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The rank of the diagonal and submetrizability

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Abstract: Several topological properties lying between the submetrizability and the G_δ -diagonal property are studied. We are mostly interested in their relationship to each other and to the submetrizability. The first example of a Tychonoff space with a regular G_δ -diagonal but without a zero-set diagonal is given. The same example shows that a Tychonoff separable space with a regular G_δ -diagonal need not be submetrizable. We give a necessary and sufficient condition for submetrizability of a regular separable space. The rank 5-diagonal plays a crucial role in this criterion. Every closed bounded subset of a Tychonoff space with a G_δ -diagonal is shown to be Čech-complete. Under a slightly stronger condition, any such subset is shown to be a Moore space. We also establish that every closed bounded subset of a Tychonoff space with a regular G_δ -diagonal is metrizable by a complete metric and, therefore, has the Baire property. Some further results are obtained, and new open problems are posed.

Keywords: G_δ -diagonal, rank k -diagonal, submetrizability, condensation, regular G_δ -diagonal, zero-set diagonal, Čech-completeness, pseudocompact space, Moore space, Mrowka space, bounded subset, extent, Souslin number

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