ²	102	A ₁	103	A ₂	104	A ₁ A ₂ ²	105	A ₂ A ₂	106	A ₁ A ₂ ²	107	A ₁ A ₂	108	A ₁ A ₂ A ₂ A ₂ ²		109-110	A ₁ A ₂		124	A ₁		
111	A ₂	112	A ₁	113	A ₁ A ₂ A ₂ A ₂ ²		114	A ₁	115	A ₁	116	A ₁ A ₂ ²	117	A ₂ A ₂	118	A ₂ A ₂	119	A ₁ A ₂ ²	120	A ₁ A ₂ ²	121	A ₁ A ₂			
122-23	A ₂	124	A ₂ A ₂	125	A ₁ A ₂	126	A ₂	136	A ₁	137	A ₁ A ₂ ²	138	A ₁	139	A ₁ A ₂ ²	140	A ₁	141	A ₁ A ₂ ²	142	A ₁	143	A ₂		
152	A ₁	153	A ₁ A ₂	154	A ₁ A ₂ ²	155	A ₂	156	A ₁	157	A ₁ A ₂ A ₁ A ₂ A ₁ A ₂		173	A ₂ A ₂	174	A ₂ A ₂	175	A ₂	176-80	A ₁	181	A ₁ A ₂ A ₁ A ₂			
167	A ₁ A ₂	168	A ₁ A ₂ ²	169	A ₂ A ₂	170	A ₂ A ₂	171	A ₁ A ₂	200	A ₁	201-203	A ₁ A ₂		204	A ₁ A ₂ ²	205-206	A ₁ A ₂		207	A ₂	208	A ₁		
182	A ₁	183	A ₁ A ₂	184	A ₁ A ₂ ²	185	A ₁ A ₂	186-91	A ₂	224	A ₁	225	A ₁ A ₂ ²	226	A ₁	227	A ₁ A ₂ ²	228	A ₁	229	A ₂	230	A ₁ A ₂ ²		
216-218	A ₁ A ₂			219	A ₂	220	A ₁ A ₂	221-22	A ₂	234	A ₂	235	A ₂	236	A ₂ A ₂	237-39	A ₂	240	A ₁	241	A ₁ A ₂ ²	242-46	A ₂		
231	A ₂	232	A ₁ A ₂ ^{2</}																						

200-202	A ₁ A ₂	203	A ₂	204	A ₁ A ₂	205-7	A ₂	208-212	A ₁ A ₂	213	A ₂	214	A ₁ A ₂	215	A ₁ A ₂ ⁴	216-218	A ₁ A ₂																																																																																																																																																																																																																																																																																						
219	A ₂	220	A ₁ A ₂	221-23	A ₂	224-226	A ₁ A ₂	227-29	A ₂	232-234	A ₁ A ₂	235-37	A ₂	240	A ₂	241	A ₁ A ₂ A ₁ ⁷																																																																																																																																																																																																																																																																																						
242-45	A ₂	248	A ₁ A ₂	249	A ₁ A ₂ ³	250-53	A ₂	258-60	A ₁	261	A ₁ A ₂ ³	262	A ₁	263	A ₁ A ₂	264-66	A ₁	267	A ₂	268	A ₁	269	A ₁ A ₂																																																																																																																																																																																																																																																																																
270	A ₁	271	A ₂	274	A ₁	275	A ₁ A ₂ ²	276	A ₁	277	A ₁ A ₂ ²	278	A ₁	279	A ₁ A ₂	280	A ₁	281	A ₁ A ₂ ²	282-83	A ₂	284	A ₁																																																																																																																																																																																																																																																																																
285	A ₁ A ₂ ²	286-87	A ₂	288-92	A ₁	293	A ₁ A ₂	294	A ₁	295	A ₁ A ₂ ²	296	A ₂ A ₂	297	A ₁ A ₂	298	A ₂ A ₂	299	A ₂	300	A ₂ A ₂	301	A ₁ A ₂																																																																																																																																																																																																																																																																																
302	A ₁ A ₂ ²	303	A ₁ A ₂ ⁴	304	A ₁	305-306	A ₁ A ₂ ²	307	A ₁ A ₂ ²	308	A ₁ A ₂ ²	309	A ₁ A ₂	310	A ₁ A ₂ ²	311	A ₁ A ₂ ³	312	A ₁ A ₂ ²	313	A ₁ A ₂																																																																																																																																																																																																																																																																																		
