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Rings of continuous functions vanishing at infinity

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Abstract: We prove that a Hausdorff space X is locally compact if and only if its topology coincides with the weak topology induced by $C_\infty(X)$. It is shown that for a Hausdorff space X, there exists a locally compact Hausdorff space Y such that $C_\infty(X) \cong C_\infty(Y)$. It is also shown that for locally compact spaces X and Y, $C_\infty(X) \cong C_\infty(Y)$ if and only if $X \cong Y$. Prime ideals in $C_\infty(X)$ are uniquely represented by a class of prime ideals in $C^*(X)$. ∞-compact spaces are introduced and it turns out that a locally compact space X is ∞-compact if and only if every prime ideal in $C_\infty(X)$ is fixed. The existence of the smallest ∞-compact space in βX containing a given space X is proved. Finally some relations between topological properties of the space X and algebraic properties of the ring $C_\infty(X)$ are investigated. For example we have shown that $C_\infty(X)$ is a regular ring if and only if X is an ∞-compact P_∞-space.

Keywords: σ-compact, pseudocompact, ∞-compact, ∞-compactification, P_∞-space, P-point, regular ring, fixed and free ideals
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