

A Summary on Pricing American Call Options Under the Assumption of a Lognormal Framework in the Korn-Rogers Model

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Abstract. In accordance with a variety of option pricing models and the economic approach of the dividend discount model, Korn and Rogers [Stocks paying discrete dividends: modelling and option pricing, *Journal of Derivatives* **13** (2005), 44–49] have introduced a general dividend model preserving the stock price to follow an exponential Lévy process and to be equal to the sum of all its discounted dividends. In this paper we use the model of Korn and Rogers in a Black-Scholes framework to derive a closed-form solution for the pricing of American Call options under the assumption of a possibly known next dividend followed by several stochastic dividend payments during the option's time to maturity.

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1. Introduction

In financial mathematics stock prices are typically assumed to follow a semi-martingale. Usually there is no reference to the economic value of future cash flows such as dividend payments obtained by possessing the stock. But modeling the stock price as a stochastic process and preserving the economic approach of the so-called dividend discount model does not necessarily lead to a contradiction.

Taking the following facts into account leads to a consistent dividend model based on stochastic processes:

- (1) The stock price equals the present value of all future dividend payments.
- (2) There is a close relation between the next dividend payment and the stock price.
- (3) An investor expects the dividend yield to be higher than the yield of a risk less bond.

- (4) The closer the dividend payment the more the randomness of the dividend payment reduces.

In their paper Korn and Rogers (2005) use these facts to derive the price of a stock $S(t)$ paying dividends $D(t_i)$ at future times $t_i > t$ by

$$(1.1) \quad S(t) = E \left[\sum_{i=1}^{\infty} e^{-r(t-t_i)} D(t_i) \right],$$

no matter whether the early dividends are known or unknown.

The case of known dividends and the valuation of European options as well as American call options have been widely discussed in the literature before. Roll [18], Geske [10, 11, 12] and Whaley [20, 21] have solved the pricing problem of an American call on a stock with a known dividend payment during its time to maturity. Sterk [19] has verified the fit of the Roll-Geske-Whaley formula to American call prices. In mathematical finance, Geske [9] was the first to consider uncertain dividends leading to a closed-form solution in an adjusted Black-Scholes framework. Following this introduction of an unknown dividend, Broadie *et al.* [4] as well as Chance *et al.* [5] examined the influence of stochastic dividend payments on the price of a European option. Besides, there is a number of publications that are concerned with dividends and the derivation of a market opinion, for example, Décamps and Villeneuve [6]. Professionals, see for example Bos *et al.* [2], Bos and Vandermark [3], Frishling [7] or Haug *et al.* [14], still take a great interest in the question which model reflects reality the best and offers consistent option pricing — especially with American options. Zhu [22] has just tackled the problem of giving a closed-form solution for the price of an American put option.

In this paper we use the basic idea of Korn and Rogers [16] under the assumption of a lognormal framework in order to price American options on a dividend paying stock. The main contributions of the paper are the following:

- (i) We transfer the model of Korn and Rogers into a lognormal, Black-Scholes-type framework and show its consistency with the Black-Scholes formula.
- (ii) We discuss closed-form solutions for an American Call in the presence of just one dividend payment during the option's time to maturity.
- (iii) We develop a recursive algorithm to derive closed-form solutions to the pricing problem of American call options in the multi-dividend case.

We note that this paper is a summary of Kruse and Müller [17] which we refer the reader to for explicit proofs.

2. Stochastic dividends in the Korn-Rogers Model under the assumption of a lognormal dividend process

We assume that the stock pays dividends at equidistant times $0 < t_1, t_1 + h, \dots, t_1 + h(l-1), t_1 + hl, \dots$. The first l dividends D_1, \dots, D_l are known and paid at times $0 < t_1, t_1 + h, \dots, t_1 + h(l-1)$. The later dividends D_{l+1}, D_{l+2}, \dots are stochastic and paid at times $t_1 + hl, t_1 + h(l+1), \dots$. Korn and Rogers assume the stochastic dividends to be given by $D_{l+1} = X(t_1 + hl), D_{l+2} = X(t_1 + h(l+1)), \dots, D_n = X(t_1 + h(n-1))$ where X is an exponential Lévy process scaled by some constant. Furthermore they assume that for some $\mu < r$, where r is the risk free interest rate, and $0 \leq s \leq t$ holds,

