Spaces with countable sn-networks

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Abstract. In this paper, we prove that a space X is a sequentially-quotient π-image of a metric space if and only if X has a point-star sn-network consisting of cs^*-covers. By this result, we prove that a space X is a sequentially-quotient π-image of a separable metric space if and only if X has a countable sn-network, if and only if X is a sequentially-quotient compact image of a separable metric space; this answers a question raised by Shou Lin affirmatively. We also obtain some results on spaces with countable sn-networks.

Keywords: separable metric space, sequentially-quotient (π, compact) mapping, point-star sn-network, $cs*$-cover

Classification: Primary 54C05, 54C10; Secondary 54D65, 54E40

1. Introduction

In his book ([8]), Shou Lin proved that a space X is a quotient compact image of a separable metric space if and only if X is a quotient π-image of a separable metric space, if and only if X has a countable weak base. Then, are there similar results on sequentially-quotient images of separable metric spaces? Related to this question, Shou Lin and Yan proved that a space X is a sequentially-quotient compact image of a separable metric space if and only if X has a countable sn-network ([10]). But they do not know whether sequentially-quotient π-images of separable metric spaces and sequentially-quotient compact images of separable metric spaces are equivalent. So Shou Lin raised the following question ([12]).

Question 1.1. *Has a sequentially-quotient π-image of a separable metric space a countable sn-network?*

In this paper, we give a characterization of sequentially-quotient π-images of metric spaces to prove that a space X is a sequentially-quotient π-image of a separable metric space if and only if X has a countable sn-network, which answers the above question affirmatively. We also obtain some results on spaces with countable sn-networks.

Throughout this paper, all spaces are assumed to be regular T_1, and all mappings are continuous and onto. $\mathbb{N}$ and ω denote the set of all natural numbers and

The first author was supported in part by NSF of the Education Committee of Jiangsu Province in China #02KJB110001.

the first infinite ordinal respectively. Let $x \in X, \mathcal{U}$ be a collection of subsets of X, f be a mapping. Then $st(x,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : x \in U\}$, $f(\mathcal{U}) = \{f(U) : U \in \mathcal{U}\}$. The sequence $\{x_n : n \in \mathbb{N}\}$, the sequence $\{P_n : n \in \mathbb{N}\}$ of subsets and the sequence $\{\mathcal{P}_n : n \in \mathbb{N}\}$ of collections of subsets are abbreviated to $\{x_n\}$, $\{P_n\}$ and $\{\mathcal{P}_n\}$ respectively. We use the convention that every convergent sequence contains its limit point. For example, if we say that a sequence converging to x is eventually in A, or frequently in A, it is to be understood that $x \in A$. (M,d) denotes a metric space with metric d, $B(a,\varepsilon) = \{b \in M : d(a,b) < \varepsilon\}$. (α_n) denotes a point of a Tychonoff-product space, the n-th coordinate is α_n. For terms which are not defined here we refer to [1].

Definition 1.2 ([16])**.** Let (M,d) be a metric space. $f : M \longrightarrow X$ is said to be a π-mapping, if $d(f^{-1}(x), X - f^{-1}(U)) > 0$ for every $x \in X$ and every open neighborhood U of x.

Definition 1.3 ([10])**.** Let $f : X \longrightarrow Y$ be a mapping. f is a quotient mapping if whenever $f^{-1}(U)$ is open in X, then U is open in Y; f is a sequentially-quotient mapping, if for every convergent sequence S in Y, there is a convergent sequence L in X such that $f(L)$ is a subsequence of S; f is a compact mapping, if $f^{-1}(y)$ is compact in X for every $y \in Y$; f is a perfect mapping, if f is a closed and compact mapping.

Remark 1.4. (1) compact mappings defined on metric spaces are π-mappings.

(2) If the domain is sequential, then quotient mapping $\Longrightarrow$ sequentially-quotient mapping ([8, Proposition 2.1.16]).

