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A note on propagation of singularities of semiconcave functions of two variables

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Abstract: P. Albano and P. Cannarsa proved in 1999 that, under some applicable conditions, singularities of semiconcave functions in $\mathbb{R}^n$ propagate along Lipschitz arcs. Further regularity properties of these arcs were proved by P. Cannarsa and Y. Yu in 2009. We prove that, for $n = 2$, these arcs are very regular: they can be found in the form (in a suitable Cartesian coordinate system) $\psi(x) = (x, y_1(x) - y_2(x))$, $x \in [0, \alpha]$, where y_1, y_2 are convex and Lipschitz on $[0, \alpha]$. In other words: singularities propagate along arcs with finite turn.

Keywords: semiconcave functions, singularities
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