314-15	A ₂	316	A ₁ A ₂ ²	317	A ₁ A ₂ A ₁ ²	318-19	A ₂	320	A ₁	321	A ₁ A ₂ ²	322	A ₁	323	A ₁ A ₂ ²	324-27	A ₂	328-330	A ₁ A ₂																																																																																																																																																																																																																																																																																				
331	A ₁ A ₂ ²	332-35	A ₂	336	A ₁	337-338	A ₁ A ₂	339	A ₁ A ₂ ³	340-43	A ₂	344-345	A ₁ A ₂	346	A ₁ A ₂ A ₁ ⁷	347-51	A ₂	352	A ₁ A ₂ ²	353	A ₁ A ₂ ²	354-57	A ₂	360	A ₁ A ₂ ²	361	A ₁ A ₂	362-65	A ₂	368	A ₂ A ₂ ²	369	A ₁ A ₂ ³	370-73	A ₂	376	A ₁ A ₂	377	A ₁ A ₂ ²	378-79	A ₂																																																																																																																																																																																																																																																														
380	A ₁ A ₂ ²	381	A ₂	384-86	A ₁	387	A ₁ A ₂	388	A ₁	389	A ₁ A ₂ ²	390-391	A ₁ A ₂	392	A ₁	393-394	A ₁ A ₂ ²	395-99	A ₂	400	A ₁	401	A ₁ A ₂ ²	402	A ₁ A ₂ ³	403	A ₁ A ₂	404	A ₁ A ₂ ²	405-406	A ₁ A ₂	407	A ₁ A ₂ ²	408	A ₁ A ₂ A ₁ ³	409	A ₁ A ₂ ³																																																																																																																																																																																																																																																																		
410	A ₁ A ₂ ²	411-22	A ₂	423	A ₁ A ₂ ⁴	424-47	A ₂	448-451	A ₁ A ₂	452-55	A ₂	456-458	A ₁ A ₂	459	A ₁ A ₂ ²	460-63	A ₂	464-466	A ₁ A ₂	467	A ₁ A ₂ ²	468	A ₁ A ₂ ²	469	A ₁ A ₂ ²	470	A ₁ A ₂ ²	471	A ₂	472	A ₁ A ₂	473	A ₁ A ₂ ²	474	A ₁ A ₂ ²	475	A ₁ A ₂ ⁴	476-80	A ₂	481	A ₁ A ₂ ²	482-85	A ₂	488	A ₂	489	A ₁ A ₂ ²	490-93	A ₂	496	A ₁	497	A ₁ A ₂ ³	498-91	A ₂	504	A ₂	505	A ₁ A ₂ ⁴	506-9	A ₂	A₁ = 103	104-6	A ₁	107	A ₁ A ₂																																																																																																																																																																																																																																					
108	A ₁	109	A ₁ A ₂	110	A ₁	112-18	A ₁	120	A ₁	121	A ₁ A ₂	122	A ₁	123	A ₂	124	A ₁	125	A ₁ A ₂	126	A ₂	128-54	A ₁	129	A ₁ A ₂	130	A ₁	131	A ₁ A ₂	132	A ₁	133	A ₁ A ₂	134	A ₁ A ₂	135	A ₁ A ₂	136-82	A ₁	143	A ₁ A ₂	144-50	A ₁	151	A ₁ A ₂	152-54	A ₁	155	A ₂	156-58	A ₁	159	A ₂	160-66	A ₁	167	A ₁ A ₂	168-74	A ₁	175	A ₂	176-85	A ₁	186-87	A ₂	188	A ₁	189-91	A ₂	192-98	A ₁	201	A ₁ A ₂	202	A ₁	203	A ₁ A ₂	204	A ₁	205	A ₁ A ₂	206	A ₁	207	A ₂	208-14	A ₁	215	A ₁ A ₂	216	A ₁	217	A ₁ A ₂	218	A ₁	219	A ₂	220	A ₁	221-23	A ₂	224-30	A ₁	232	A ₁	233	A ₁ A ₂	234	A ₁	235	A ₂	236	A ₁	237-38	A ₂	240-42	A ₁	243	A ₂	244	A ₁	245-46	A ₂	248	A ₁	249	A ₁ A ₂	250-54	A ₂	258-71	A ₁	274-82	A ₁	283	A ₂	284-86	A ₁	287	A ₂	288-2	A ₁	303	A ₂ A ₂	304-14	A ₁	315	A ₂	316	A ₁	317	A ₁ A ₂	318	A ₁	319	A ₂	320-24	A ₁	325	A ₂	326	A ₁	327	A ₂	328-30	A ₁	331	A ₂	332	A ₁	333	A ₂	334	A ₁	335	A ₂	344	A ₁	345	A ₁ A ₂	346	A ₁	347	A ₂	348	A ₁	349-51	A ₂	352-56	A ₁	357	A ₂	358	A ₁	360	A ₁	361	A ₂	362	A ₁	363	A ₂	364	A ₁	365-66	A ₂	368-70	A ₁	371	A ₂	372	A ₁	373-74	A ₂	376	A ₁	377	A ₁ A ₂	378-79	A ₂	381	A ₁ A ₂ ²	382	A ₂	384-98	A ₁	399	A ₂	400-10	A ₁	411	A ₂	412	A ₁	413-15	A ₂	416-26	A ₁	427	A ₂	428	A ₁	429-47	A ₂	448-50	A ₁	451	A ₁ A ₂	452	A ₁	453	A ₂	455	A ₂	456	A ₁	457	A ₁ A ₂	458	A ₁	459	A ₁ A ₂	460	A ₁	461-63	A ₂	464-66	A ₁	467	A ₁ A ₂	468	A ₁	469	A ₂	470-471	A ₁ A ₂ ²	472	A ₁	473	A ₁ A ₂	474	A ₂	475	A ₁ A ₂ ²	476-79	A ₂	480-82	A ₁	483-86	A ₂	488	A ₁	489	A ₁ A ₂	490-94	A ₂