$$(2.1) \quad E[X(s)|F_t] = e^{\mu(t-s)} \cdot X(t).$$

Hence the stock price, if assumed equal to the present value of the sum of its future dividends, is given by

$$(2.2) \quad S(t) = E \left[\sum_{i=1}^{\infty} e^{-r(t_i-t)} D_i \right],$$

and furthermore can be written on taking into account the geometric series in Equation (2.2) and the martingale property in (2.1) as

$$(2.3) \quad S(t) = \begin{cases} \sum_{m=1}^l e^{-r(t_1+(m-1)h-t)} D_m + X(t) \frac{e^{-(r-\mu)(t_1+lh-t)}}{1-e^{-(r-\mu)h}} \\ \text{for } t \in [0, t_1]; \\ \sum_{m=k+1}^l e^{-r(t_1+(m-1)h-t)} D_m + X(t) \frac{e^{-(r-\mu)(t_1+lh-t)}}{1-e^{-(r-\mu)h}} \\ \text{for } t \in [t_1+h(k-1), t_1+hk), 1 \leq k < l; \\ X(t) \frac{e^{-(r-\mu)(t_1+kh-t)}}{1-e^{-(r-\mu)h}} \\ \text{for } t \in [t_1+h(k-1), t_1+hk), l \leq k. \end{cases}$$

We specify the dynamics of the dividend process X to be given by a geometric Brownian motion

$$(2.4) \quad dX(t) = \mu X(t) dt + \sigma X(t) dW_t,$$

where $\mu < r$, r is the risk free interest rate and σ is the volatility of the dividend process. Note that this assumption leads directly to following representation of the stock price:

$$(2.5) \quad S(t) = \begin{cases} \left(S_0 - \sum_{m=1}^l D_m e^{-r(t_1+(m-1)h)} \right) e^{(r-1/2\sigma^2)t+\sigma W_t} + \sum_{m=1}^l D_m e^{-r(t_1+(m-1)h-t)} \\ \text{for } t \in [0, t_1]; \\ \left(S_0 - \sum_{m=1}^l D_m e^{-r(t_1+(m-1)h)} \right) e^{(r-1/2\sigma^2)t+\sigma W_t} + \sum_{m=k+1}^l D_m e^{-r(t_1+(m-1)h-t)} \\ \text{for } t \in [t_1+h(k-1), t_1+hk), 1 \leq k < l; \\ \left(S_0 - \sum_{m=1}^l D_m e^{-r(t_1+(m-1)h)} \right) e^{(r-\mu)(k-l)h} e^{(r-1/2\sigma^2)t+\sigma W_t} \\ \text{for } t \in [t_1+h(k-1), t_1+hk), l \leq k. \end{cases}$$

Of course this representation of the stock price relies on the fact that the announcement dates and the payment dates of the dividends coincide. We note that it is possible to include a time difference between the announcement and the payment of dividends, which would lead to higher dimensional distribution functions. Nevertheless due to the fact that the dividend payment date is the important date at which we distinguish between exercising and holding the option, we neglect this difference. If we set $l = 0$, we are in the case of strictly stochastic dividends, as well as we can focus on the case of n known dividends in some fixed time interval $[0, T]$ with $0 < t_1 < \dots < t_n = t_1 + (n - 1)h < T$. Furthermore we can directly deduce that these assumptions are consistent with the Black-Scholes formula for European stock options:

Theorem 2.1. *We assume that the underlying stock to a European call option with strike K and maturity T is paying n dividends D_1, \dots, D_n during the option's time to maturity. The first l dividend payments at times $t_1, \dots, t_1 + h(l - 1)$ are known while the later dividends*

$D_{l+1} = X(t_1 + hl), \dots, D_n = X(t_1 + h(n-1))$ follow a geometric Brownian motion. The market price of the stock is given by S_0 . Under these assumptions, the price of the European call with maturity $T > t_1$ is given by

$$(2.6) \quad \tilde{S}_0 N(\tilde{d}_1) - K e^{-rT} N(\tilde{d}_2),$$

where $N(\cdot)$ is the standard normal cumulative distribution function as well as

$$(2.7) \quad \tilde{d}_1 = \frac{\ln\left(\frac{\tilde{S}_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \text{ and } \tilde{d}_2 = \tilde{d}_1 - \sigma\sqrt{T},$$

and

$$(2.8) \quad \tilde{S}_0 = \left(S_0 - \sum_{m=1}^l D_m e^{-r(t_1+h(m-1))} \right) e^{-(r-\mu)(n-l)h}.$$

The proof of Theorem 2.1. is straightforward by using the representation of $S(t)$ as in Equation (2.5).

