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On the number of Russell's socks or $2 + 2 + 2 + \dots = ?$

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Abstract: The following question is analyzed under the assumption that the Axiom of Choice fails badly: Given a countable number of pairs of socks, then how many socks are there? Surprisingly this number is not uniquely determined by the above information, thus giving rise to the concept of Russell-cardinals. It will be shown that:

- some Russell-cardinals are even, but others fail to be so;
- no Russell-cardinal is odd;
- no Russell-cardinal is comparable with any cardinal of the form \aleph_α or 2^{\aleph_α} ;
- finite sums of Russell-cardinals are Russell-cardinals, but finite products — even squares — of Russell-cardinals may fail to be so;
- some countable unions of pairwise disjoint Russell-sets are Russell-sets, but others fail to be so;
- for each Russell-cardinal a there exists a chain consisting of 2^{\aleph_0} Russell-cardinals between a and 2^a ;
- there exist antichains consisting of 2^{\aleph_0} Russell-cardinals;
- there are neither minimal nor maximal Russell-cardinals;
- no Russell-graph has a chromatic number.

Keywords: Bertrand Russell, Axiom of Choice, Generalized Continuum Hypothesis, Dedekind-finite sets, Dedekind-cardinals, Russell-cardinals, odd and (almost) even cardinals, cardinal arithmetic, coloring of graphs, chromatic number, socks

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