(3) If the image is sequential, then sequentially-quotient mapping $\Longrightarrow$ quotient mapping ([8, Proposition 2.1.16]).

Definition 1.5 ([3])**.** Let X be a space.

(1) Let $x \in X$. A subset P of X is a sequential neighborhood of x (called a sequence barrier at x in [9]) if every sequence $\{x_n\}$ converging to x is eventually in P, i.e., $x \in P$ and there is $k \in \mathbb{N}$ such that $x_n \in P$ for all $n > k$.

(2) A subset P of X is sequentially open if P is a sequential neighborhood of x for every $x \in P$. X is sequential if every sequentially open subset of X is open.

Remark 1.6. P is a sequential neighborhood of x if and only if every sequence $\{x_n\}$ converging to x is frequently in P, i.e., $x \in P$ and for every $k \in \mathbb{N}$, there is $n > k$ such that $x_n \in P$.

Definition 1.7 ([10])**.** Let $\mathcal{P}$ be a cover of a space X.

(1) $\mathcal{P}$ is a k-network of X, if whenever K is a compact subset of an open set U, there is a finite $\mathcal{F} \subset \mathcal{P}$ such that $K \subset \bigcup\mathcal{F} \subset U$.

(2) $\mathcal{P}$ is a cs-network of X, if every convergent sequence S converging to a point $x \in U$ with U open in X, is eventually in $P \subset U$ for some $P \in \mathcal{P}$.

(3) $\mathcal{P}$ is a cs^*-network of X, if every convergent sequence S converging to a point $x \in U$ with U open in X, is frequently in $P \subset U$ for some $P \in \mathcal{P}$.

Definition 1.8 ([10])**.** Let $\mathcal{P} = \bigcup\{\mathcal{P}_x : x \in X\}$ be a cover of a space X. Assume that $\mathcal{P}$ satisfies the following (a) and (b) for every $x \in X$.

(a) $\mathcal{P}$ is a network of X, that is, whenever $x \in U$ with U open in X, then $x \in P \subset U$ for some $P \in \mathcal{P}_x$, where $\mathcal{P}_x$ is called a network at x.

(b) If $P_1, P_2 \in \mathcal{P}_x$, then there exists $P \in \mathcal{P}_x$ such that $P \subset P_1 \cap P_2$.

(1) $\mathcal{P}$ is called a weak base of X, if for $G \subset X$, G is open in X if and only if for every $x \in G$ there exists $P \in \mathcal{P}_x$ such that $P \subset G$, where $\mathcal{P}_x$ is called a weak neighborhood base at x.

(2) $\mathcal{P}$ is called an sn-network of X, if every element of $\mathcal{P}_x$ is a sequential neighborhood of x for every $x \in X$, where $\mathcal{P}_x$ is called an sn-network at x.

Definition 1.9 ([4], [5], [13], [17])**.** A space X is a g-metric space (resp. an sn-metric space) if X has a σ-locally finite weak base (resp. a σ-locally finite sn-network). A space X is called g-first countable (resp. sn-first countable), if X has a weak base (resp. an sn-network) $\mathcal{P} = \bigcup\{\mathcal{P}_x : x \in X\}$ such that $\mathcal{P}_x$ is countable for every $x \in X$. A space X is an $\aleph$-space if X has a σ-locally finite k-network; X is an $\aleph_0$-space if X has a countable k-network.

Remark 1.10. (1) In [9], sn-networks are said to be universal cs-networks; sn-first countable is said to be universally csf-countable; sn-metric spaces are said to be spaces with σ-locally finite universal cs-networks.

(2) $\aleph_0$-spaces $\Longrightarrow$ $\aleph$-spaces, $\aleph$-spaces $\Longleftrightarrow$ spaces with a σ-locally finite cs-network [2, Theorem 4], $\aleph_0$-spaces $\Longleftrightarrow$ spaces with a countable cs^*-network (see [18, Proposition C]).