3. Pricing American options

We are now going to focus on the pricing of American call options. In contrast to European options, American options can be executed over the whole time interval up to their maturity. If the underlying stock pays no dividends, there is no need to exercise the American option before its expiry date, see e.g., Hull [15]. In contrast, an American call on a dividend paying stock might be exercised early since it might be optimal to exercise the call immediately prior to a dividend date.

3.1. American call options in the one-dividend case

In order to underline the basic idea of our solution in the multi-dividend case, we first restrict ourselves to the case of just one dividend during the option's time to maturity. Let us first focus on the American call options on a stock that pays an unknown dividend during the option's time to maturity:

Theorem 3.1. *We assume that the underlying stock pays a stochastic dividend $D_1 = X(t_1)$ at time t_1 with $0 < t_1 < T < t_2 = t_1 + h$ during the American call's time to maturity T . Under these assumptions, the price of the American call option with strike K is given by*

$$(3.1) \quad C_D^{0,1}(S_0, 0, T, K) = S_0 \Pi_1^{0,1}(S_0, 0) - K e^{-rT} \Pi_2^{0,1}(S_0, 0),$$

where

$$\Pi_1^{0,1}(S_0, 0) = N(d_1^1) + e^{-(r-\mu)h} N\left(d_2^1, -d_1^1, -\sqrt{\frac{t_1}{T}}\right)$$

and

$$(3.2) \quad \Pi_2^{0,1}(S_0, 0) = e^{r(T-t_1)} N(d_1^2) + N\left(d_2^2, -d_1^2, -\sqrt{\frac{t_1}{T}}\right)$$

for

$$(3.3) \quad d_1^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{S^*}\right) + \left(r + \frac{1}{2}\sigma^2\right)t_1}{\sigma\sqrt{t_1}} \text{ and } d_1^2 = d_1^1 - \sigma\sqrt{t_1}$$

as well as

$$(3.4) \quad d_2^1 = \frac{\ln\left(\frac{S_0 e^{-(r-\mu)h}}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2^2 = d_2^1 - \sigma\sqrt{T}.$$

$N(\cdot)$ is the standard normal cumulative distribution function (cdf) while $N(\cdot, \cdot, \rho)$ is the bivariate normal cdf with correlation ρ , S_0 is today's stock price and S^* is the unique stock price such that at time t_1 of the dividend payment, the following holds:

$$(3.5) \quad C_{BlackScholes}(S^*, t_1, T - t_1, K) = S^* + D^* - K$$

with

$$(3.6) \quad D^* = S^* \frac{1 - e^{-(r-\mu)h}}{e^{-(r-\mu)h}}.$$

Now assume that we want to price an American call with just one known dividend payment during its time to maturity. These assumptions are the same as for the well-known Roll-Geske-Whaley formula (see Hull [15] or Roll [18], Geske [10, 11, 12] and Whaley [20, 21]). We can show that our approach is leading to the same option price.

Theorem 3.2. *We assume that the underlying stock pays a known dividend D_1 at time t_1 with $0 < t_1 < T < t_2 = t_1 + h$ during the American call's time to maturity T . Under these assumptions, the price of the American call option with strike K is given by*

$$(3.7) \quad C_D^{1,0}(S_0, 0, T, K) = (S_0 - D_1 e^{-rt_1}) \Pi_1^{1,0}(S_0, 0) - K e^{-rT} \Pi_2^{1,0}(S_0, 0) + D_1 e^{-rt_1} N(d_1^2)$$

where

$$\Pi_1^{1,0}(S_0, 0) = N(d_1^1) + N\left(d_2^1, -d_1^1, -\sqrt{\frac{t_1}{T}}\right)$$

