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***Powers and alternative laws***

Comment.Math.Univ.Carolin. 48,1 (2007) 25-40.

**Abstract:** A groupoid is alternative if it satisfies the alternative laws $x(xy) = (xx)y$ and $x(yy) = (xy)y$. These laws induce four partial maps on $\mathbb{N}^+ \times \mathbb{N}^+$

$$(r, s) \mapsto (2r, s - r), \quad (r - s, 2s), \quad (r/2, s + r/2), \quad (r + s/2, s/2),$$

that taken together form a dynamical system. We describe the orbits of this dynamical system, which allows us to show that $n$th powers in a free alternative groupoid on one generator are well-defined if and only if $n \leq 5$. We then discuss some number theoretical properties of the orbits, and the existence of alternative loops without two-sided inverses.

**Keywords:** alternative laws, alternative groupoid, powers, dynamical system, alternative loop, two-sided inverse
**AMS Subject Classification:** Primary 20N02; Secondary 20N05, 37E99