(3) For a space, weak base $\Longrightarrow$ sn-network $\Longrightarrow$ cs-network ([10]). So g-metric spaces $\Longrightarrow$ sn-metric spaces $\Longrightarrow$ $\aleph$-spaces, and g-first countable space $\Longrightarrow$ sn-first countable space. Spaces with countable weak base $\Longrightarrow$ spaces with countable sn-networks $\Longrightarrow$ $\aleph_0$-spaces.

(4) An sn-network for a sequential space is a weak base ([10]). Notice that g-first countable $\Longrightarrow$ sequential. g-first countable space $\Longleftrightarrow$ sequential, sn-first countable space. Spaces with countable weak base $\Longleftrightarrow$ sequential, spaces with countable sn-network.

(5) g-metric space $\Longleftrightarrow$ k, sn-metric space (see [9, Theorem 3.15 and Corollary 3.16]). So every k, sn-metric space is sequential.

Definition 1.11 ([11])**.** Let $\{\mathcal{P}_n\}$ be a sequence of covers of a space X.

(1) $\{\mathcal{P}_n\}$ is a point-star network of X, if $\{st(x, \mathcal{P}_n)\}$ is a network at x for every $x \in X$;

(2) $\{\mathcal{P}_n\}$ is a point-star sn-network, if $\{st(x, \mathcal{P}_n)\}$ is an sn-network at x for every $x \in X$.

Remark 1.12. Spaces with a point-star sn-network are sn-first countable.

Definition 1.13 ([11])**.** Let $\mathcal{P}$ be a cover of a space X. $\mathcal{P}$ is a cs^*-cover, if every convergent sequence in X is frequently in P for some $P \in \mathcal{P}$.

2. Main results

At first, we give a characterization of spaces with countable sn-networks. We have known that having countable weak bases, Lindelöf and separable are equivalent for g-metric spaces ([17]), and $\aleph_0$, (hereditarily) Lindelöf and hereditarily separable are equivalent for $\aleph$-spaces ([14], [15]). We have the following analogue for sn-metric spaces.

Theorem 2.1. *The following are equivalent for a space X:*
(1) *X has a countable sn-network;*
(2) *X is an sn-first countable, $\aleph_0$-space;*
(3) *X is a (hereditarily) Lindelöf, sn-metric space;*
(4) *X is a hereditarily separable, sn-metric space;*
(5) *X is an ω_1-compact, sn-metric space.*

Proof: (1) $\Longrightarrow$ (2) follows from Remark 1.10(3).

(2) $\Longrightarrow$ (3): sn-first countable, $\aleph$-spaces are sn-metric spaces ([9]), so X is an sn-metric space. $\aleph_0$-spaces are hereditarily Lindelöf (see [7, Theorem 3.4]), so X is hereditarily Lindelöf.

(3) $\Longrightarrow$ (4): sn-metric spaces are $\aleph$-spaces, and Lindelöf and hereditarily separable are equivalent for $\aleph$-spaces (see [7, Theorem 3.4]), so X is hereditarily separable.

(4) $\Longrightarrow$ (5): It is clear that every Lindelöf space is ω_1-compact.

(5) $\Longrightarrow$ (1): Let $\mathcal{P} = \bigcup\{\mathcal{P}_n : n \in \mathbb{N}\}$ be an sn-network of X. We can assume $\mathcal{P}_n$ is a closed discrete collection of subsets of X for every $n \in \mathbb{N}$ ([4]). We claim that $|\mathcal{P}_n| \leq \omega$ for every $n \in \mathbb{N}$. If not, there is $n \in \mathbb{N}$ such that $|\mathcal{P}_n| > \omega$. We pick $x_P \in P$ for every $P \in \mathcal{P}_n$. Then $\{x_P : P \in \mathcal{P}_n\}$ is a closed discrete subspace of X. This contradicts the ω_1-compactness of X. So X has a countable sn-network. □

Since perfect mappings inversely preserve sn-metric spaces if the domain spaces have G_δ-diagonal ([4]), and inversely preserve Lindelöf spaces, we obtain the following result by the above theorem.