and

$$(3.8) \quad \Pi_2^{1,0}(S_0, 0) = e^{r(T-t_1)} N(d_1^2) + N\left(d_2^2, -d_1^2, -\sqrt{\frac{t_1}{T}}\right)$$

for

$$(3.9) \quad d_1^1 = \frac{\ln\left(\frac{S_0 - D_1 e^{-rt_1}}{S^*}\right) + \left(r + \frac{1}{2}\sigma^2\right) t_1}{\sigma\sqrt{t_1}} \quad \text{and} \quad d_1^2 = d_1^1 - \sigma\sqrt{t_1}$$

as well as

$$(3.10) \quad d_2^1 = \frac{\ln\left(\frac{S_0 - D_1 e^{-rt_1}}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2^2 = d_2^1 - \sigma\sqrt{T}.$$

$N(\cdot)$ is the standard normal cdf while $N(\cdot, \cdot, \rho)$ is the bivariate normal cdf with correlation ρ , S_0 is today's stock price and S^* is the unique stock price such that at time t_1 of the dividend payment the following holds:

$$(3.11) \quad C_{BlackScholes}(S^*, t_1, T - t_1, K) = S^* + D_1 - K.$$

For the proofs of Theorem 3.1 and Theorem 3.2, we refer the reader to Kruse and Müller [17].

3.2. American call options in the multi-dividend case

In this section we derive the price of American call options with n dividend payments during their time to maturity. Since it might be optimal to exercise the option prior to each of the n dividend dates, we are able to calculate the option price recursively by going back in time from the option's time to maturity T to the trading date $t = 0$. This leads to the calculation of $n - 1$ multivariate normal distributions and thereby to the fact that the steps of the overall calculation process for the option price are of order $O(n^4)$. We note that our pricing formula does not cover the case of several known dividends during the option's time to maturity. Alternatively one could use the arguments as in Hull [15] that only the last of the known dividends is relevant and use its payment date as the time t_1 in the pricing formula below. Of course the option price resulting from this argument would have to take all other known dividend payments into account as in the representation of the stock in Equation (2.5).

Theorem 3.3. *We suppose that the underlying stock S of an American call with strike K and maturity T pays n unknown dividends $D_1 = X(t_1), D_2 = X(t_1 + h), \dots, D_n = X(t_1 + h(n - 1))$ at times $t_1 + h < \dots < t_1 + h(n - 1) < T$ during the option's time to maturity T . The market price of the stock is given by S_0 . Hence the price of the American call is equal to*

$$(3.12) \quad C_D^{0,n}(S_0, 0, T, K) = S_0 \Pi_1^{0,n}(S_0, 0) - Ke^{-rT} \Pi_2^{0,n}(S_0, 0)$$

where

$$(3.13) \quad \Pi_1^{0,n}(S_0, 0) = N(d_1^1) + \sum_{i=1}^n e^{-(r-\mu)ih} N_{i+1} \left(d_{i+1}^1, -d_i^1, \dots, -d_1^1; C^{(i+1)} \right)$$

and

$$(3.14) \quad \begin{aligned} \Pi_2^{0,n}(S_0, 0) &= e^{r(T-t_1)} N(d_1^2) \\ &\quad + \sum_{i=1}^{n-1} e^{r(T-(t_1+ih))} N_{i+1} \left(d_{i+1}^2, -d_i^2, \dots, -d_1^2; C^{(i+1)} \right) \\ &\quad + N_{n+1} \left(d_{n+1}^2, -d_n^2, \dots, -d_1^2; C^{(n+1)} \right) \end{aligned}$$

with

$$(3.15) \quad d_i^1 = \frac{\ln \left(\frac{S_0 e^{-(r-\mu)ih}}{S_i^*} \right) + \left(r + \frac{1}{2} \sigma^2 \right) (t_1 + (i-1)h)}{\sigma \sqrt{t_1 + (i-1)h}},$$

$$d_i^2 = d_i^1 - \sigma \sqrt{t_1 + (i-1)h}$$

for $i = 1, \dots, n$, and

$$(3.16) \quad d_{n+1}^1 = \frac{\ln \left(\frac{S_0 e^{-(r-\mu)nh}}{K} \right) + \left(r + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \quad \text{and} \quad d_{n+1}^2 = d_{n+1}^1 - \sigma \sqrt{T}$$