496-2	A ₂	504	A ₂	505	A ₁ A ₂ ²	506-10	A ₂	A₁ = 105	106	A ₂	110	A ₂	114-15	A ₂	118-19	A ₂	122	A ₂	126	A ₂	128-54	A ₁	129	A ₂	140	A ₁ A ₂ A ₁ ⁴	141	A ₁	142-143	A ₁ A ₂ ²	146-147	A ₁ A ₂	148-149	A ₁ A ₂	150-151	A ₁ A ₂	154-55	A ₂	157-59	A ₂	161	A ₁ A ₂	163	A ₁ A ₂	164-66	A ₁	167	A ₁ A ₂	168-70	A ₁	171	A ₂	172-74	A ₁	175	A ₂	176-85	A ₁	186-87	A ₂	188	A ₁	189-91	A ₂	192-98	A ₁	201	A ₁ A ₂	202	A ₁	203	A ₁ A ₂	204	A ₁	205	A ₁ A ₂	206	A ₁	207	A ₂	208-14	A ₁	215	A ₁ A ₂	216	A ₁	217	A ₁ A ₂	218	A ₁	219	A ₂	220	A ₁	221-23	A ₂	224-30	A ₁	232	A ₁	233	A ₁ A ₂	234	A ₁	235	A ₂	236	A ₁	237-38	A ₂	240-42	A ₁	243	A ₂	244	A ₁	245-46	A ₂	248	A ₁	249	A ₁ A ₂	250-54	A ₂	258-71	A ₁	274-82	A ₁	283	A ₂	284-86	A ₁	287	A ₂	288-2	A ₁	303	A ₂ A ₂	304-14	A ₁	315	A ₂	316	A ₁	317	A ₁ A ₂	318	A ₁	319	A ₂	320-24	A ₁	325	A ₂	326	A ₁	327	A ₂	328-30	A ₁	331	A ₂	332	A ₁	333	A ₂	334	A ₁	335	A ₂	344	A ₁	345	A ₁ A ₂	346	A ₁	347	A ₂	348	A ₁	349-51	A ₂	352-56	A ₁	357	A ₂	358	A ₁	360	A ₁	361	A ₂	362	A ₁	363	A ₂	364	A ₁	365-66	A ₂	368-70	A ₁	371	A ₂	372	A ₁	373-74	A ₂	376	A ₁	377	A ₁ A ₂	378-79	A ₂	381	A ₁ A ₂ ²	382	A ₂	384-98	A ₁	399	A ₂	400-10	A ₁	411	A ₂	412	A ₁	413-15	A ₂	416-26	A ₁	427	A ₂	428	A ₁	429-47	A ₂	448-50	A ₁	451	A ₁ A ₂	452	A ₁	453	A ₂	455	A ₂	456	A ₁	457	A ₁ A ₂	458	A ₁	459	A ₁ A ₂	460	A ₁	461-63	A ₂	464-66	A ₁	467	A ₁ A ₂	468	A ₁	469	A ₂	470-471	A ₁ A ₂ ²	472	A ₁	473	A ₁ A ₂	474	A ₂	475	A ₁ A ₂ ²	476-79	A ₂	480-82	A ₁	483-86	A ₂	488	A ₁	489	A ₁ A ₂	490-94	A ₂					
130-131	A ₁ A ₂ ²	142-143	A ₁ A ₂ ²	146-147	A ₁ A ₂	166-167	A ₁ A ₂	170-75	A ₂	178	A ₂	179	A ₁ A ₂ ³	180-82	A ₂	183	A ₁ A ₂ ²	186-91	A ₂	205	A ₂	206	A ₁ A ₂	207	A ₂	208	A ₁ A ₂	209	A ₁	210	A ₁ A ₂	211	A ₁ A ₂	212-14	A ₁	215	A ₁ A ₂	216-18	A ₁	219	A ₂	220	A ₁	221-23	A ₂	224-30	A ₁	232	A ₁	233	A ₁ A ₂	234	A ₁	235	A ₂	236	A ₁	237-38	A ₂	240-42	A ₁	243	A ₂	244	A ₁	245-46	A ₂	248	A ₁	249	A ₁ A ₂	250-54	A ₂	258-71	A ₁	274-82	A ₁	283	A ₂	284-86	A ₁	287	A ₂	288-2	A ₁	303	A ₂ A ₂	304-14	A ₁	315	A ₂	316	A ₁	317	A ₁ A ₂	318	A ₁	319	A ₂	320-24	A ₁	325	A ₂	326	A ₁	327	A ₂	328-30	A ₁	331	A ₂	332	A ₁	333	A ₂	334	A ₁	335	A ₂	344	A ₁	345	A ₁ A ₂	346	A ₁	347	A ₂	348	A ₁	349-51	A ₂	352-56	A ₁	357	A ₂	358	A ₁	360	A ₁	361	A ₂	362	A ₁	363	A ₂	364	A ₁	365-66	A ₂	368-70	A ₁	371	A ₂	372	A ₁	373-74	A ₂	376	A ₁	377	A ₁ A ₂	378-79	A ₂	381	A ₁ A ₂ ²	382	A ₂	384-98	A ₁	399	A ₂	400-10	A ₁																																																																																																																										

318 A ₁ ² A ₂ ²	319 A ₂	320-22 A ₁	323 A ₁ ² A ₂	324 A ₁	325 A ₂	326 A ₁	327 A ₂	328-30 A ₁	331 A ₁ A ₂ ²	332 A ₁	333-35 A ₂
336-38 A ₁	339 A ₁ A ₂ ²	340 A ₁	341 A ₂	342 A ₁ A ₂	343 A ₂	344 A ₁	345-346 A ₁ A ₂	364 A ₁	347 A ₂	348 A ₁	349-51 A ₂
352-54 A ₁	355 A ₂	356 A ₁	357-59 A ₂	360 A ₁	361 A ₁ A ₂	362 A ₁	363 A ₂	364 A ₁	365 A ₂	368 A ₁	369 A ₁ A ₂ ²
370-71 A ₂	372 A ₁ A ₂ ²	373-75 A ₂	376 A ₁	377 A ₁ A ₂ ²	378-79 A ₂	380 A ₁ A ₂ ²	381 A ₂	384-86 A ₁	387 A ₁ A ₂	388-89 A ₁	405 A ₁ A ₂ ²
390-391 A ₁ A ₂	392-94 A ₁	408 A ₁	395 A ₁ A ₂	396 A ₁	397 A ₂	398 A ₁ A ₂	399 A ₂	400-2 A ₁	403 A ₁ A ₂	404 A ₁	405 A ₁ A ₂ ²
406-407 A ₁ A ₂	408 A ₁	409-410 A ₁ A ₂ ²	440-47 A ₂	448 A ₁	449-451 A ₁ A ₂	452 A ₁ A ₂ ²	453 A ₂	470-471 A ₁ A ₂ ²	454 A ₁ A ₂	455 A ₂	456 A ₁
424-25 A ₁	426-38 A ₂	439 A ₁ A ₂ ²	464 A ₁	465-467 A ₁ A ₂	482-87 A ₂	488 A ₁	489 A ₁ A ₂ ²	490-93 A ₂	496-4 A ₂	505 A ₁ A ₂ ²	506-9 A ₂
457-459 A ₁ A ₂	460-63 A ₂	464 A ₁	476 A ₂	480 A ₁	481 A ₁ A ₂ ²	482-87 A ₂	488 A ₁	490-93 A ₂	496-4 A ₂	505 A ₁ A ₂ ²	506-9 A ₂
474-475 A ₁ A ₂ ²	476-79 A ₂	480 A ₁	481 A ₁ A ₂ ²	482-87 A ₂	488 A ₁	489 A ₁ A ₂ ²	490-93 A ₂	496-4 A ₂	505 A ₁ A ₂ ²	506-9 A ₂	506-9 A ₂
A ₁ = 111	112-22 A ₁	123 A ₂	124-26 A ₁	128-58 A ₁	159 A ₂	160-86 A ₁	187 A ₂	188-90 A ₁	191 A ₂	192-6 A ₁	207 A ₁ A ₂
