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Metrics with homogeneous geodesics on flag manifolds

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Abstract: A geodesic of a homogeneous Riemannian manifold $(M = G/K, g)$ is called homogeneous if it is an orbit of an one-parameter subgroup of G. In the case when $M = G/H$ is a naturally reductive space, that is the G-invariant metric g is defined by some non degenerate biinvariant symmetric bilinear form B, all geodesics of M are homogeneous. We consider the case when $M = G/K$ is a flag manifold, i.e. an adjoint orbit of a compact semisimple Lie group G, and we give a simple necessary condition that M admits a non-naturally reductive invariant metric with homogeneous geodesics. Using this, we enumerate flag manifolds of a classical Lie group G which may admit a non-naturally reductive G-invariant metric with homogeneous geodesics.

Keywords: homogeneous Riemannian spaces, homogeneous geodesics, flag manifolds
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