with $S_1^*, S_2^*, \dots, S_n^*$ such that for $i = 1, \dots, n$

$$(3.17) \quad C_D^{0,n-i}(S_i^*, t_1 + (i-1)h, T - (t_1 + (i-1)h), K) = S_i^* + D_i^* - K,$$

where $C_D^{0,0}(\dots)$ is equal to the Black-Scholes price at time $t_1 + (n - 1)h$ and D_i^* are the unknown dividends at times $t_1 + (i - 1)h$ such that

$$(3.18) \quad D_i^* = S_i^* \frac{1 - e^{-(r-\mu)h}}{e^{-(r-\mu)h}}.$$

Furthermore $N(\cdot)$ is the standard normal cdf and $N_{i+1}(\cdot, \dots, \cdot, C^{(i+1)})$ is the $i + 1$ -dimensional normal cdf with correlation matrix

$$(3.19) \quad C^{(i+1)} = \left(c_{kj}^{(i+1)} \right) \text{ for } i = 1, \dots, n \text{ and } k, j = 1, \dots, i + 1.$$

The correlation matrices $C^{(2)}, \dots, C^{(n+1)}$ can be recursively derived by defining

$$(3.20) \quad c_{jk}^{(i+1)} = \begin{cases} \frac{\hat{c}_{jk}^{(i)} \sqrt{(i-j+1)h} \sqrt{(i-k+1)h+t_1}}{\sqrt{t_1+(i-j+1)h} \sqrt{t_1+(i-k+1)h}} & \text{for } i = 2, \dots, n-1; j = 1, \dots, i \text{ and } k = j+1, \dots, i \\ -\sqrt{\frac{t_1}{t_1+(i-j+1)h}} & \text{for } i = 1, \dots, n-1; j = 1, \dots, i \text{ and } k = i+1 \\ \frac{\hat{c}_{jk}^{(n)} \sqrt{T-t_1} \sqrt{(n-k+1)h+t_1}}{\sqrt{T} \sqrt{t_1+(n-k+1)h}} & \text{for } i = n; j = 1 \text{ and } k = 2, \dots, n \\ \frac{\hat{c}_{jk}^{(n)} \sqrt{(n-j+1)h} \sqrt{(n-k+1)h+t_1}}{\sqrt{t_1+(n-j+1)h} \sqrt{t_1+(n-k+1)h}} & \text{for } i = n; j = 2, \dots, n \text{ and } k = j+1, \dots, n \\ -\sqrt{\frac{t_1}{T}} & \text{for } i = n; j = 1 \text{ and } k = n+1 \\ -\sqrt{\frac{t_1}{t_1+(n-j+1)h}} & \text{for } i = n; j = 2, \dots, n \text{ and } k = n+1 \end{cases}$$

with $\hat{C}^{(i+1)} = \left(\hat{c}_{kj}^{(i+1)} \right)$ being the correlation matrices from the calculation of the American call price in the case of $n - 1$ dividends paid at times $t_1 + h < \dots < t_1 + h(n - 1) < T$.

Hence we are able to price American call options under the assumption that all future dividends are unknown. By the same technique, it is just one more step to derive a closed-form solution if the next dividend is known and all other future dividends are unknown.

Corollary 3.1. We suppose that the underlying stock S of an American call with strike K and maturity T pays a known dividend D_1 at time t_1 and further unknown $n - 1$ dividends $D_2 = X(t_1 + h), \dots, D_n = X(t_1 + h(n - 1))$ at times $t_1 + h < \dots < t_1 + h(n - 1) < T$ during the option's time to maturity. The market price of the stock is given by S_0 . Hence the price of the American call is equal to

$$(3.21) \quad C_D^{1,n-1}(S_0, 0, T, K) = (S_0 - D_1 e^{-rt_1}) \Pi_1^{1,n-1}(S_0, 0) - Ke^{-rT} \Pi_2^{1,n-1}(S_0, 0) + D_1 e^{-rt_1} N(d_1^2),$$

where

$$(3.22) \quad \Pi_1^{1,n-1}(S_0, 0) = N(d_1^1) + \sum_{i=1}^n e^{-(r-\mu)(i-1)h} N_{i+1} \left(d_{i+1}^1, -d_i^1, \dots, -d_1^1; C^{(i+1)} \right)$$

