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Characterizations of almost transitive superreflexive Banach spaces

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Abstract: Almost transitive superreflexive Banach spaces have been considered in [7] (see also [4] and [6]), where it is shown that such spaces are uniformly convex and uniformly smooth. We prove that convex transitive Banach spaces are either almost transitive and superreflexive (hence uniformly smooth) or extremely rough. The extreme roughness of a Banach space X means that, for every element u in the unit sphere of X, we have

$$limsup_{||h||\to 0}\frac{||u+h||+||u-h||-2}{||h||}=2.$$

We note that, in general, the property of convex transitivity for a Banach space is weaker than that of almost transitivity.

Keywords: convex transitive, almost transitive, superreflexive, uniformly smooth, rough norm
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