Corollary 2.2. *Let $f : X \longrightarrow Y$ be a perfect mapping. If Y has a countable sn-network and X has a G_δ-diagonal, then X has a countable sn-network.*

We give an example to show that "hereditarily separable" in Theorem 2.1 cannot relax to "separable".

Example 2.3. There is a separable, sn-metric space, which has not any countable sn-network.

Proof: Let Y be a space in [6, Example 1]. Then Y is a separable, $\aleph$-space, and is not an $\aleph_0$-space, hence Y has not any countable sn-network. Notice that every convergent sequence in Y is a finite subset of Y and Y has a σ-locally finite

cs-network, so Y has a σ-locally finite sn-network. That is, Y is an sn-metric space. □

Lemma 2.4. *Let $\{\mathcal{P}_n\}$ be a sequence of cs^*-covers of a space X, and S be a sequence converging to a point $x \in X$. Then there is a subsequence L of S such that for every $n \in \mathbb{N}$, there is $P_n \in \mathcal{P}_n$ such that L is eventually in P_n.*

Proof: $\mathcal{P}_n$ is a cs^*-cover of X for every $n \in \mathbb{N}$, so there is $P_n \in \mathcal{P}_n$ such that S is frequently in P_n. Since S is frequently in P_1, there is a subsequence S_1 of S such that $S_1 \subset P_1$. Put x_{n_1} is the first term of S_1. Similarly, S_1 is frequently in P_2, there is a subsequence S_2 of S_1 such that $S_2 \subset P_2$. Put x_{n_2} is the second term of S_2. By the inductive method, for every $k \in \mathbb{N}$, since S_{k-1} is frequently in P_k, there is a subsequence S_k of S_{k-1} such that $S_k \subset P_k$. Put x_{n_k} is the k-th term of S_k. Let $L = \{x_{n_k} : k \in \mathbb{N}\} \cup \{x\}$. Then L, which is a subsequence of S, is eventually in P_n for every $n \in \mathbb{N}$. In fact, for every $n \in \mathbb{N}$, if $k > n$, then $x_{n_k} \in S_k \subset S_n \subset P_n$. □

Theorem 2.5. *The following are equivalent for a space X:*

(1) *X is a sequentially-quotient π-image of a metric space;*
(2) *X has a point-star sn-network consisting of cs^*-covers;*
(3) *X has a point-star network consisting of cs^*-covers.*

Proof: (1) $\Longrightarrow$ (2). Let $f : M \longrightarrow X$ be a sequentially-quotient π-mapping, (M, d) be a metric space. For every $n \in \mathbb{N}$, put $\mathcal{B}_n = \{B(a, 1/n) : a \in M\}$ and $\mathcal{P}_n = f(\mathcal{B}_n)$. Then $\{\mathcal{P}_n\}$ is a sequence of covers of X. Obviously, $\{st(x, \mathcal{P}_n)\}$ satisfies Definition 1.8(b) by the construction of $\{\mathcal{P}_n\}$.

(i) $\{\mathcal{P}_n\}$ is a point-star network of X: Let $x \in U$ with U open in X. f is a π-mapping, so there is $n \in \mathbb{N}$ such that $d(f^{-1}(x), M - f^{-1}(U)) > 2/n$, thus $st(x, \mathcal{P}_n) \subset U$. In fact, if $y \in st(x, \mathcal{P}_n)$, then there is $P = f(B(a, 1/n)) \in \mathcal{P}_n$ for some $a \in M$ such that $x, y \in P$. Let $b, c \in B(a, 1/n)$ such that $f(b) = x$ and $f(c) = y$. Then $d(c, f^{-1}(x)) \leq d(c, b) < 2/n$, so $c \notin M - f^{-1}(U)$, thus $y = f(c) \in U$.