and

$$\begin{aligned}
 \Pi_2^{1,n-1}(S_0, 0) &= e^{r(T-t_1)}N(d_1^2) \\
 &+ \sum_{i=1}^{n-1} e^{r(T-(t_1+ih))}N_{i+1}\left(d_{i+1}^2, -d_i^2, \dots, -d_1^2; C^{(i+1)}\right) \\
 &+ N_{n+1}\left(d_{n+1}^2, -d_n^2, \dots, -d_1^2; C^{(n+1)}\right)
 \end{aligned}
 \tag{3.23}$$

with

$$d_i^1 = \frac{\ln\left(\frac{(S_0 - D_1 e^{-rt_1})e^{-(r-\mu)(i-1)h}}{S_i^*}\right) + (r + \frac{1}{2}\sigma^2)(t_1 + (i-1)h)}{\sigma\sqrt{t_1 + (i-1)h}},$$

$$d_i^2 = d_i^1 - \sigma\sqrt{t_1 + (i-1)h}$$

for $i = 1, \dots, n$ and

$$d_{n+1}^1 = \frac{\ln\left(\frac{(S_0 - D_1 e^{-rt_1})e^{-(r-\mu)(n-1)h}}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_{n+1}^2 = d_{n+1}^1 - \sigma\sqrt{T}$$

Furthermore $S_1^*, S_2^*, \dots, S_n^*$ are the critical stock prices, such that for $i = 1, \dots, n$,

$$C_D^{0,n-i}(S_i^*, t_1 + (i-1)h, T - (t_1 + (i-1)h), K) = S_i^* + D_i^* - K,$$

and $D_1^* = D_1$ is the known first dividend while $D_i^*, i = 2, \dots, n$, are defined by Equation (3.18). Furthermore $N(\cdot)$ is the standard normal cdf and $N_{i+1}(\cdot, \dots, \cdot, C^{(i+1)})$ is the $i+1$ -dimensional normal cdf with correlation matrix $C^{(i+1)} = (c_{kj}^{(i+1)})$ for $i = 1, \dots, n$ and $k, j = 1, \dots, i+1$ defined by Equation (3.20).

Sketch of proof of Theorem 3.1.: The basic idea of the proof is by induction over the number of dividend payments during the option’s time to maturity using some results on the recursive calculation of the correlation matrices. The derivation of the correlation matrices is illustrated in Figure 1.

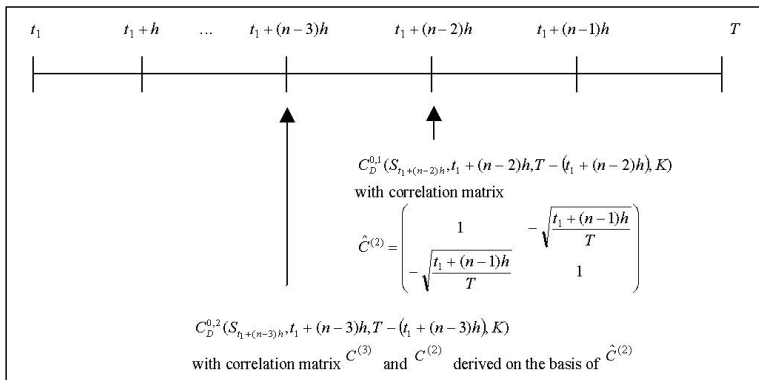


Figure 1. Recursive derivation of the correlation matrices.

For a detailed proof of Theorem 3.1., we refer the reader to Kruse and Müller [17].

4. Conclusion

The pricing of European options under the assumption that the stock price follows a geometric Brownian motion is well developed. The Black-Scholes model has become a market standard for European options. Hence most professionals are familiar with this pricing formula and the underlying model. In our paper we use the general dividend model of Korn and Rogers in a Black-Scholes framework to price American call options in the presence of dividend payments during the option's time to maturity. We present closed-form solutions to American call prices in the case of multiple stochastic dividends, which might be also following a known first dividend payment. Furthermore we show that the model is consistent with the well-known Roll-Geske-Whaley formula for the price of an American call on stock paying just one known dividend during the options time to maturity.

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