208-14 A ₁	215 A ₁ A ₂	216-18 A ₁	219 A ₁ A ₂	220-22 A ₁	223 A ₂	224-30 A ₁	231 A ₁ A ₂	232-34 A ₁	235 A ₂	236-38 A ₁	240-42 A ₁
243 A ₁ A ₂	244-46 A ₁	247 A ₁ A ₂	248-50 A ₁	251 A ₂	252 A ₁	253-54 A ₂	258-71 A ₁	274-34 A ₁	335 A ₂	336-42 A ₁	343 A ₁ A ₂
344-46 A ₁	347 A ₁ A ₂	348-50 A ₁	351 A ₂	352-58 A ₁	359 A ₂	360-66 A ₁	368-74 A ₁	375 A ₂	376-78 A ₁	379 A ₂	380-81 A ₁
382 A ₂	384-14 A ₁	415 A ₂	416-30 A ₁	431 A ₂	432-38 A ₁	439 A ₁ A ₂	440-41 A ₁	442 A ₂	444-47 A ₂	448-54 A ₁	455 A ₁ A ₂
456-58 A ₁	459 A ₁ A ₂	460 A ₁	461 A ₂	462 A ₁	463 A ₂	464-66 A ₁	467 A ₁ A ₂	468-70 A ₁	471 A ₁ A ₂	472-74 A ₁	475 A ₁ A ₂
476 A ₁	477 A ₂	478 A ₁ A ₂ ²	479 A ₂	480-82 A ₁	483 A ₂	484 A ₁	485 A ₂	486 A ₁	487 A ₂	488-90 A ₁	491 A ₁ A ₂
492 A ₁	493 A ₂	496-97 A ₁	498-0 A ₂	502-3 A ₂	504 A ₁	505 A ₁ A ₂	506-9 A ₂	A ₁ = 113	114 A ₂	118 A ₂	122 A ₂
130 A ₁ ² A ₂	131 A ₁ ² A ₂	132 A ₁ ² A ₂ ²	133 A ₁ ² A ₂	134 A ₁ ² A ₂	135 A ₁ ² A ₂	138 A ₁ ² A ₂	139 A ₂	140 A ₁ ² A ₂ ²	141 A ₂	142 A ₁ ² A ₂	143 A ₂
146-147 A ₁ ² A ₂	A ₁ ² A ₂	148-149 A ₁ ² A ₂	A ₁ A ₂	150 A ₁ ² A ₂	151 A ₁ ² A ₂	A ₁ ² A ₂ A ₁ A ₂	154 A ₂	156-59 A ₂	162 A ₁ ² A ₂	163 A ₁ ² A ₂	164 A ₁ ² A ₂
165 A ₁ A ₂ A ₁ A ₂ A ₁ A ₂ A ₁ A ₂	166 A ₁ ² A ₂	167 A ₁ ² A ₂	171-74 A ₂	178 A ₂	179 A ₂	178 A ₂	179 A ₂	A ₁ A ₂ ² A ₁ A ₂ ² A ₁ A ₂ ²	203-7 A ₂	207 A ₂	207 A ₁ A ₂
187-88 A ₂	190-91 A ₂	194 A ₁ ³ A ₂	195 A ₁ ² A ₂	196 A ₁ ² A ₂	197 A ₂	198 A ₁ ² A ₂	199 A ₂	202 A ₁ A ₂	203-7 A ₂	210-211 A ₁ A ₂	212 A ₂
212 A ₁ A ₂ A ₁ A ₂ A ₁ A ₂ A ₁ A ₂	213 A ₂	214 A ₁ A ₂	215 A ₁ ² A ₂	218 A ₁ ² A ₂	219 A ₂	220 A ₂	223 A ₂	226 A ₁ A ₂	226 A ₁ A ₂	227 A ₂	229-30 A ₂
234 A ₂	236 A ₂	242 A ₂	244 A ₂	250 A ₂	258 A ₁ ² A ₂	259 A ₁ ² A ₂	262 A ₁ ² A ₂	263 A ₁ A ₂	266 A ₁ ² A ₂	270 A ₁ A ₂	271 A ₁ A ₂ ³
274 A ₁ ² A ₂	275 A ₁ ² A ₂	278 A ₁ ² A ₂	279 A ₁ A ₂	286 A ₂	287 A ₂	290 A ₁ ² A ₂	291 A ₁ A ₂	294-295 A ₁ A ₂	A ₁ A ₂	298 A ₁ ² A ₂	299 A ₁ A ₂ ⁷
302 A ₁ A ₂	303 A ₁ A ₂ ²	306 A ₁ A ₂	307 A ₁ A ₂ ²	310 A ₁ A ₂	311 A ₁ A ₂ ²	314 A ₁ A ₂	318 A ₁ A ₂ ²	319 A ₂	322 A ₁ ² A ₂	323 A ₁ A ₂	326 A ₂
330 A ₁ ² A ₂	331 A ₁ ² A ₂	334-35 A ₂	338 A ₁ A ₂	339 A ₁ A ₂	342 A ₂	346 A ₁ A ₂	354-55 A ₂	358-59 A ₂	362 A ₂	367 A ₂	370 A ₂
382 A ₂	386 A ₁ ³ A ₂	387 A ₁ ² A ₂	388 A ₁ ³ A ₂	389 A ₁ ² A ₂	390 A ₁ ² A ₂	391 A ₁ A ₂	394 A ₁ A ₂	395-98 A ₂	402-403 A ₁ A ₂	420 A ₂	426 A ₂
404-405 A ₁ A ₂	406-407 A ₁ A ₂	442 A ₂	444-47 A ₂	450-451 A ₁ A ₂	471 A ₂	474 A ₁ A ₂ ²	475 A ₂	477-79 A ₂	482 A ₂	485-87 A ₂	491-93 A ₂
428 A ₂	431 A ₂	439 A ₂	442 A ₂	444-47 A ₂	471 A ₂	474 A ₁ A ₂ ²	475 A ₂	477-79 A ₂	482 A ₂	485-87 A ₂	491-93 A ₂
463 A ₂	466 A ₁ A ₂	467 A ₁ A ₂	468 A ₁ A ₂ ⁴	470 A ₁ A ₂ ²	471 A ₂	474 A ₁ A ₂ ²	475 A ₂	477-79 A ₂	482 A ₂	485-87 A ₂	491-93 A ₂
498 A ₂	500 A ₂	502 A ₂	506 A ₂	508 A ₂	A ₁ = 115	116-18 A ₁	120 A ₁	121 A ₁ ² A ₂	122 A ₂	124 A ₁ ² A ₂	125 A ₁ ² A ₂
128-42 A ₁	143 A ₂	144-54 A ₁	156 A ₁	157 A ₁ A ₂	158-59 A ₂	160-70 A ₁	171 A ₂	172 A ₁	173 A ₂ ² A ₁	174 A ₁	176-82 A ₁
183 A ₁ A ₂ A ₁ A ₂ A ₁ A ₂	206 A ₁	207 A ₂	208-14 A ₁	215 A ₂	A ₁ ² A ₂ A ₁ A ₂ A ₁ A ₂	187-88 A ₂	190-91 A ₂	192-2 A ₁	174 A ₁	203 A ₁ ² A ₂	204 A ₁
205 A ₂	229 A ₂	230 A ₁	232 A ₁	233 A ₁ ³ A ₂	234 A ₁	236 A ₁	240-41 A ₁	242 A ₂	244 A ₂	248 A ₁ ² A ₂	249 A ₁ ² A ₂
224-28 A ₁	253 A ₂	258-70 A ₁	271 A ₁ A ₂	274-82 A ₁	284 A ₁	285 A ₁ A ₂	286 A ₁	288-98 A ₁	299 A ₁ A ₂	300-2 A ₁	303 A ₁ A ₂
250 A ₂	311 A ₁ A ₂	312 A ₁	313 A ₁ ² A ₂	314 A ₁ ² A ₂	316 A ₁	317 A ₁	A ₁ ² A ₂ A ₁ A ₂	318 A ₁ ² A ₂	319 A ₁ A ₂	320-24 A ₁	326 A ₁
304-10 A ₁	331 A ₁ ² A ₂	332 A ₁	334-35 A ₂	336-40 A ₁	342 A ₁	344 A ₁	345 A ₁ ² A ₂	346 A ₁ A ₂	348 A ₁	352-56 A ₁	358-59 A ₂