(ii) $\mathcal{P}_n$ is a cs^*-cover of X for every $n \in \mathbb{N}$: Let S be a sequence in X converging to the point $x \in X$. f is sequentially-quotient, so there is a sequence L in M converging to a point $a \in f^{-1}(x)$ such that $f(L) = S_1$ is a subsequence of S. L is eventually in $B(a, 1/n)$, so $S_1 = f(L)$ is eventually in $P = f(B(a, 1/n)) \in \mathcal{P}_n$. Thus S is frequently in P.

(iii) $st(x, \mathcal{P}_n)$ is a sequential neighborhood of x for every $x \in X$ and $n \in \mathbb{N}$: Let S be a sequence converging to the point $x \in X$. By the proof in the above (ii), S is frequently in some $P \in \mathcal{P}_n$ and $x \in P$, so S is frequently in $st(x, \mathcal{P}_n)$. By Remark 1.6, $st(x, \mathcal{P}_n)$ is a sequential neighborhood of x.

By the above (i)–(iii), X has a point-star sn-network consisting of cs^*-covers.

(2) $\Longrightarrow$ (3) is obvious.

(3) $\Longrightarrow$ (1). Let $\{\mathcal{P}_n\}$ be a point-star network consisting of cs^*-covers of X. We can assume $\mathcal{P}_n$ is a collection of closed subsets of X for every $n \in \mathbb{N}$.

Put $\mathcal{P}_n = \{P_\alpha : \alpha \in A_n\}$ for every $n \in \mathbb{N}$, the topology on A_n is the discrete topology. Put $M = \{a = (\alpha_n) \in \Pi_{n\in\mathbb{N}} A_n : \{P_{\alpha_n}\}$ is a network at some $x_a \in X\}$. Then M, which is a subspace of the product space $\Pi_{n\in\mathbb{N}} A_n$, is a metric space with metric d defined as follows:

Let $a = (\alpha_n), b = (\beta_n) \in M$. Then $d(a, b) = 0$ if $a = b$, and $d(a, b) = 1/\min\{n \in \mathbb{N} : \alpha_n \neq \beta_n\}$ if $a \neq b$.

Define $f : M \longrightarrow X$ by $f(a) = x_a$ for every $a = (\alpha_n) \in M$, where $\{P_{\alpha_n}\}$ is a network at x_a. It is easy to see that x_a is unique for every $a \in M$ by T_1-property of X, so f is a function.

(i) f is onto: Let $x \in X$. For every $n \in \mathbb{N}$, there is $\alpha \in A_n$ such that $x \in P_{\alpha_n}$. For $\{\mathcal{P}_n\}$ is a point-star network of X, $\{P_{\alpha_n}\}$ is a network for x. Put $a = (\alpha_n)$, then $f(a) = x$.

(ii) f is continuous: Let $a = (\alpha_n) \in M$, U be a neighborhood of $x = f(a)$. Then there is $k \in \mathbb{N}$ such that $P_{\alpha_k} \subset U$. Put $V = \{b = (\beta_n) \in M : \beta_k = \alpha_k\}$. Then V is open in M containing a and $f(V) \subset P_{\alpha_k} \subset U$, thus f is continuous.

(iii) f is a π-mapping: Let $x \in U$ with U open in X. For $\mathcal{P}_n$ is a point-star network of X, there is $n \in \mathbb{N}$ such that $st(x, \mathcal{P}_n) \subset U$. Then $d(f^{-1}(x), M - f^{-1}(U)) \geq 1/2n > 0$. In fact, let $a = (\alpha_n) \in M$ such that $d(f^{-1}(x), a) < 1/2n$. Then there is $b = (\beta_n) \in f^{-1}(x)$ such that $d(a, b) < 1/n$, so $\alpha_k = \beta_k$ if $k \leq n$. Notice that $x \in P_{\beta_n} \in \mathcal{P}_n$, $P_{\alpha_n} = P_{\beta_n}$, so $f(a) \in P_{\alpha_n} = P_{\beta_n} \subset st(x, \mathcal{P}_n) \subset U$, hence $a \in f^{-1}(U)$. Thus $d(f^{-1}(x), a) \geq 1/2n$ if $a \in M - f^{-1}(U)$, so $d(f^{-1}(x), M - f^{-1}(U)) \geq 1/2n > 0$.

