

Fidel Casarrubias-Segura, Fernando Hernández-Hernández,
Ángel Tamariz-Mascarúa
Martin's Axiom and ω -resolvability of Baire spaces

Comment.Math.Univ.Carolin. 51,3 (2010) 519–540.

Abstract: We prove that, assuming MA, every crowded T_0 space X is ω -resolvable if it satisfies one of the following properties: (1) it contains a π -network of cardinality $< \mathfrak{c}$ constituted by infinite sets, (2) $\chi(X) < \mathfrak{c}$, (3) X is a T_2 Baire space and $c(X) \leq \aleph_0$ and (4) X is a T_1 Baire space and has a network \mathcal{N} with cardinality $< \mathfrak{c}$ and such that the collection of the finite elements in it constitutes a σ -locally finite family. Furthermore, we prove that the existence of a T_1 Baire irresolvable space is equivalent to the existence of a T_1 Baire ω -irresolvable space, and each of these statements is equivalent to the existence of a T_1 almost- ω -irresolvable space. Finally, we prove that the minimum cardinality of a π -network with infinite elements of a space $\text{Seq}(u_t)$ is strictly greater than \aleph_0 .

Keywords: Martin's Axiom, Baire spaces, resolvable spaces, ω -resolvable spaces, almost resolvable spaces, almost- ω -resolvable spaces, infinite π -network

AMS Subject Classification: Primary 54E52, 54A35; Secondary 54D10, 54A10

REFERENCES

1. Alas O., Sanchis M., Tkačhenko M.G., Tkachuk V.V., Wilson R.G., *Irresolvable and submaximal spaces: Homogeneity versus σ -discreteness and new ZFC examples*, Topology Appl. **107** (2000), 259–273.
2. Angoa J., Ibarra M., Tamariz-Mascarúa Á., *On ω -resolvable and almost- ω -resolvable spaces*, Comment. Math. Univ. Carolin. **49** (2008), 485–508.
3. Bell M., Kunen K., *On the π -character of ultrafilters*, C.T. Math. Rep. Acad. Sci. Canada **3** (1981), 351–356.
4. Biernias J., Terepeta M., *A sufficient condition for maximal resolvability of topological spaces*, Comment. Math. Univ. Carolin. **41** (2004), 139–144.
5. Bolstein R., *Sets of points of discontinuity*, Proc. Amer. Math. Soc. **38** (1973), 193–197.
6. Comfort W.W., Feng L., *The union of resolvable spaces is resolvable*, Math. Japonica **38** (1993), 413–414.
7. Comfort W.W., García-Ferreira S., *Resolvability: a selective survey and some new results*, Topology Appl. **74** (1996), 149–167.
8. van Douwen E.K., *Applications of maximal topologies*, Topology Appl. **51** (1993), 125–139.
9. El'kin A.G., *On the maximal resolvability of products of topological spaces*, Soviet Math. Dokl. **10** (1969), 659–662.
10. El'kin A.G., *Resolvable spaces which are not maximally resolvable*, Moscow Univ. Math. Bull. **24** (1969), 116–118.
11. Feng L., *Strongly exactly n -resolvable spaces of arbitrary large dispersion character*, Topology Appl. **105** (2000), 31–36.
12. Foran J., Liebuits P., *A characterization of almost resolvable spaces*, Rend. Circ. Mat. Palermo (2) **40** (1991), 136–141.
13. Feng L., Masaveu O., *Exactly n -resolvable spaces and ω -resolvability*, Math. Japonica **50** (1999), 333–339.
14. Hewitt E., *A problem of set-theoretic topology*, Duke Math. J. **10** (1943), 306–333.
15. Illanes A., *Finite and ω -resolvability*, Proc. Amer. Math. Soc. **124** (1996), 1243–1246.
16. Katětov M., *On topological spaces containing no disjoint dense sets*, Mat. Sb. **21** (1947), 3–12.
17. Kunen K., *Set Theory. An Introduction to Independence Proofs*, Studies in Logic and the Foundations of Mathematics, 102, North Holland, sixth impression, Amsterdam, London, New York, Tokyo, 1995.

18. Kunen K., Tall F., *On the consistency of the non-existence of Baire irresolvable spaces*, Topology Atlas, <http://at.yorku.ca/v/a/a/a/27.htm> (1998).
19. Kunen K., Szymansky A., Tall F., *Baire irresolvable spaces and ideal theory*, Ann. Math. Silesiana **2** (14) (1986), 98-107.
20. Lindgren W.F., Szymanski A.A., *A non-pseudocompact product of countably compact spaces via Seq*, Proc. Amer. Math. Soc. **125** (1997), 3741-3746.
21. Malykhin V.I., *On extremally disconnected topological groups*, Soviet Math. Dokl. **16** (1975), 21-25.
22. Malykhin V.I., *On the resolvability of the product of two spaces and a problem of Katětov*, Dokl. Akad. Nauk SSSR **222** (1975), 765-729.
23. Malykhin V.I., *Irresolvable countable spaces of weight less than \mathfrak{c}* , Comment. Math. Univ. Carolin. **40** (1999), no. 1, 181-185.
24. Pavlov O., *On resolvability of topological spaces*, Topology Appl. **126** (2002), 37-47.
25. Pytke'ev E.G., *On maximally resolvable spaces*, Proc. Steklov. Inst. Math. **154** (1984), 225-230.
26. Tamariz-Mascarúa Á., Villegas-Rodríguez H., *Spaces of continuous functions, box products and almost- ω -resolvable spaces*, Comment. Math. Univ. Carolin. **43** (2002), no. 4, 687-705.
27. Vaughan J.E., *Two spaces homeomorphic to Seq(p)*, manuscript.
28. Villegas L.M., *On resolvable spaces and groups*, Comment. Math. Univ. Carolin. **36** (1995), 579-584.
29. Villegas L.M., *Maximal resolvability of some topological spaces*, Bol. Soc. Mat. Mexicana **5** (1999), 123-136.