328-30 A ₁	361 A ₁ ² A ₂	362 A ₁	364 A ₁	367 A ₂	368-70 A ₁	372 A ₁	376 A ₁ ³ A ₂	377 A ₁ ² A ₂	380 A ₁ ² A ₂	381 A ₁ ² A ₂ ⁴	382 A ₂
360 A ₁	400-6 A ₁	407 A ₁ A ₂	408 A ₂	409 A ₁ A ₂	410 A ₁ A ₂	412 A ₂	415 A ₂	416-22 A ₁	423 A ₁ A ₂	424-25 A ₁	426 A ₂
384-98 A ₁	431 A ₂	439 A ₂	442 A ₂	444-47 A ₂	448-52 A ₁	454 A ₁	456-58 A ₁	459 A ₁ ² A ₂	460-61 A ₂	463 A ₂	464-66 A ₁
428 A ₂	468 A ₁	470 A ₁ A ₂ ²	471 A ₂	472 A ₁	473 A ₁ ² A ₂ A ₁ A ₂	474 A ₁ A ₂	474 A ₁ A ₂	475 A ₁ ² A ₂	477-79 A ₂	480-81 A ₁	482 A ₂
467 A ₁ A ₂	488 A ₁	489 A ₁ ² A ₂	491-93 A ₂	498 A ₂	500 A ₂	502 A ₂	505 A ₁ ² A ₂	506 A ₂	508-9 A ₂	A ₁ = 117	118 A ₁
485-87 A ₂	488 A ₁	489 A ₁ ² A ₂	491-93 A ₂	498 A ₂	500 A ₂	502 A ₂	505 A ₁ ² A ₂	506 A ₂	508-9 A ₂	A ₁ = 117	118 A ₁
122-23 A ₂	128-30 A ₁	131 A ₁ ² A ₂	132-34 A ₁	135 A ₁ ² A ₂	136-38 A ₁	139 A ₁ ² A ₂	140-42 A ₁	143 A ₂	144-46 A ₁	147 A ₁ ² A ₂	148-50 A ₁
151 A ₁ ² A ₂	152-53 A ₁	154 A ₁ ² A ₂	156 A ₁	157-59 A ₂	160-62 A ₁	163 A ₁ ² A ₂	164-66 A ₁	167 A ₂	A ₁ ² A ₂ A ₁ ² A ₂	168-70 A ₁	171 A ₂
172-74 A ₁	176-78 A ₁	179 A ₁ ² A ₂	180-82 A ₁	183 A ₁ ² A ₂	184 A ₁	185 A ₁	A ₁ A ₂ A ₁ ³ A ₂	187-88 A ₂	190-91 A ₂	192 A ₁	198 A ₁
193 A ₁	A ₁ ² A ₂ A ₁ A ₂	194 A ₁	195 A ₁ A ₂	196 A ₁ A ₂	197 A ₁ A ₂	198 A ₁ ² A ₂	199 A ₁ A ₂	200 A ₁	201-203 A ₁ A ₂	A ₁ A ₂	204 A ₁
205 A ₂	206 A										

275 A_1A_2	278-279 A_1A_2	306-307 A_1A_2	282 A_1A_2	286 A_1A_2	287 A_2	290 $A_1^2A_2$	291 A_1A_2	294-295 A_1A_2	298 $A_1^2A_2$	299 A_1A_2	302-303 A_1A_2	306-307 A_1A_2	282 A_1A_2	286 A_1A_2	287 A_2	290 $A_1^2A_2$	291 A_1A_2	294-295 A_1A_2	298 $A_1^2A_2$	299 A_1A_2	302-303 A_1A_2	306-307 A_1A_2		
326 A_1A_2	327 A_2	330 $A_1^2A_2$	331 A_1A_2	334-35 A_2	338-339 A_1A_2	320-34 A_1	315 A_2	318 A_1A_2	319 $A_1A_2^2$	322 $A_1^2A_2$	323 A_1A_2	324-35 A_2	331 A_1A_2	334-35 A_2	338-339 A_1A_2	320-34 A_1	315 A_2	318 A_1A_2	319 $A_1A_2^2$	322 $A_1^2A_2$	323 A_1A_2	324-35 A_2		
355 A_2	358-59 A_2	362-63 A_2	366-67 A_2	370 A_2	374-75 A_2	378 A_2	382 A_2	386 $A_1^2A_2$	387 A_1A_2	388 $A_1^2A_2^2$	390-391 A_1A_2	394-395 A_1A_2	366-67 A_2	370 A_2	374-75 A_2	378 A_2	382 A_2	386 $A_1^2A_2$	387 A_1A_2	388 $A_1^2A_2^2$	390-391 A_1A_2	394-395 A_1A_2		
405 A_1A_2	422-423 A_1A_2	426 A_2	428-31 A_2	434-38 A_2	439 $A_1A_2^2$	442 A_2	444-47 A_2	450-451 A_1A_2	452-53 A_2	454 A_1A_2	455 A_2	458-459 A_1A_2	428-31 A_2	434-38 A_2	439 $A_1A_2^2$	442 A_2	444-47 A_2	450-451 A_1A_2	452-53 A_2	454 A_1A_2	455 A_2	458-459 A_1A_2		
476-79 A_2	482-83 A_2	485-87 A_2	490-93 A_2	498-0 A_2	502-3 A_2	506 A_2	508 A_2	$A_1 = 123$	124-26 A_1	128-58 A_1	159 A_2	187 A_2	188-90 A_1	191 A_2	192-18 A_1	219 A_1A_2	220-22 A_1	223 A_2	224-38 A_1	240-46 A_1	247 A_1A_2	248-50 A_1		
160-86 A_1	177 A_2	179 A_1A_2	181 A_2	182 A_1A_2	183 $A_1A_2^2$	185 A_1A_2	205 A_2	209 A_1A_2	210-86 A_1	211 A_1A_2	212 A_1A_2	213 A_1A_2	214-18 A_1	215 A_2	216-18 A_1	217-18 A_1	218-18 A_1	219-18 A_1	220-22 A_1	221-23 A_2	222-23 A_2	223-23 A_2	224-38 A_1	
143 A_2	145-147 A_1A_2	148 A_2	149-151 A_1A_2	152 A_1	153-154 A_1A_2	155 A_2	156 $A_1A_2^2$	157 A_1A_2	158 A_1A_2	159 A_2	160 A_1	161 A_1A_2	162 A_1A_2	163 A_1A_2	164 A_1	165 A_1	166 A_1A_2	167 A_1A_2	168 A_1	169 A_1A_2	170 A_1A_2	171 A_1A_2	172 A_1	
182 A_1A_2	183 $A_1A_2^2$	185 A_1A_2	187 A_2	189 A_1A_2	190-91 A_2	193 A_1A_2	195 A_1A_2	197 A_2	198-199 A_1A_2	201 A_1A_2	202 A_1A_2	203 A_1A_2	204 A_1A_2	205-207 A_1A_2	208 A_1	209 A_1A_2	210 A_1A_2	211 A_1A_2	212 A_1A_2	213 A_1A_2	214 A_1A_2	215 A_2	216 A_1A_2	217 A_1A_2