(iv) f is sequentially-quotient: Let S be a sequence converging to a point $x \in X$. Notice that $\{\mathcal{P}_n\}$ is a sequence of cs^*-covers of X. By Lemma 2.4, there is a subsequence $L = \{x_k : k \in \mathbb{N}\} \cup \{x\}$ of S such that for every $n \in \mathbb{N}$, there is $\alpha_n \in A_n$ such that L is eventually in P_{α_n}. Put $a = (\alpha_n)$. Since $\{\mathcal{P}_n\}$ is a point-star network, $a \in M$ and $f(a) = x$. We pick $b_k \in f^{-1}(x_k)$ for every $x_k \in L$ as follows. For every $n \in \mathbb{N}$, if $x_k \in P_{\alpha_n}$, put $\beta_{k_n} = \alpha_n$; if $x_k \notin P_{\alpha_n}$, pick $\alpha_{k_n} \in A_n$ such that $x_k \in P_{\alpha_{k_n}}$, and put $\beta_{k_n} = \alpha_{k_n}$. Put $b_k = (\beta_{k_n}) \in \Pi_{n\in\mathbb{N}} A_n$. Obviously, $b_k \in M$ and $f(b_k) = x_k$. It is easy to prove that $L' = \{b_k : k \in \mathbb{N}\} \cup \{a\}$ is a sequence in M converging to the point a. In fact, let U be open in M containing a. By the definition of Tychonoff-product spaces, we can assume there is $m \in \mathbb{N}$ such that $U = ((\Pi\{\{\alpha_n\} : n \leq m\}) \times (\Pi\{A_n : n > m\})) \cap M$. For every $n \leq m$, L is eventually in P_{α_n}, so there is $k(n) \in \mathbb{N}$ such that $x_k \in P_{\alpha_n}$ if $k > k(n)$, thus $\beta_{k_n} = \alpha_n$. Put $k_0 = \max\{k(1), k(2), \ldots, k(m), m\}$. It is easy to see that $b_k \in U$ if $k > k_0$, so L' converges to a. Thus there is a converging sequence L' in M such that $f(L') = L$ is a subsequence of S, so f is sequentially-quotient. □

The following lemma belongs to Shou Lin ([8, Proposition 3.7.14(2)]).

Lemma 2.6. *Let $f : X \longrightarrow Y$ be a sequentially-quotient mapping, and X be an $\aleph_0$-space. Then Y is an $\aleph_0$-space.*

PROOF: Since X is an $\aleph_0$-space, it is easy to prove that X has a countable cs-network $\mathcal{P}$(also see [8, Proposition 1.6.7]). Put $\mathcal{P}' = f(\mathcal{P})$. By Remark 1.10(2), we need only prove that $\mathcal{P}'$ is a cs^*-network of Y.

Let S be a sequence in Y converging to a point $y \in U$ with U open in Y. f is sequentially-quotient, so there is a sequence L in X converging to a point $x \in f^{-1}(y) \subset f^{-1}(U)$ such that $f(L)$ is a subsequence of S. Since $\mathcal{P}$ is a cs-network of X, there exists $P \in \mathcal{P}$ such that L is eventually in P and $P \subset f^{-1}(U)$. Thus $f(L)$ is eventually in $f(P) \subset U$, and so S is frequently in $f(P) \subset U$. Notice that $f(P) \in \mathcal{P}'$. So $\mathcal{P}'$ is a cs^*-network of Y. □

Theorem 2.7. *The following are equivalent for a space X:*

(1) *X has a countable sn-network;*
(2) *X is a sequentially-quotient compact image of a separable metric space;*
(3) *X is a sequentially-quotient π-image of a separable metric space.*

PROOF: (1) $\Longleftrightarrow$ (2) from [10], and (2) $\Longrightarrow$ (3) from Remark 1.4. We only need to prove (3) $\Longrightarrow$ (1).