252 A_1A_2	253 A_2	258-71 A_1	259 A_2	260 A_1A_2	261 A_1A_2	262 A_1A_2	263 A_1A_2	264 A_1	265 A_1A_2	266 A_1A_2	267 A_1A_2	268 A_1A_2	269 A_1A_2	270 A_1A_2	271 A_1A_2	272 A_1A_2	273 A_1A_2	274 A_1A_2	275 A_1A_2	276 A_1A_2	277 A_1A_2	278 A_1A_2	279 A_1A_2	280 A_1A_2
376-78 A_1	380-81 A_1	382 A_2	384-14 A_1	387 A_2	389-91 A_2	392 A_1	393 A_1A_2	394 A_1A_2	395 A_1A_2	396 A_1A_2	397 A_1A_2	398 A_1A_2	399 A_1A_2	400 A_1A_2	401 A_1A_2	402 A_1A_2	403 A_1A_2	404 A_1A_2	405 A_1A_2	406 A_1A_2	407 A_1A_2	408 A_1A_2	409 A_1A_2	410 A_1A_2
445-47 A_2	448-62 A_1	463 A_2	464-70 A_1	471 A_1A_2	472-74 A_1	475 A_1A_2	476 A_1	477-79 A_2	480-84 A_1	485 A_2	486 A_1	487 A_2	488-90 A_1	491 A_2	492 A_1	493 A_2	496-98 A_1	499-0 A_2	502-3 A_2	504-5 A_2	506 A_2	508 A_2	$A_1 = 124$	124-26 A_1
487 A_2	488-90 A_1	491 A_2	492 A_1	493 A_2	496-98 A_1	499-0 A_2	502-3 A_2	504-5 A_2	506 A_2	508 A_2	$A_1 = 124$	124-26 A_1	128-58 A_1	159 A_2	187 A_2	188-90 A_1	191 A_2	192-18 A_1	219 A_1A_2	220-22 A_1	223 A_2	224-38 A_1	240-46 A_1	
129 A_1A_2	130-131 A_1A_2	133-134 A_1A_2	135 A_1A_2	137 A_1A_2	138-139 A_1A_2	141-142 A_1A_2	143 A_1A_2	144 A_1	145-146 A_1A_2	147 A_1A_2	148 A_1A_2	149-150 A_1A_2	151 A_1A_2	152 A_1	153-155 A_1A_2	156 A_1A_2	157 A_1A_2	158 A_1A_2	159 A_2	160 A_1	161 A_1A_2	162 A_1A_2	163 A_1A_2	164 A_1
163 A_1A_2	165-167 A_1A_2	169-170 A_1A_2	171 A_2	173 A_2	174 A_1A_2	175 A_1A_2	176 A_1	177 A_1A_2	178 A_1A_2	179 A_1A_2	180 A_1A_2	181 A_2	182 A_1A_2	183 A_1A_2	184 A_1	185 A_1A_2	186-91 A_2	192 A_1	193 A_1A_2	194 A_1A_2	195 A_1A_2	196 A_1A_2	197 A_1A_2	198 A_1A_2
182 A_1A_2	183 $A_1A_2^2$	185 A_1A_2	187 A_2	189 A_1A_2	190-91 A_2	193 A_1A_2	195 A_1A_2	197 A_2	198-199 A_1A_2	201 A_1A_2	202 A_1A_2	203 A_1A_2	204 A_1A_2	205-207 A_1A_2	208 A_1	209 A_1A_2	210 A_1A_2	211 A_1A_2	212 A_1A_2	213 A_1A_2	214 A_1A_2	215 A_2	216 A_1A_2	217 A_1A_2
205 A_2	206 A_1A_2	207 A_2	209 A_1A_2	211 A_1A_2	213-215 A_1A_2	217 A_2	219 A_2	221-23 A_2	222-23 A_2	223 A_2	224 A_1	225 A_1A_2	226 A_1A_2	227 A_1A_2	228 A_1A_2	229 A_1A_2	230 A_1A_2	231 A_1A_2	232 A_1	233-235 A_1A_2	236 A_1A_2	237 A_2	238 A_1A_2	239 A_1A_2
229 A_2	233 A_1A_2	235 A_2	237 A_2	241 A_1A_2	243 A_2	247 A_2	249 A_1A_2	251 A_2	252 A_1A_2	253 A_1A_2	254 A_1A_2	255 A_1A_2	256 A_1A_2	257 A_1A_2	258 A_1A_2	259 A_1A_2	260 A_1A_2	261 A_1A_2	262 A_1A_2	263 A_1A_2	264 A_1A_2	265 A_1A_2	266 A_1A_2	267 A_2
271 A_1A_2	275 A_1A_2	279 A_1A_2	335 A_2	339 $A_1A_2^2$	347 A_2	355 A_2	359 A_2	363 A_2	367 A_2	375 A_2	379 A_2	387 A_1A_2	391 A_1A_2	395 A_1A_2	403 A_1A_2	407 A_1A_2	411 A_2	415 A_2	423 A_1A_2	431 A_2	439 A_1A_2	447 A_2	463 A_2	467 A_2
331 A_1A_2	$A_1A_2^2A_1A_2^2A_1A_2^2A_1A_2^2$	$A_1A_2^4$	335 A_2	339 $A_1A_2^2$	347 A_2	355 A_2	359 A_2	363 A_2	367 A_2	375 A_2	379 A_2	387 A_1A_2	391 A_1A_2	395 A_1A_2	403 A_1A_2	407 A_1A_2	411 A_2	415 A_2	423 A_1A_2	431 A_2	439 A_1A_2	447 A_2	463 A_2	467 A_2
387 A_1A_2	391 A_1A_2	395 A_1A_2	403 A_1A_2	407 A_1A_2	411 A_2	415 A_2	423 A_1A_2	431 A_2	439 A_1A_2	447 A_2	463 A_2	467 A_2	471 $A_1A_2^2$	$A_1 = 125$	126 A_2	128 A_1	129 A_1A_2	130 A_1A_2	131 A_1A_2	132 A_1	133-134 A_1A_2	135 A_1A_2	136 A_1A_2	137 A_1A_2
471 $A_1A_2^2$	479 A_2	$A_1 = 125$	126 A_2	128 A_1	129 A_1A_2	130 A_1A_2	131 A_1A_2	132 A_1	133-134 A_1A_2	135 A_1A_2	136 A_1A_2	137 A_1A_2	138 A_1A_2	139 A_1A_2	140 A_1	141-142 A_1A_2	143 A_1A_2	144 A_1	145-146 A_1A_2	147 A_1A_2	148 A_1A_2	149 A_1A_2	150 A_1A_2	151 A_1A_2
136 A_1A_2	137 $A_1^2A_2$	138 A_1A_2	139 A_1A_2	140 A_1	141-142 A_1A_2	143 A_1A_2	144 A_1	145-146 A_1A_2	147 A_1A_2	148 A_1A_2	149 A_1A_2	150 A_1A_2	151 A_1A_2	152 A_1	153-155 A_1A_2	156 A_1A_2	157 A_1A_2	158 A_1A_2	159 A_2	160 A_1	161 A_1A_2	162 A_1A_2	163 A_1A_2	164 A_1