Let $f : M \longrightarrow X$ be a sequentially-quotient π-mapping, M be a separable metric space. Then X is sn-first countable from Theorem 2.5 and Remark 1.12. Sequentially-quotient mappings preserve $\aleph_0$-spaces by Lemma 2.6, so X is an $\aleph_0$-space. Thus X has a countable sn-network by Theorem 2.1.

□

Every k-space with a countable sn-network is sequential by Remark 1.10(5), so it has a countable weak base. Thus we have the following corollary.

Corollary 2.8. *The following are equivalent for a k-space X:*

(1) *X has a countable weak base;*
(2) *X has a countable sn-network;*
(3) *X is a quotient compact image of a separable metric space;*
(4) *X is a sequentially-quotient compact image of a separable metric space;*
(5) *X is a quotient π-image of a separable metric space;*
(6) *X is a sequentially-quotient π-image of a separable metric space.*

Acknowledgment. The author would like to thank the referee for his valuable amendments.

References

[1] Engelking R., *General Topology*, Polish Scientific Publishers, Warszawa, 1977.
[2] Foged L., *Characterizations of $\aleph$-spaces*, Pacific J. Math. **110** (1984), 59–63.
[3] Franklin S.P., *Spaces in which sequence suffice*, Fund. Math. **57** (1965), 107–115.
[4] Ge Y., *On sn-metrizable spaces*, Acta Math. Sinica **45** (2002), 355–360 (in Chinese).
[5] Gruenhage G., *Generalized metric spaces*, in: K. Kunen and J.E. Vaughan, Eds., Handbook of Set-Theoretic Topology, Amsterdam, North-Holland, pp. 423–501.
[6] Lin S., *On normal separable $\aleph$-space*, Questions Answers Gen. Topology **5** (1987), 249–254.

[7] Lin S., *A survey of the theory of* $\aleph$*-space*, Questions Answers Gen. Topology **8** (1990), 405–419.
[8] Lin S., *Generalized Metric Spaces and Mappings*, Chinese Science Press, Beijing, 1995 (in Chinese).
[9] Lin S., *A note on the Arens' spaces and sequential fan*, Topology Appl. **81** (1997), 185–196.
[10] Lin S., Yan P., *Sequence-covering maps of metric spaces*, Topology Appl. **109** (2001), 301–314.
[11] Lin S., Yan P., *On sequence-covering compact mappings*, Acta Math. Sinica **44** (2001), 175–182 (in Chinese).
[12] Lin S., *Point-star networks and* π*-mappings*, Selected Papers of Chinese Topology Symposium, Chinese Fuzhou Teachers University Publ., Chinese Fuzhou, 2001 (in Chinese).
[13] Michael E.A., $\aleph_0$*-spaces*, J. Math. Mech. **15** (1966), 983–1002.
[14] O'Meara P., *A new class of topological spaces*, Univ. of Alberta Dissertation, 1966.
[15] O'Meara P., *On paracompactness in function spaces with the compact-open topology*, Proc. Amer. Math. Soc. **29** (1971), 183–189.
[16] Ponomarev V.I., *Axioms of countability and continuous mappings*, Bull. Pol. Acad. Math. **8** (1960), 127–133 (in Russian).
[17] Tanaka Y., σ*-Hereditarily closure-preserving* k*-networks and* g*-metrizability*, Proc. Amer. Math. Soc. **112** (1991), 283–290.
[18] Tanaka Y., *Theory of* k*-networks*, Questions Answers Gen. Topology **12** (1994), 139–164.

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(*Received* August 25, 2002, *revised* December 11, 2003)