148 $A_1A_2^2$	149-150 A_1A_2	151 A_1A_2	152 A_1	153-155 A_1A_2	156 A_1A_2	157 A_1A_2	158 A_1A_2	159 A_2	160 A_1	161 A_1A_2	162 A_1A_2	163 A_1A_2	164 A_1	165 A_1	166 A_1A_2	167 A_1A_2	168 A_1	169 A_1A_2	170 A_1A_2	171 A_1A_2	172 A_1	173-174 A_1A_2	174 A_1A_2	175 A_1A_2
159 A_2	160 A_1	161 A_1A_2	162 A_1A_2	163 A_1A_2	164 A_1	165 A_1	166 A_1A_2	167 A_1A_2	168 A_1	169 A_1A_2	170 A_1A_2	171 A_1A_2	172 A_1	173-174 A_1A_2	174 A_1A_2	175 A_1A_2	176 A_1	177 A_1A_2	178 A_1A_2	179 A_1A_2	180 A_1A_2	181 A_2	182 A_1A_2	183 A_1A_2
170 A_1A_2	171 A_1A_2	172 A_1	173-174 A_1A_2	174 A_1A_2	175 A_1A_2	176 A_1	177 A_1A_2	178 A_1A_2	179 A_1A_2	180 A_1A_2	181 A_2	182 A_1A_2	183 A_1A_2	184 A_1	185 A_1A_2	186-91 A_2	192 A_1	193 A_1A_2	194 A_1A_2	195 A_1A_2	196 A_1A_2	197 A_1A_2	198 A_1A_2	199 A_1A_2
181 A_1A_2	182 A_1A_2	183 A_1A_2	184 A_1	185 A_1A_2	186-91 A_2	192 A_1	193 A_1A_2	194 A_1A_2	195 A_1A_2	196 A_1A_2	197 A_1A_2	198 A_1A_2	199 A_1A_2	200 A_1	201-203 A_1A_2	204 A_1A_2	205-207 A_1A_2	208 A_1	209 A_1A_2	210 A_1A_2	211 A_1A_2	212 A_1A_2	213 A_1A_2	214 A_1A_2
197 A_1A_2	198 A_1A_2	199 A_1A_2	200 A_1	201-203 A_1A_2	204 A_1A_2	205-207 A_1A_2	208 A_1	209 A_1A_2	210 A_1A_2	211 A_1A_2	212 A_1A_2	213 <												

342 A_1A_2	343 A_2	344 $A_1^2A_2$	345-350 A_1A_2	352 $A_1^3A_2$	353 $A_1^2A_2^2$	354 $A_1^2A_2$	355 $A_1A_2^2$	356 $A_1^2A_2$	357-59 A_2	360 $A_1^2A_2$
361-362 A_1A_2	A_1A_2	363 $A_1A_2^2$	364 $A_1^2A_2$	365-67 A_2	368 $A_1^2A_2$	369-370 A_1A_2	371 $A_1A_2^2$	372 A_1A_2	373-74 A_2	
376-378 A_1A_2	A_1A_2	379 A_2	380-381 A_1A_2	382 A_2	392 $A_1^2A_2A_1A_2$		393-394 A_1A_2	395 $A_1A_2A_1A_2^2$		
396-398 A_1A_2	A_1A_2	399 A_2	408 A_1A_2	409 $A_1A_2A_1A_2A_1A_2^2$	410 $A_1A_2^2$	411 $A_1A_2^3$	412 $A_1A_2A_1A_2^2$	413 $A_1A_2^2$		
414-15 A_2	424 A_1A_2	425 $A_1A_2A_1A_2^2$	426 $A_1A_2^2$	427-31 A_2	440 A_2	441 $A_1A_2^3$	442 A_2	443-450 $A_1A_2A_1A_2^2$	444-46 A_2	448-450 A_1A_2
451 A_1A_2	452 $A_1^2A_2$	453 A_2	454 A_1A_2	455 A_2	456 $A_1^2A_2$	457-460 A_1A_2	461 A_2	462 A_1A_2	463 A_2	
464-476 A_1A_2	A_1A_2	477 A_2	478 A_1A_2	479 A_2	480-482 A_1A_2	483 $A_1A_2^2$	484-87 A_2	488-490 A_1A_2	491 $A_1A_2^2$	
492-93 A_2	496-497 A_1A_2		498-0 A_2	502 A_2	504-505 A_1A_2	506-9 A_2	$A_1 = 319$	320-26 A_1	327 $A_1^3A_2^2$	328-30 A_1
331 $A_1^2A_2$	332-34 A_1	335 A_2	336-38 A_1	339 $A_1^2A_2$	340-42 A_1	343 A_1A_2	344 A_1	345-346 $A_1^2A_2$	347 A_1A_2	348 A_1
349-350 A_1A_2	A_1A_2	351 A_2	352-54 A_1	355 $A_1^2A_2$	356-58 A_1	359 A_2	360-62 A_1	363 $A_1^2A_2$	364-66 A_1	367 A_2
371 A_1A_2	372-74 A_1	375 A_2	376 A_1	377-378 A_1A_2	A_1A_2	379 $A_1A_2^2$	380 A_1	381 A_1A_2	382 A_2	383 A_1
449-450 $A_1^3A_2$	$A_1^3A_2$	451 $A_1^2A_2$	452 A_1	453-454 $A_1^2A_2$	$A_1^2A_2$	455 A_1A_2	457-458 $A_1^2A_2$	459 A_1A_2	460 $A_1^2A_2$	461 A_1
461-462 A_1A_2	A_1A_2	463 A_2	465 $A_1^2A_2$	467 A_1A_2	468 $A_1^2A_2$	469-471 A_1A_2	475-478 A_1A_2	479 A_2	480 A_1	481 A_1
485 A_2	486 A_1A_2	487 A_2	493-94 A_2	501 A_2	503 A_2	$A_1 = 351$	381 A_1	438-39 A_1	446 A_1	447 A_2
381 A_1A_2	415 A_2	447 A_2	$A_1 = 375$	376-78 A_1	379 A_1A_2	380 A_1	381 $A_1^2A_2$	382 A_1	384-42 A_1	444-45 A_1
448-62 A_1	463 A_2	464-70 A_1	471 $A_1^2A_2$	472-74 A_1	475 A_1A_2	476 A_1	477-478 A_1A_2	479 A_2	480-86 A_1	487 A_2
488-90 A_1	491 A_1A_2	492 A_1	493 A_2	496-98 A_1	499 A_1A_2	500 A_1	502 A_2	504 A_1	505 A_1A_2	506-9 A_2
387 $A_1^2A_2$	391 $A_1^2A_2$	395 $A_1^2A_2$	399 A_1A_2	403 $A_1^2A_2$	407 A_1A_2	411 A_1A_2	415 A_1A_2	423 A_1A_2	427 $A_1^2A_2$	431 A_1A_2
443 A_2	447 A_2	455 A_1A_2	463 A_1A_2	471 A_1A_2	479 A_2	$A_1 = 383$	384-78 A_1	479 A_1A_2	480-94 A_1	496-2 A_1
504-6 A_1	507 A_1A_2	508-9 A_1								503 A_1A_2

Acknowledgements

Thanks to the anonymous referees which pointed out some technical and mathematical mistakes in the paper.

References

- [1] Thierry Bousch, Jean Mairesse, *Asymptotic height optimization for topical IFS, Tetris heaps, and the finiteness conjecture*, J. Amer. Math. Soc., 15 (2002), 77–111, doi: 10.1090/S0894-0347-01-00378-2.
- [2] Vincent D. Blondel, Jacques Theys, Alexander A. Vladimirov, 2003. *An elementary counterexample to the finiteness conjecture*, SIAM J. Matrix Anal. Appl., 24 (2003), 963–970, doi: 10.1137/S0895479801397846.
- [3] Antonio Cicone, Nicola Guglielmi, Stefano Serra-Capizzano, Marino Zennaro, *Finiteness property of pairs of 2×2 sign-matrices via real extremal polytope norm*, Linear Alg. Appl., 432 (2010) 01, 796–816, doi: 10.1016/j.laa.2009.09.022.
- [4] Ingrid Daubechies, Jeffrey C. Lagarias, *Two-scale difference equations. ii. local regularity, infinite products of matrices and fractals*, SIAM J. Math. Anal. 23 (1992) 4, 1031–1079, doi: 10.1137/0523059.
- [5] Gustav Gripenberg, *Computing the joint spectral radius*, Linear Alg. Appl., 234 (1996), 43–60, doi: 10.1016/0024-3795(94)00082-4.
- [6] Leonid Gurvits, *Stability of discrete linear inclusion*, Linear Alg. Appl., 231 (1995), 47–85, doi: 10.1016/0024-3795(95)90006-3.
- [7] Nicola Guglielmi, Vladimir Yu. Protasov, *Exact computation of joint spectral characteristics of linear operators*, Found. Comput. Math., 13 (2013) 1, 37–39, doi: 10.1007/s10208-012-9121-0.
- [8] Nicola Guglielmi, Vladimir Yu. Protasov, *Invariant polytopes of sets of matrices with applications to regularity of wavelets and subdivisions*, SIAM J. Matr. Anal. Appl., 37 (2016) 1, 18–52, doi: 10.1137/15M1006945.
- [9] Kevin G. Hare, Ian D. Morris, Nikita Sidorov, Jacques Theys, *An explicit counterexample to the Lagarias–Wang finiteness conjecture*, Adv. Math., 226 (2011) 6, 4667-4701, doi: 10.1016/j.aim.2010.12.012.
- [10] Raphael M. Jungers, *The joint spectral radius. Theory and applications*, Lecture Notes in Control and Information Sciences (2009), Springer.
- [11] Raphael M. Jungers, Vincent D. Blondel, *On the finiteness property for rational matrices*, Linear Algebra Appl., 428 (2008), 2283–2295, doi: 10.1016/j.laa.2007.07.007.
- [12] Victor Kozyakin, *A dynamical systems construction of a counterexample to the finiteness conjecture*, Proceedings of the 44th IEEE Conference on Decision and Control, (2005), 2338–2343, doi: 10.1109/CDC.2005.1582511.
- [13] Jeffrey C. Lagarias, Wang Yang, *The finiteness conjecture for the generalized spectral radius of a set of matrices*, Lin. Algebra Appl., 214 (1995), 17–42, doi: 10.1016/0024-3795(93)00052-2.
- [14] Thomas Mejstrik, *Improved invariant polytope algorithm and applications*, ACM Trans. Math. Softw., 46 (2020) 3 (29), 1–26, doi: 10.1145/3408891.
- [15] Thomas Mejstrik, *t-toolboxes for Matlab*, Gitlab, (2018), gitlab.com/tommsch/ttoolboxes, 2022-02-23.
- [16] Bruce E. Moision, Alon Orlitsky, Paul H. Siegel, *On codes that avoid specified differences*, IEEE Trans. Inf. Theory, 47 (2001), 433–442.
- [17] Thomas Mejstrik, Vladimir Yu. Protasov, *Elliptic polytopes and Lyapunov norms of linear operators*, arXiv: 2107.02610.
- [18] Gian-Carlo Rota, Gilbert Strang, *A note on the joint spectral radius*, Kon. Nederl. Acad. Wet. Proc. 63 (60).
- [19] Fabian Wirth, *The generalized spectral radius is strictly increasing*, Lin. Algebra Appl., 395 (2005), 141–153, doi: 10.1016/j.laa.2004.07.013.